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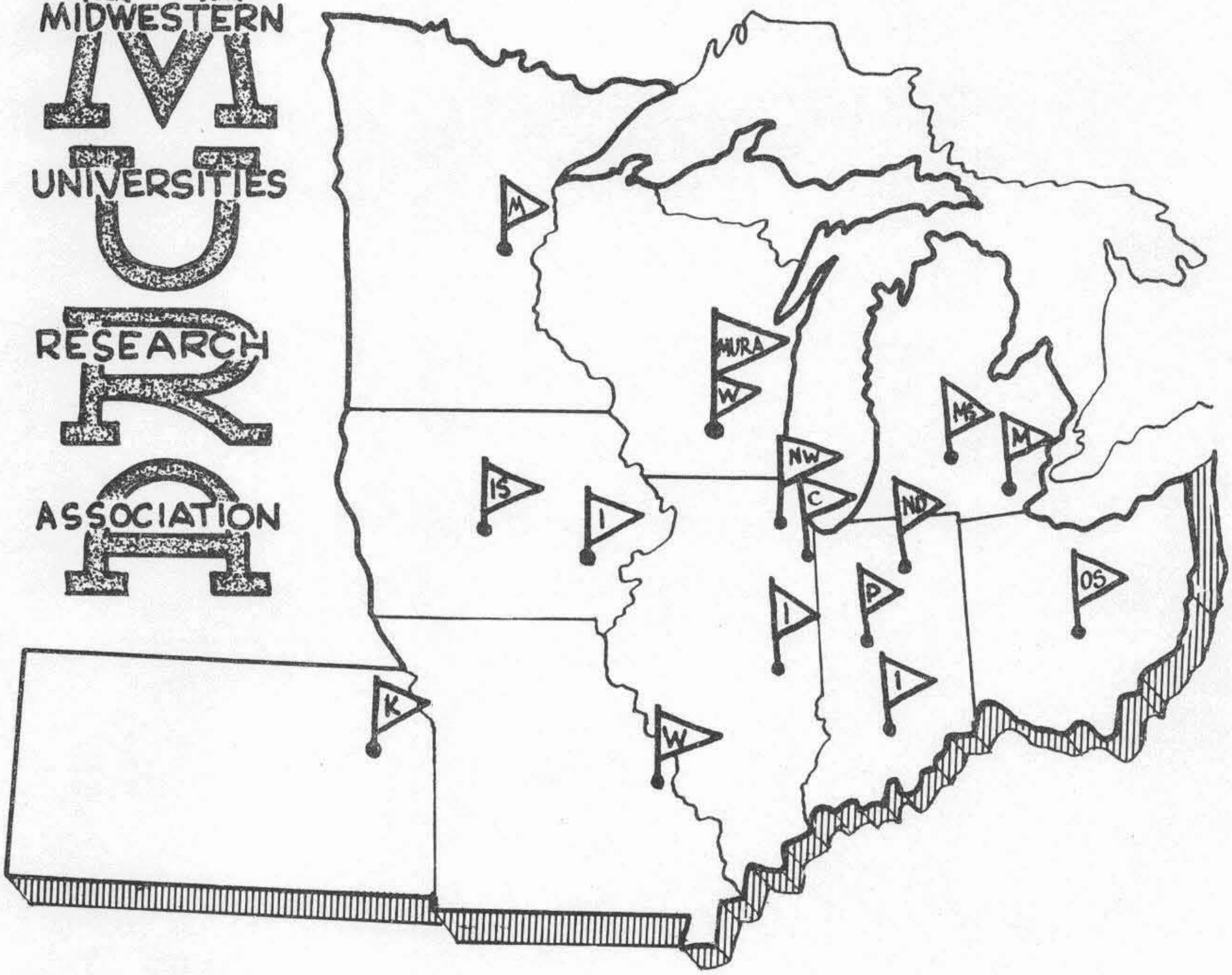

CONSTANT FREQUENCY CYCLOTRONS WITH SPIRALLY RIDGED POLES

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CONSTANT FREQUENCY CYCLOTRONS WITH SPIRALLY RIDGED POLES

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This report on spirally ridged cyclotrons has required revision in the manner discussed at the January 1955 New York meeting of the American Physical Society. This is due to Laslett's determination of the influence of the scalloped orbit on focussing.

