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Bose-Einstein study of position-momentum correlations of charged pions in hadronic Z⁰ decays

The OPAL Collaboration

A bstract

A study of Bose-Einstein correlations in pairs of identically charged pions produced in $e^+e^$ annihilations at the Z⁰ peak has been performed for the rst time assuming a non-static emitting source. The results are based on the high statistics data obtained with the OPAL detector at LEP. The correlation functions have been analyzed in intervals of the average pair transverse momentum and of the pair rapidity, in order to study possible correlations between the pion production points and their momenta (position-momentum correlations). The Yano-Koonin and the Bertsch-Pratt parameterizations have been tted to the measured correlation functions to estimate the geometrical parameters of the source as well as the velocity of the source elements with respect to the overall centre-of-mass frame. The source rapidity is found to scale approximately with the pair rapidity, and both the longitudinal and transverse source dimensions are found to decrease for increasing average pair transverse momenta.

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The OPAL Collaboration

G.Abbiendi², C.Ainsley⁵, P.F.Akesson⁷, G.Alexander²¹, G.Anagnostou¹, K.J.Anderson⁸, S.Asai²², D.Axen²⁶, I.Bailey²⁵, E.Barberio⁷, T.Barillari³¹, R.J.Barlow¹⁵, R.J.Batley⁵, P.Bechtle²⁴, T.Behnke²⁴, K.W. Bell¹⁹, P.J.Bell¹, G.Bella²¹, A.Bellerive⁶, G.Benelli⁴, S.Bethke³¹, O.Biebel³⁰, O.Boeriu⁹, P.Bock¹⁰, M.Bouten eur³⁰, S.Braibant², R.M.Brown¹⁹, H J.Burckhart⁷, S.Campana⁴, P.Capiluppi², R.K.Camegie⁶, A.A.Carter¹², J.R.Carter⁵, C.Y. Chang¹⁶, D.G. Charlton¹, C.Ciocca², A.Csilling²⁸, M.Cu ani², S.Dado²⁰, G M.Dallavalle², A.DeRoeck⁷, E.A.DeWolf²^s, K.Desch²⁴, B.Dienes²⁹, J.Dubbert³⁰, E.Duchovni²³, G.Duckeck³⁰, I.P.Duerdoth¹⁵, E.Etzion²¹, F.Fabbri², P.Ferrari⁷, F.Fiedler³⁰, I.Fleck⁹, M.Ford¹⁵, A.Frey⁷, P.Gaqnon¹¹, J.W.Gary⁴, C.Geich-Gimbel³, G.Giacomelli², P.G iacom elli², M.G iunta⁴, J.G oldberg²⁰, E.G ross²³, J.G runhaus²¹, M.G ruw e⁷, A.G upta⁸, C.Hajdu²⁸, M.Hamann²⁴, G.G.Hanson⁴, A.Harel²⁰, M.Hauschild⁷, C.M.Hawkes¹, R.Hawkings⁷, G.Herten⁹, R.D.Heuer²⁴, J.C.Hill⁵, D.Horvath²⁸^{*c*}, P.Igo-Kemenes¹⁰, K.Ishii²², H.Jerem ie¹⁷, P.Jovanovic¹, T.R.Junk^{6,i}, J.K.anzaki^{22,u}, D.K.arlen²⁵, K.K.awagoe²², T.Kawamoto²², R.K.Keeler²⁵, R.G.Kellogg¹⁶, B.W.Kennedy¹⁹, S.Kluth³¹, T.Kobayashi²², M.Kobe^{3,t}, S.Kom am iya²², T.Kram er²⁴, A.Krasznahorkay Jr.^{29,e}, P.Krieger^{6,1}, J.von Krogh¹⁰, T.Kuhl²⁴, M.Kupper²³, G.D.La erty¹⁵, H.Landsman²⁰, D.Lanske¹³, D.Lelbuch²³, J.Letts^o, L.Levinson²³, J.Lillich⁹, S.L.Lloyd¹², F.K.Loebinger¹⁵, J.Lu^{26,b}, A.Ludwig^{3,t}, J.Ludwig⁹, W.Mader^{3,t}, S.Marcellin², A.J.Martin¹², T.Mashimo²², P.Mattig^m, J.M cK enna²⁶, R.A.M cPherson²⁵, F.M eijers⁷, W.M enges²⁴, F.S.M erritt⁸, H.M es⁶^a, N.Meyer²⁴, A.Michelini², S.Mihara²², G.Mikenberg²³, D.J.Miller¹⁴, W.Mohr⁹, T.Mori²², A.M utter⁹, K.Nagai¹², I.Nakam ura^{22,w}, H.Nan jo²², H.A.Neal³², S.W. O.Neale¹, A.Oh⁷, M.J.O reglia⁸, S.O rito²²; , C.Pahl³¹, G.Pasztor⁴¹⁷, J.R.Pater¹⁵, J.E.Pilcher⁸, J.Pinfold²⁷, D.E.Plane⁷, O.Pooth¹³, M.Przybycien^{7,n}, A.Quadt³¹, K.Rabbertz^{7,r}, C.Rembser⁷, P.Renkel²³, J.M. Roney²⁵, A.M. Rossi², Y. Rozen²⁰, K. Runge⁹, K. Sachs⁶, T. Saeki²², EKG.Sarkisyan^{7;j}, AD.Schaile³⁰, O.Schaile³⁰, P.Schar Hansen⁷, J.Schieck³¹, T.Schomer-Sadenius^{7,z}, M.Schroder⁷, M.Schumacher³, R.Seuster^{13,f}, T.G.Shears^{7,h} B.C. Shen⁴, P. Sherwood¹⁴, A. Sku ja¹⁶, A.M. Sm ith⁷, R. Sobie²⁵, S. Soldner-Rem bold¹⁵, F.Spano⁸^x, A.Stahl¹³, D.Strom¹⁸, R.Strohm er³⁰, S.Tarem²⁰, M.Tasevsky⁷^{z1}, R.Teuscher⁸, M A. Thom son⁵, E. Torrence¹⁸, D. Toya²², I. Trigger⁷, Z. Trocsany f^{29e} , E. Tsur²¹, M.F.Tumer-Watson¹, I.Ueda²², B.U jvari²⁹^p, C.F.Vollmer³⁰, P.Vannerem⁹, R.Vertesi²⁹^p, M.Verzocchi¹⁶, H.Voss⁷^H, J.Vossebeld^{7,h}, C.P.W ard⁵, D.R.W ard⁵, P.M.W atkins¹, A.T.Watson¹, N.K.Watson¹, P.S.Wells⁷, T.Wengler⁷, N.Wermes³, G.W.Wilson^{15,*}, JA.W ilson¹, G.W olf²³, T.R.W yatt¹⁵, S.Yamashita²², D.Zer-Zion⁴, L.Zivkovic²⁰

