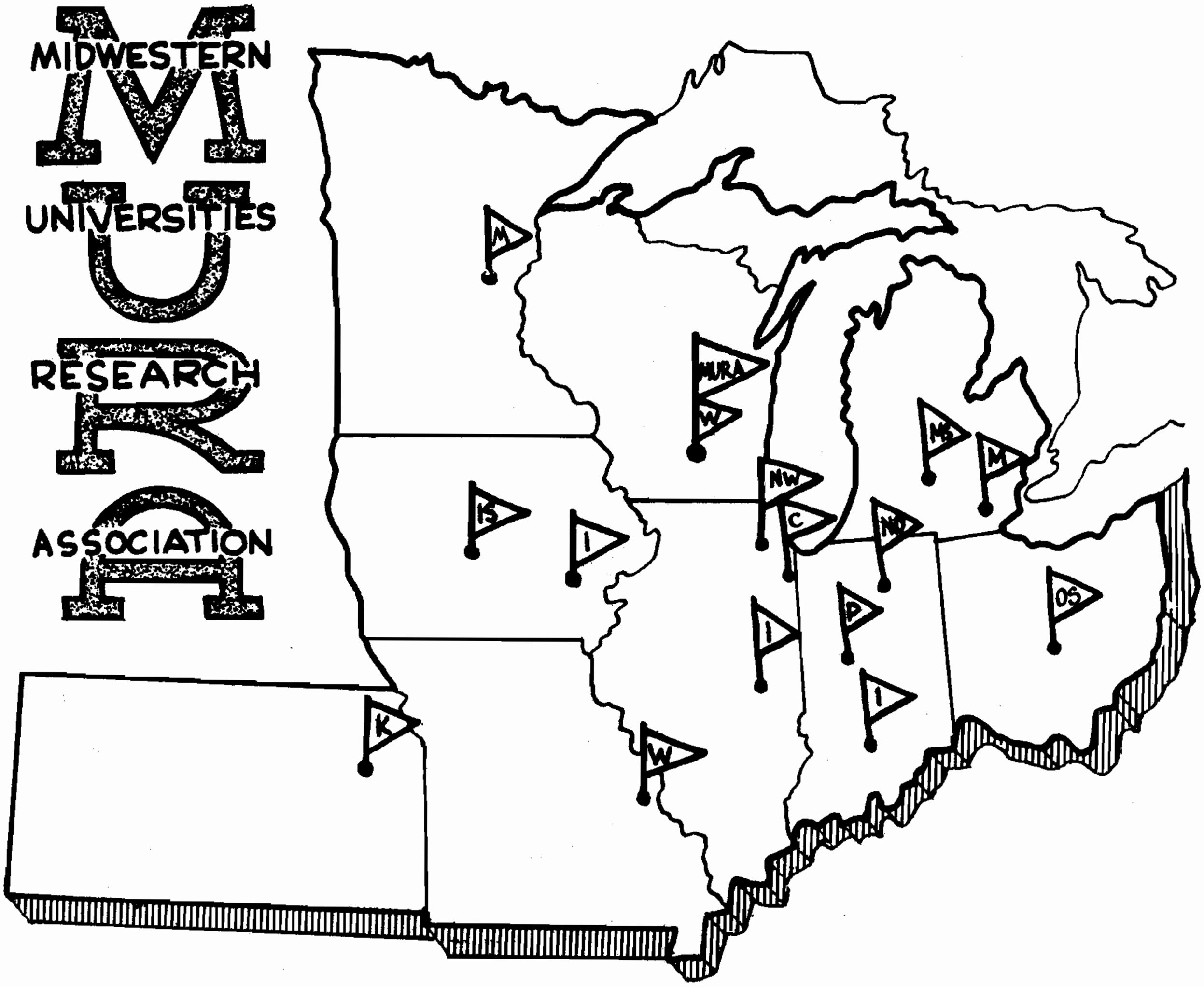

MAGNET APERTURE AS A FUNCTION OF n

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MAGNET APERTURE AS A FUNCTION OF n

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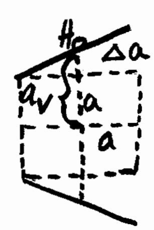
The use of nonlinear magnetic fields to limit the effect of the resonances crossing the stability region appears to indicate again a high value of n and a correspondingly small aperture. The purpose of this note is to show the aperture variation with n, with the assumption that the blow-ups are limited.