To make semi-relativistic particles circulate in a cyclotron at constant frequency and inorbits that are approximately circles, it is necessary to have the average magnetic field increase with radius. This gives rise to vertical (axial) defocussing. The possibility of overcoming this defocussing by the addition of alternating gradient effects is considered here. The spirally ridged (Mark V) type of pole is tried and we will follow the course of trying to avoid integral, half integral, and sum resonances regardless of the structural difficulties to exhibit the problems. Subsequent possible compromises for easing these difficulties will then become evident.

The condition on the average field gradient for constant rotational frequency is

$$\left(\frac{p}{mc}\right)^2 = \frac{r}{\bar{H}}\frac{d\bar{H}}{dr} \equiv k = \frac{\beta^2}{1-\beta^2} \qquad (1)$$

*Work supported in part by the National Science Foundation.

where $\bar{H} = \frac{1}{\Theta}\int_0^{\Theta} H d\theta$

This gives the differential equation

$$\frac{r}{H}\frac{d\bar{H}}{dr} = \left(\frac{e}{mc^2}\bar{H}r\right)^2$$

or (2)

$$\frac{d\bar{H}}{H^3} = \left(\frac{e}{mc^2}\right)^2 r\,dr$$

(3)

$$\text{So}\left(\frac{1}{\bar{H}_c^{\,2}} - \frac{1}{\bar{H}_o^{\,2}}\right) = \left(\frac{e}{mc^2}\right)^2 r_o^{\,2}$$

where $\bar{H}_c$ is the central field strength at r = 0

If we do not inject at the center

$$\left(\frac{1}{\bar{H}_i^{\,2}} - \frac{1}{\bar{H}_o^{\,2}}\right) = \left(\frac{e}{mc^2}\right)^2 \left(r_o^{\,2} - r_i^{\,2}\right) \tag{4}$$

and

$$\bar{H}(r) = \frac{1}{\sqrt{\frac{1}{\bar{H}_i^{\,2}} - \left(\frac{e}{mc^2}\right)^2 \left(r^2 - r_i^{\,2}\right)}} \tag{5}$$

with

$$k = \left(\frac{\bar{H}_o}{\bar{H}_c}\right)^2 - 1 \tag{6}$$

In terms of the final energy the ratio of the field at outer edge to the field at center is

$$\frac{\bar{H}_o}{\bar{H}_c} = \frac{KE}{mc^2} + 1 \tag{7}$$

The design features will now be found using the smooth approximation. By ν_r and ν_z we mean the number of radial and axial betatron oscillations around a circumference. The result of using Laslett's scalloped orbit as described in Revision MURA-DWK-7 is to change the usual rule of Terwilliger,

$$\left.\begin{aligned} \nu_r^2 &= 1 + k(r) + (A.G.) \\ \nu_z^2 &= -k(r) + (A.G.) \end{aligned}\right\} \quad (8)$$

which holds if the r and z differential equations for displacements from the equilibrium orbit have the non-alternating part of the force constant 1-k and -k respectively. However, the r and z differential equations for a Mark V orbit have (-A.G.) added to k for the non-alternating part of the force constant giving:

$$\left.\begin{aligned} \nu_r^2 &= 1 + k(r) \\ \nu_z^2 &= -k(r) + 2(A.G.) \end{aligned}\right\} \quad (9)$$

The scalloping of the orbit makes the construction of a cyclotron easier than was originally expected because the amount of alternating gradient focussing can be less.

The rigid requirement on the variation of k(r) shown in (1) and equation (9) show that

$$\nu_r = \sqrt{1 + \left(\frac{p}{mc}\right)^2} = \frac{KE}{mc^2} + 1 = \bar{H}_o / \bar{H}_c \quad (12)$$

and

$$k = \left[\frac{KE}{mc^2} + 1\right]^2 - 1 \qquad (13)$$

Combining equations (9) we have

$$\nu_r^2 + \nu_z^2 = 1 + 2(A.G.) \qquad (14)$$
$$\nu_r^2 = 1 + k$$

On the ν_r, ν_z plane (14) is a circle. Figure I shows these quantities and the location of various resonances which are encountered if the working point moves.