¹School of Physics and A stronom y, University of Birm ingham, Birm ingham B15 2TT, UK ²D ipartim ento di Fisica dell'Universita di Bologna and INFN, I-40126 Bologna, Italy

³Physikalisches Institut, Universitat Bonn, D-53115 Bonn, Germany

- ⁴Department of Physics, University of California, Riverside CA 92521, USA
- ⁵Cavendish Laboratory, Cam bridge CB3 OHE, UK

⁶O ttawa-Carleton Institute for Physics, Department of Physics, Carleton University, O ttawa, O ntario K 1S 5B6, Canada

⁷CERN, European Organisation for Nuclear Research, CH–1211 Geneva 23, Switzerland ⁸Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago IL 60637, USA

⁹Fakultat fur Physik, Albert-Ludwigs-Universitat Freiburg, D-79104 Freiburg, Germany

 $^{10}{\rm P}$ hysikalisches Institut, U niversitat H eidelberg, D –69120 H eidelberg, G erm any

 11 Indiana U niversity, D epartm ent of P hysics, B bom ington IN $\,47405\,$, U SA

 $^{12}\mbox{Q}$ usen M ary and W est eld College, University of London , London E1 4N S, UK

 $^{13}{\rm T}$ echnische Hochschule Aachen, III Physikalisches Institut, Sommerfeldstrasse 26–28, D–52056 Aachen, Germany

 $^{14}\text{University}$ College London , London W C1E 6BT , UK

 15 School of Physics and A stronom y, Schuster Laboratory, The University of M anchester M 13 9PL, UK

¹⁶D epartm ent of Physics, University of Maryland, College Park, MD 20742, USA

¹⁷Laboratoire de Physique Nucleaire, Universite de Montreal, Montreal, Quebec H3C 3J7, Canada

