



NUMBER MURA-24

REQUISITE ENERGY TOLERANCE AT INJECTION

A summary is given of the theoretical basis for estimating the energy homogeneity desirable in the beam injected into an A.G. synchrotron. Some numerical examples are given for proton A.G. synchrotrons of 50 Mev injection energy.

Motivation:

As mentioned in previous notes [May-June, 1954], the question has been raised concerning the requisite energy tolerances at injection and concerning the estimates of this quantity reputedly made at the Brookhaven National Laboratory.

Method of Approach:

The approach may be considered to be similar in principle to that of K. Johnson [CERN proton-synchrotron lectures, Sect. III-3 (October, 1953)], as reported earlier [previous notes and LJL (MAC-3) (February, 1954)]. In this approach the requirement considered is that the initial momentum spread shall be no greater than that acceptable into synchrotron phase oscillations, but, in the interests of efficient capture, a more conservative specification may be imposed. Only injection for a single turn or less is considered.

Derivation of Formulas:

The equation of phase oscillations in the steady state, with ϕ representing the electrical phase angle, may be written [cf. LJL (MAC)-3, and references therein]:

$$\ddot{\phi} + A^2 (\sin \phi - \sin \phi_0) = 0,$$

where

$$A^2 = \omega_0^2 h (-\gamma) \frac{e V_0 I}{2 \pi E_s} \quad \text{and} \quad -\gamma = \left(\frac{E_0}{E} \right)^2 - \frac{1}{1 + \frac{\Sigma L}{2 \pi R}}.$$

This equation has the first integral

$$\dot{\phi}^2 = 2 A^2 [\cos \phi + \cos \phi_0 - (\pi - \phi - \phi_0) \sin \phi_0]$$

and

$$\dot{\phi}_{\max}^2 = 4 A^2 [\cos \phi_0 - (\frac{\pi}{2} - \phi_0) \sin \phi_0].$$

For

$$\phi_0 = \frac{\pi}{6}, \quad \dot{\phi}_{\max} = 4 A^2 \left[\frac{\sqrt{3}}{2} - \frac{\pi}{6} \right] = 2.37 A^2;$$

$$\dot{\phi}_{\max} = 1.17 A.$$

For efficient capture, however, we may write

$$\dot{\phi}_{\max} = 1.17 f A,$$

where f is a numerical factor which might be taken to be about 0.3 (see Fig. 1).

The associated momentum variation is

$$\frac{dp}{p} = \frac{1}{\gamma} \frac{d\omega}{\omega} = \frac{1}{\omega \gamma h} \dot{\phi} = \frac{1}{\beta \omega_0 \gamma h} \dot{\phi} = \frac{1.17 f}{A} \sqrt{\frac{1}{h |\gamma|} \frac{e V_0 / 2 \pi}{E_s}}.$$

The required peak R.F. voltage per turn, V_0 , is estimated by

$$\frac{1}{2\pi} (\delta E/\text{Turn}) = \frac{R \left[1 + \frac{\Sigma L}{2\pi R} \right] E_f}{C \cdot (\text{acc. time})}$$

$$\frac{eV_0}{2\pi} = \frac{R \left[1 + \frac{\Sigma L}{2\pi R} \right] E_f}{C \cdot (\text{acc. time}) \cdot \sin \phi_0}$$

Numerical Values:

We include in the table below results computed for (i) a machine similar to that planned by the CERN group, (ii) a machine of the type believed under design at Brookhaven, and (iii) a similar machine with characteristics discussed in the mid-west group. The values listed for the CERN machine are based on what appears to be Johnsen's assumption of $f = 1$ and $|\gamma| \approx 1$; the value of $eV_0/2\pi$ for the CERN machine is estimated from the quoted $B = 12$ kilogauss/sec and leads to results similar to those stated by Johnsen in the absence of frequency error. The value taken for $eV_0/2\pi$ in the BNL case is obtained from the formula of the preceding section as

$$\frac{85.34 \times 1.5 \times 25 \times 10^9}{3 \times 10^8 \times 1 \times 0.5} = 21.4 \times 10^3 \text{ ev/radian,}$$

for $\phi_0 = \frac{\pi}{6}$, although this value seems somewhat less than that suggested by the Brookhaven ADD minutes No. 62 (May 12, 1954). 50 Mev injection is assumed in each case.