It is evident from the graph that the working point can move from $\nu_r = 1$, $\nu_z = 0$ at the center of the cyclotron poles (ordinary cyclotron) to $\nu_r = 1½$, $\nu_z = 0$ to ½ which is at $mc^2/2$ or 466 Mev for protons before the first half integral resonance is encountered. If the "D" voltage is large enough, this resonance might be charged through allowing the working point to go up to $\nu_r = 2$, $\nu_z = 0$ or 932 Mev where an integral and a sum resonance occur.

In general the working point must pass KE/mc^2 integral, Ke/mc^2 sum, and $2KE/mc^2$ half-integral resonances to reach the kinetic energy KE. This totals about 4 x E resonances which must be passed if E is measured in billion electron volts.

The parameters in the quantity (A.G.) for spirally ridged poles making a field and

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It is evident from the graph that the working point can move from $\nu_r = 1$, $\nu_z = 0$ at the center of the cyclotron poles (ordinary cyclotron) to $\nu_r = 1\frac{1}{2}$, $\nu_z = 0$ to $\frac{1}{2}$ which is at $mc^2/2$ or 466 Mev for protons before the first half integral resonance is encountered. If the "D" voltage is large enough, this resonance might be charged through allowing the working point to go up to $\nu_r = 2$, $\nu_z = 0$ or 932 Mev where an integral and a sum resonance occur.

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The parameters in the quantity (A.G.) for spirally ridged poles making a field

$$H = H_c \left(1 + f \sin[x/\lambda(r) - N\theta]\right) \Big/ \sqrt{1 - \left(\frac{eH_c r}{mc^2}\right)^2} \quad (10)$$

For a constant frequency cyclotron are

$$2(\text{A.G.}) = \left(\frac{f}{\lambda N}\right)^2 \quad (11)$$

Where $x = \frac{r - r_0}{r}$ is the radial displacement from r_0, $\lambda(r)$ is the radial separation of ridges at r in units of r, N is the number of ridges encountered on one revolution and f is the fractional variation of the field in the median plane due to the ridges.

As an example, suppose one wanted a kinetic energy of $mc^2/2$. Then at the high energy orbit $\frac{r}{\bar{H}}\frac{d\bar{H}}{dr} = k$ must be 1.25 and in the center k = 0 could be used as in an ordinary cyclotron. Then $\nu_r = \sqrt{1 - k} = 1.5$ betatron oscillations around the orbit. It is necessary for stability in the Mathieu differential equation describing small oscillations that $\sigma < \pi$, that is that $2\nu < N$. (Cases of stability with $\sigma_r > \pi$ for small oscillations have been observed with the digital computor if the amplitude is large enough to cause non-linear effects.) Thus $N > 3$ so choose N = 5 ridges around circumference. Then using (9) and (11)

$$\left(\frac{f}{5\lambda}\right)^2 = 1.25 + \nu_z^2$$

We want $\nu_z < \frac{1}{2}$, the first z resonance.

Choose $\nu_z = 1/3$. Then

$$f/\lambda = 5\left(\frac{5}{4} + \frac{1}{9}\right)^{1/2}/2\pi = .925 \quad (15)$$

will provide enough alternating gradient focussing at the high energy orbit to overcome the defocussing action of the radially increasing field and to produce the axial focussing required to give $\nu_z = 1/3$

The problem now becomes one of the best choice of f, λ, , and G, the minimum gap from the pole to pole between ridges. Figure II shows* relations between these quantities when adjacent ridges produce an essentially 2 dimensional problem.

If G/λ is too large, infinite crevices open in the pole between ridges and additional reverse inter-ridge poles or coils are needed to generate the flutter in the median plane. One curve shows the largest values of G/λ which do not require extra coils or poles. The other curve is derived from G/λ by multiplying by $2\pi f$. It shows $G\, f/\lambda$ which is proportional to the greatest gap between ridge tops with f/λ any constant value which is desired. The largest gap can be achieved with $f = .25$ where $G/\lambda = .28$. By (15) $\lambda = .27$ in units of r - that is ridges are more than a quarter of the radius apart. So G = (.28) (.27) = .075 in units of

*See MURA-DWK-10 for further discussion.