 18 University of O regon, D epartm ent of Physics, Eugene OR $\,97403\,$, USA

 $^{19}\mathrm{R}$ utherford Appleton Laboratory, Chilton, D idcot, O xfordshire O X 11 0Q X , U K

²⁰D epartm ent of Physics, Technion-Israel Institute of Technology, Haifa 32000, Israel

²¹D epartm ent of Physics and Astronom y, TelAviv University, TelAviv 69978, Israel

²² International C entre for E lem entary Particle Physics and D epartm ent of Physics, U niversity of Tokyo, Tokyo 113-0033, and K obe U niversity, K obe 657-8501, Japan

²³Particle Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel

 24 U niversitat H am burg/D E SY , Institut fur Experim entalphysik , N otkestrasse 85 , D –22607 H am – burg , G erm any

²⁵University of Victoria, Department of Physics, POBox 3055, Victoria BCV8W 3P6, Canada ²⁶University of British Columbia, Department of Physics, Vancouver BCV6T 121, Canada

²⁷University of Alberta, Department of Physics, Edmonton AB T 6G 2J1, Canada

²⁸R esearch Institute for Particle and Nuclear Physics, H-1525 Budapest, P O Box 49, Hungary ²⁹Institute of Nuclear R esearch, H-4001 D ebrecen, P O Box 51, Hungary

³⁰Ludwig-Maximilians-Universitat Munchen, Sektion Physik, Am Coulombwall 1, D-85748 Garching, Germany

³¹M ax-P lanck-Institute fur Physik, Fohringer R ing 6, D -80805 M unchen, G em any

 32 Yale University, D epartm ent of Physics, N ew H aven, C T $\,$ 06520, U SA

 $^{\rm a}$ and at TR IUM F , Vancouver, C anada V 6T $\,$ 2A 3 $\,$

^b now at University of A lberta

^c and Institute of Nuclear Research, Debrecen, Hungary

^d now at Institute of Physics, A cademy of Sciences of the C zech R epublic 18221 Prague, C zech R epublic

 $^{\rm e}$ and D epartm ent of Experim ental P hysics, U niversity of D ebrecen , H ungary

- $^{\rm f}$ and MPIM unchen
- ^g and Research Institute for Particle and Nuclear Physics, Budapest, Hungary
- $^{\rm h}$ now at University of Liverpool, Dept of Physics, Liverpool L69 3BX , U K .
- $^{\rm i}$ now at Dept. Physics, University of Illinois at Urbana-Cham paign, U.S.A.

 $^{\rm j}$ and The University of M anchester, M 13 9PL , United K ingdom

- $^{\rm k}$ now at University of K ansas, D ept of Physics and A stronom y, Law rence, K S 66045, U .S.A .
- ¹ now at University of Toronto, Dept of Physics, Toronto, Canada

^m current address Bergische Universitat, Wuppertal, Germany

 $^{\rm n}$ now at University of M ining and M etallurgy, C racow , Poland

° now at University of California, San Diego, USA.

^p now at The University of Melbourne, Victoria, Australia

^q now at IPHE Universite de Lausanne, CH-1015 Lausanne, Switzerland

r now at IEKP Universitat Karlsruhe, Germany

^s now at University of Antwerpen, Physics Department, B-2610 Antwerpen, Belgium; supported

by Interuniversity Attraction Poles Programme { Belgian Science Policy

^t now at Technische Universitat, Dresden, Germany

^u and H igh Energy A coelerator R esearch O rganisation (K E K), T sukuba, Ibaraki, Japan

^v now at University of Pennsylvania, Philadelphia, Pennsylvania, USA

w now at TR IUM F, Vancouver, Canada

× now at Colum bia University

 $^{\rm y}$ now atCERN

 $^{\rm z}$ now atDESY

D eceased

1 Introduction

The space-time evolution of a source em itting particles can be probed using intensity interferom etry. Bose-E instein correlations (BECs) in pairs of identical bosons have been studied at di erent centre-of-m ass energies and for di erent initial states (e^+e^- [1], pp and pp [2], lepton-hadron [3], nucleus-nucleus collisions [4]). BECs manifest them selves as enhancements in the production of identical bosons which are close to one another in phase space. They can be analyzed in terms of the correlation function

$$C(p_1; p_2) = \frac{(p_1; p_2)}{_0(p_1; p_2)}$$
(1)

where p_1 and p_2 are the 4-m on enta of the two bosons, $(p_1; p_2)$ is the density of the two identical bosons and $_0(p_1; p_2)$ is the two-particle density in the absence of BECs (reference sample). From the experimental correlation function one can extract the dimension of the source element (frequently called correlation length or radius of the em itting source), i.e. the length of the region of hom ogeneity from which pions are em itted that have momenta sim ilar enough to interfere and contribute to the correlation function.