	CERN	BNL	MURA			
Inj. Energy, Kinetic	50	50	50 Mev			
Inj. Energy, Total	0.99	0.99	0.99 Gev			
β	0.314	0.314	0.314			
$ \gamma $	~ 1	0.893	0.89			
$eV_0/2\pi$	23.1×10^3	21.4×10^3	18.7×10^3	ev/radian		
h	38	12	16	36		
f	1	0.3	1	0.3	1	0.3
$\Delta p/p$	$\pm 0.29\%$	$\pm 0.16\%$	$\pm 0.43\%$	$\pm 0.13\%$	$\pm 0.29\%$	$\pm 0.086\%$
$\Delta E/E_{\text{Kin.}}$	$\pm 0.56\%$	$\pm 0.31\%$	$\pm 0.84\%$	$\pm 0.25\%$	$\pm 0.56\%$	$\pm 0.17\%$

Summary:

It appears from the foregoing table that, for many of the designs presently under consideration, it would be adequate to inject from a LINAC 50 Mev protons whose energy spread was definitely within ± 0.3 percent. Energy stability, from pulse to pulse, is also highly desirable. This tolerance may be compared with that expected for the Minnesota design, as reported by L. H. Johnston [Bull. APS, 1954 meeting in Washington, D.C., paper C1], namely $E_{\text{Kin}} = 50 \pm 0.2$ Mev or $\Delta E/E_{\text{Kin}} = \pm 0.4$ percent.

The Brookhaven group have suggested privately the objective of ± 0.1 percent, although the ADD minutes No. 57 (March 16, 1954) suggest the requirement of $\pm 1/2$ percent in energy. It is believed that $\pm 1/3$ percent in energy represents the present thinking of the group. Attention must also be given to the "emittance" of the beam, and the desirability of a beam of 1 milliamperes from the injector has been suggested.

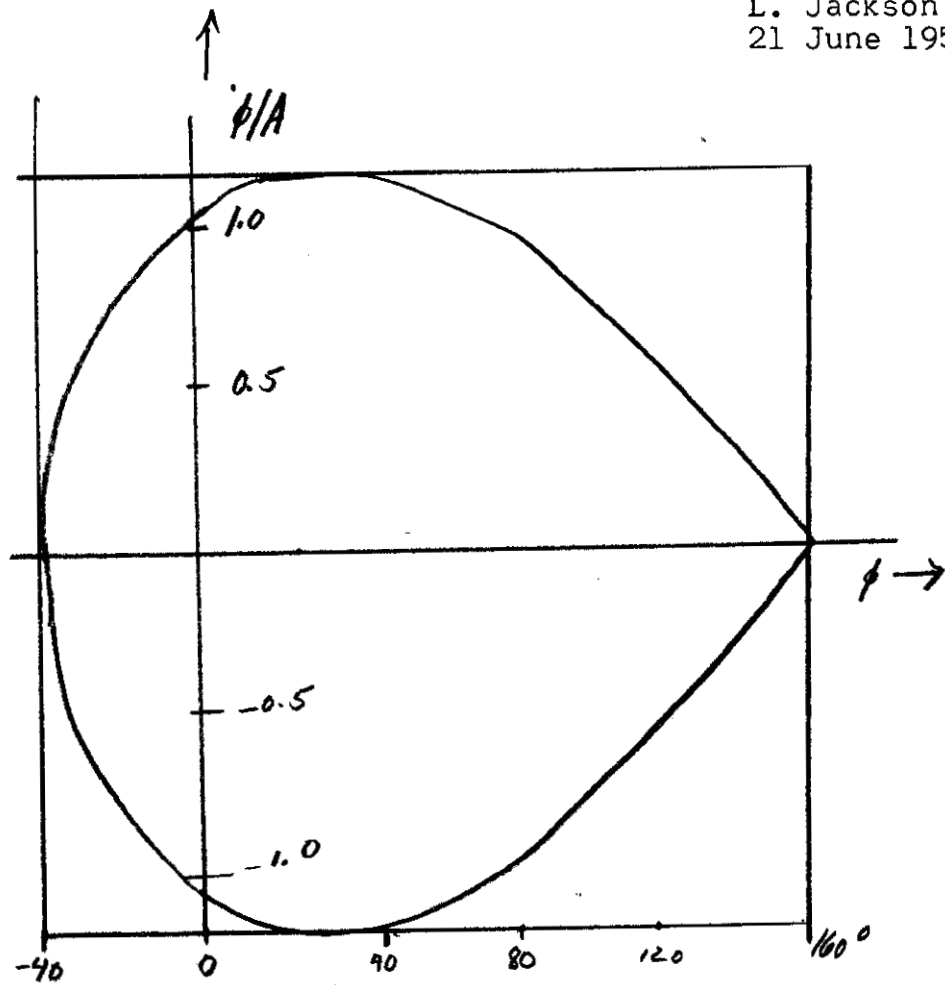


Fig. 1. Plot of region for acceptance into phase oscillations.