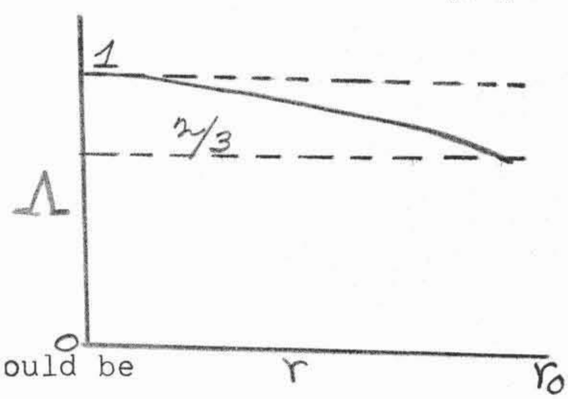
radius. But at $\bar{H}$ = 14,000 Gausses ρ = 247 cm, thus $\lambda r \equiv \Lambda$ = 62 cm. G $\equiv$ 18.6 cm at the outside rim. This small gap is one of the main difficulties with this cyclotron because it limits the voltage on the D's which should be high for a constant frequency cyclotron. At energies lower than the maximum the size of f can be decreased or the size of λ can be increased so that one ridge in from the edge, i.e. at .27 r_o in from the edge, f/λ = 0.6 is all that is needed.

As a function of radius

$$f/\lambda = N\sqrt{\frac{\left(\frac{e}{mc^2}H_c r_o\right)^2}{1-\left(\frac{e}{mc^2}H_c r_o\right)^2}+\nu_z^2} \qquad (16)$$

If we choose $\nu_z \propto r$ so that it goes to zero at the same rate as r does, we have

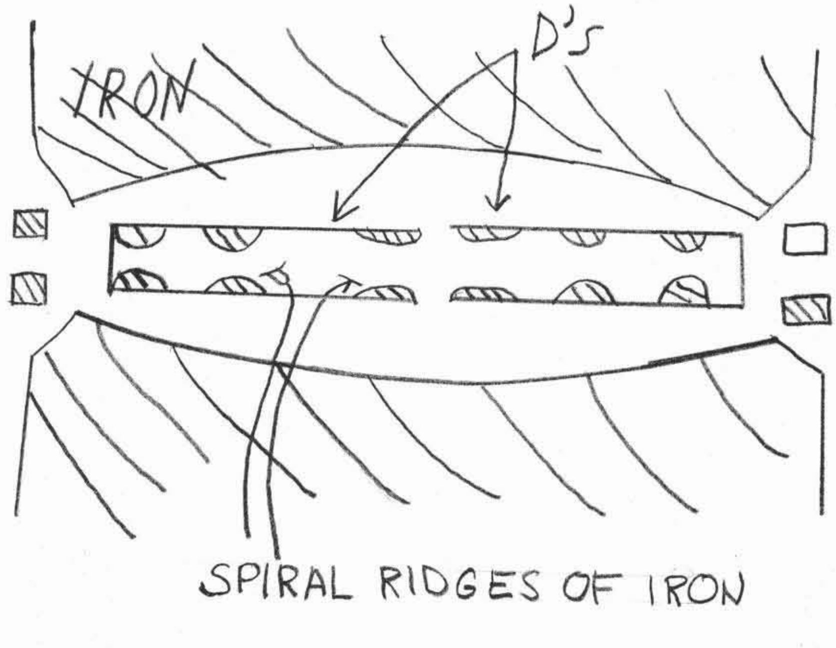
$$\lambda r_o = \frac{f}{N} A\sqrt{1-\left(\frac{e}{mc^2}H_c r\right)^2} \qquad (17)$$



where A is a constant. This product approaches a constant as $r \to 0$ if f is kept constant. Since λ is measured in units of r, Λ, the actual ridge separation, would be slightly bigger at the center than at the rim. What this means is that the ridges will turn in toward the center becoming radial and forming a Thomas type region around the center to provide some vertical focussing.