At LEP Bose-Einstein correlations were analysed extensively in Z^0 hadronic events [5-11]. Two-pion correlations were studied as a function of the relative 4-m omentum $q = (p_1$ p_2) of the pair: $C(p_1;p_2) = C(q)$. It was found that the radius of the emitting region, supposed spherical, is of the order of 1 fm and increases with the number of jets in the event [5]. No signi cant di erences were observed in the source dimensions between the and the 0 system s [6]; on the other hand, sm aller radii were measured in K K and $K^{0}K^{0}$ ($\overline{K^{0}K^{0}}$) pairs com pared with pion pairs [7]. Genuine three-pion BECswere also observed [8]. Up to fth-order genuine correlations of identically charged pions were obtained by OPAL [9], where BECs were shown to be an essential ingredient of the correlation scaling observed there, also for all-charged higher-order correlations. The hypothesis that the source is spherical was tested studying the correlations in term s of components of q: two-and three-dimensional analyses have shown that the pion emission region is elongated rather than spherical, with the longitudinal dimension, along the event thrust axis, larger than the transverse one [10, 11]. BECs were also studied in e^+e ! W^+W events: no evidence of correlations between pions originating from di erent W bosons was found [12].

All the results listed above were obtained under the hypothesis that them om entum distribution of the em itted particles is hom ogeneous throughout the source elements, as would happen if the source is static. In the case of a dynamic, i.e. expanding, source, the dimension of the regions of hom ogeneity varies with the momentum of the em itted particles. The expansion leads to correlations between the space-time em ission points and the particle 4-m om enta (position-momentum correlations) which generate a dependence of the BEC radii on the pair momenta. In this case, the correlation function is expected to depend on the average 4-m om entum of the pair K = $(p_1 + p_2)=2$ in addition to the relative 4-m om entum q: C $(p_1;p_2) = C$ (q;K) [13], so that the measured radii correspond to regions of hom ogeneity in K , i.e. excitive source elements of pairs with momentum K.

Published investigations of the source dynamics in e⁺ e⁻ collisions are available at energies lower than LEP's [1]. A dependence of the source radii on dierent components of the 4-vector K has been observed in more complex systems such as the emission region created after a highenergy collision between heavy nuclei. In particular, source radii have been found to decrease for increasing pair transverse momenta k_t (or, equivalently, transverse masses $m_t = \frac{k_t^2 + m^2}{k_t^2 + m^2}$) [4]. Hydrodynam ical models for heavy ion collisions [14] explain this correlation in terms of an expansion of the source, due to collective ows generated by pressure gradients. A similar dependence of the size parameters on m_t was measured in pp collisions [15]. Longitudinal position-momentum correlations can be expected in e^+e annihilations as a consequence of string fragmentation [16]. Models based on di erent assumptions (the Heisenberg uncertainty principle [17], the generalized B jorken-G ottfried hypothesis [18]) predict radii decreasing with the transverse mass also for sources created in e^+e collisions.

In this paper, which continues a series of OPAL studies on BECs [5,10], a measurement of three-dimensionalBose-Einstein correlation functions is presented and the correlation functions are analyzed in order to measure their dependence on K and investigate potential dynamical features of the pion-emitting source created after an e^+e^- annihilation at a centre-of-mass energy of about 91 G eV.