The aperture required for betatron oscillations for an injection angle δ is $a_b \leq \frac{4}{\sqrt{n}} R\delta$ (see C&S); that required for misalignments is $a_m \cong 20 \sqrt[4]{n} \;\Delta y$ where Δy is the displacement of the magnet sectors. The latter equation assumes the nonlinearities limit the oscillation amplitude to twice the non-resonant value. This equation can be derived by the "bump analysis" method of Kerst, or as done by C&S. The third aperture requirement, that for radial excursions due to phase oscillations, is small for high n machines and is neglected here. So the total aperture $a = a_b + a_m$.

If the magnet is symmetric with respect to horizontal and vertical oscillations, a square aperture is required; however, since n is not zero the average vertical aperture a_v must necessarily be larger than the horizontal a, as indicated on the diagram.

a_v can easily be found in terms of a, n, and R as follows:



If the magnet pole pieces are considered to have constant mmf then

$$H_0(a+\Delta a) = (H_0 + \Delta H^2)\, a$$

$$\frac{\Delta a}{a} = \frac{\Delta H}{H_0} = \frac{na}{R}$$

$$\Delta a = \frac{na^2}{R}$$

$$a_v = a + \Delta a = a\left(1 + \frac{na}{R}\right)$$

where $\frac{na}{R} \leq 1$ for the field not to change direction.

Possible values of δ and Δy appear to be 10^{-3} radians, and 0.1 mm respectively. With these tolerances, a table of apertures as a function of n was calculated for a 30 Bev machine with $R = 10^4$ cm and 2×10^4 cm. These radii correspond to median fields of about 10 KG and 5 KG.

30 Bev

$$a_m = 20^4\sqrt{n}\,\Delta y$$

$$a_b = \frac{4}{\sqrt{n}}\, R\, \delta$$

where $\Delta y = 10^{-2}$ cm

$\delta = 10^{-3}$ rad

		$R = 10^4$ cm					$R = 2 \times 10^4$ cm				
n	a_m	a_b	a	$\frac{na}{R}$	a_v	$a_v a$	a_b	a	$\frac{na}{R}$	a_v	$a_v a$
500	.9	1.8	2.7	.14	3.1	8.4	3.6	4.5	.11	5.0	22.5
750	1.0	1.4	2.4	.18	3.0	7.2	2.8	3.8	.14	4.3	16.4
1000	1.1	1.2	2.3	.23	2.8	6.5	2.4	3.5	.17	4.1	14.3
1500	1.2	1.0	2.2	.34	3.0	6.6	2.0	3.2	.24	3.9	12.5
2000	1.3	.9	2.2	.45	3.2	7.0	1.8	3.1	.31	4.0	12.4
4000	1.6	.6	2.2	.89	4.2	9.2	1.2	2.8	.56	4.4	12.4
10000	2.0	.4	2.4	2.4	8.1	19.5	.8	2.8	1.4	6.7	18.7

units cm.

Since $\frac{na}{R} \leq 1$, n = 10,000 is not feasible for either machine. The product $a_v a$ shown is a rough relative measure of the cost of the magnet, copper, and magnet power. This product is a very weak function of n. For $R = 10^4$ cm a broad minimum occurs at about n = 1000, with a total aperture area of 26 cm^2. The minimum area increases linearly with R.

The results obviously depend strongly on the tolerances allowed. As an illustration, assume the line-up could be done exactly. Then the aperture (due to injection angle) would decrease with increasing n, until n was limited by $\frac{na}{R} = 1$. This would give n = 60,000 and $a_v = .3$ cm, a = .16 cm for $R = 10^4$ cm.