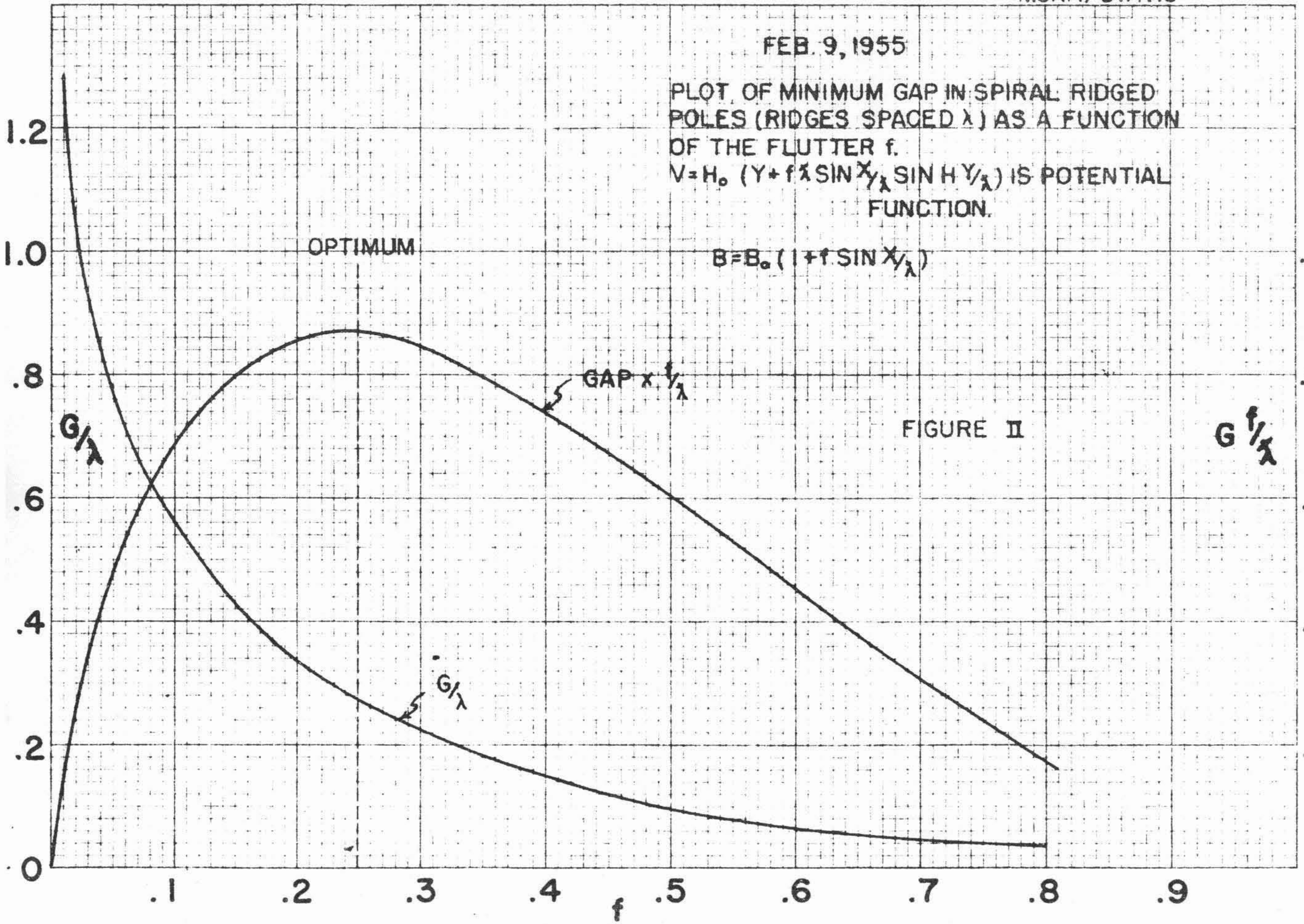
The details of what f to choose and how to bring the ridges into the center may depend to some extent on the influence of non-linearities on the useful range of oscillation amplitude. If very little of the vertical aperture can be used, an increase of f and of λ is likely to increase the possible amplitudes while sacrificing a little gap space which would be useless anyway for orbits.

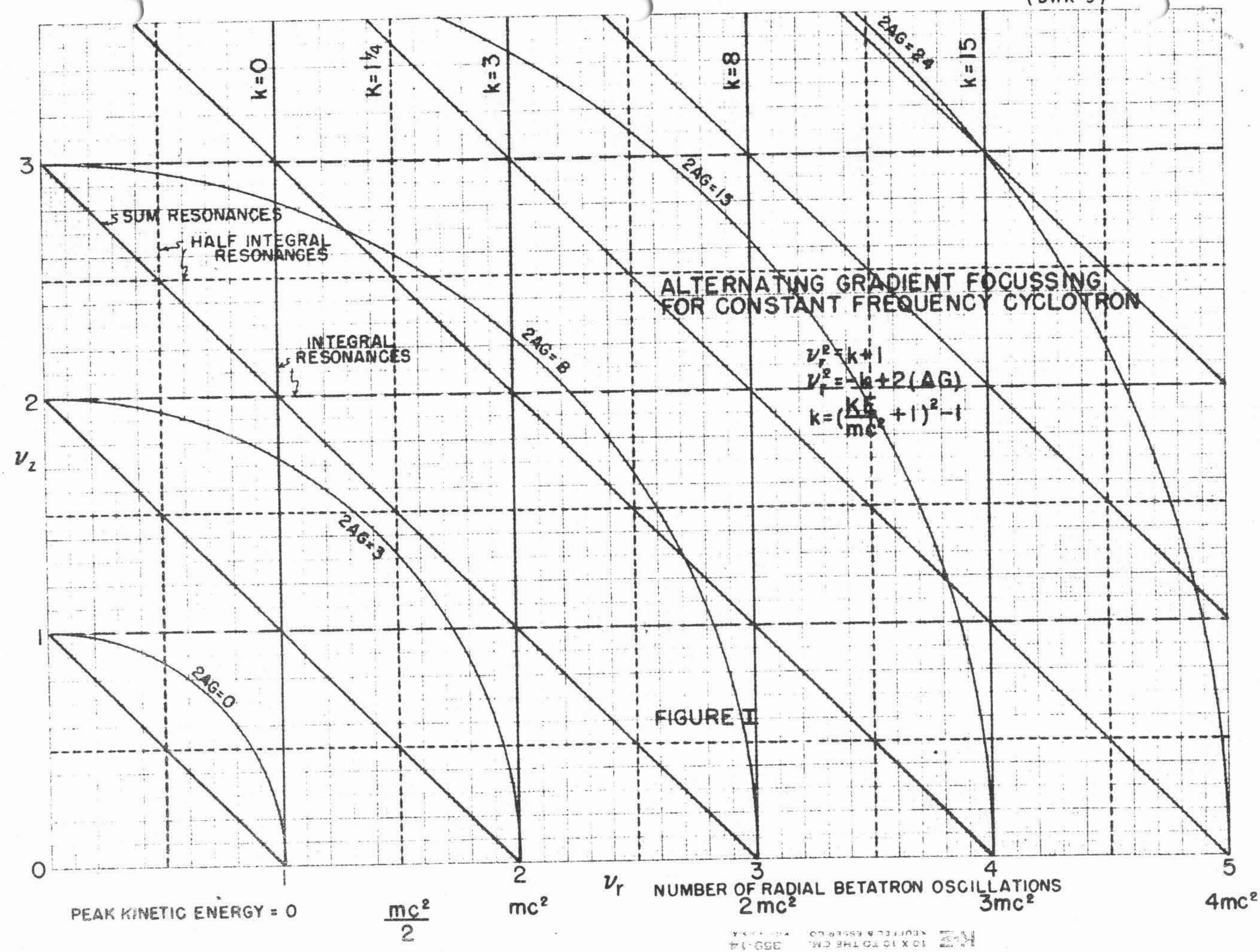
Since there is difficulty with the D's in a narrow gap, a structure might be considered which has the ridges on the poles replaced by spiral iron ridges inside the D's. These must be segmented so they transmit flux only across the gap and not along the spiral. Then the general increase of field with radius would be provided by the contour of remote poles with a large space for withstanding radiofrequency voltage.



SPIRAL RIDGES OF IRON

DWK 9
MURA/DWK10





ALTERNATING GRADIENT FOCUSSING
FOR CONSTANT FREQUENCY CYCLOTRON
$\nu_r^2 = k+1$
$\nu_z^2 = -k + 2(AG)$
$k = (\frac{KE}{mc^2} + 1)^2 - 1$
FIGURE I
k=0
k=1¼
k=3
k=8
k=15
2AG=0
2AG=3
2AG=8
2AG=15
2AG=24
SUM RESONANCES
HALF INTEGRAL RESONANCES
INTEGRAL RESONANCES
ν_z
0
1
2
3
4
5
ν_r NUMBER OF RADIAL BETATRON OSCILLATIONS
PEAK KINETIC ENERGY = 0
$\frac{mc^2}{2}$
mc^2
$2mc^2$
$3mc^2$
$4mc^2$