2 Experim ental procedure

A detailed description of the OPAL detector can be found in [19, 20]. In the present analysis, we have used the same data sample, about 4.3 m illion multihadronic events from Z^0 decays, and have applied the following selection cuts on tracks and events, identical to the ones described in [10]. First, the event thrust axis was computed, using tracks with a minimum of 20 hits in the jet chamber, a minimum transverse momentum of 150 MeV and a maximum momentum of 65 G eV. C lusters in the electrom agnetic calorin eter are used if their energies exceed 100 MeV in the barrel or 200 MeV in the endcaps. Only events well contained in the detector were accepted, requiring joos thrust j < 0.9, where thrust is the polar angle of the thrust axis with respect to the beam axis¹. Then, a set of cuts, specic to BEC analyses, were applied. Tracks were required to have a maximum momentum of 40 GeV and to originate from the interaction vertex. Electron-positron pairs from photon conversions were rejected. Events were selected if they contained a minimum number of ve tracks and if they were reasonably balanced in charge, i.e. requiring \dot{n}_{ch}^+ $n_{ch}^+ = (n_{ch}^+ + n_{ch}^-) = 0.4$, where n_{ch}^+ and n_{ch}^- are the num ber of positive and negative charge tracks, respectively. About 3.7 m illion events were left after all quality cuts. All charged particle tracks that passed the selections were used, the pion purity being approximately 90%. No corrections were applied for nalstate Coulom b interactions. All data and M onte C arb distributions presented here are given at the detector level, i.e. not corrected for e ects of detector acceptance and resolution.

The correlations were measured as functions of two dierent sets of variables, components of the pair 4-m om entum dierence q in two dierent frames.

The rst set, $(Q, Q_{t_{side}}; Q_{t_{out}})$, was evaluated in the Longitudinally CoM oving System (LCM S) [21]. For each pion pair, the LCM S is the frame, moving along the thrust axis, in which the sum of the two particle momenta, $p = (p_1 + p_2)$, lies in the plane perpendicular to the event thrust axis. The momentum di erence of the pair, $Q = (p_1 - p_2)$ is resolved into the moduli of the transverse component, Q_t , and of the longitudinal component, Q_{\prime} , where the longitudinal (²) direction coincides with the thrust axis. Q_t may in turn be resolved into \out", $Q_{t_{out}}$, and side", $Q_{t_{side}}$, components

$$Q_{t} = Q_{t_{out}} \hat{O} + Q_{t_{side}} \hat{S}$$
(2)

¹The coordinate system is de ned so that z is the coordinate parallel to the e^+ and e beam s, with positive direction along the e beam; r is the coordinate norm alto the beam axis, is the azim uthal angle and is the polar angle with respect to +z.

where δ and \hat{s} are unit vectors in the plane perpendicular to the thrust direction, such that $p = p\delta de$ nest he \out" direction and $\hat{s} = \hat{r}$ δde nest he \side" direction. It can be shown [22] that, in the LCM S, the components $Q_{t_{side}}$ and Q_{r} reject only the dijection in emission space of the two pions, while $Q_{t_{out}}$ depends on the dijection in emission time as well.

The second set, $(q_{\pm};q_{\cdot};q_{0})$, was evaluated in the event centre-ofm ass (CMS) frame. For each event, two hem ispheres are de ned by the plane perpendicular to the thrust axis. Each pair is then associated to the hem isphere containing the vector sum of the threem on enta. The pair 4-m on entum di erence q is resolved into the energy di erence $q_{0} = (E_{1} \quad E_{2})$ and the 3-m on entum di erence $q = (p_{1} \quad p_{2})$. The vector q is further decom posed into q_{\pm} and q_{\prime} , the transverse and longitudinal components, respectively, with respect to the thrust axis. In each pair, index 1 corresponds to the particle with the highest energy, so that $q_{0} = 0$. The longitudinal component, q_{\prime} , m ay be either positive, in case the vector di erence q lies in the pair hem isphere, or negative, in the opposite case. The transverse component, q_{\pm} , is positive de nite.

The experimental three-dimensional correlation functions C are dened, in a small phase space volume around each triplet of $Q \cdot Q_{t_{side}}$ and $Q_{t_{out}}$ (or q_t, q_{\cdot} and q_0) values, as the number of like-charge pairs in that volume divided by the number of unlike-charge pairs:

$$C = \frac{N_{++} + N_{-}}{N_{+}} = \frac{N_{like}}{N_{unlike}}$$
(3)

In order to have adequate statistics in each bin, a bin size of 40 M eV was chosen in each component of q, which is larger than the estimated detector resolution of 25 M eV [5].

Long-range correlations are present in the correlation function C, due to phase space limitations and charge conservation constraints. In addition, the choice of unlike-sign pairs as the reference sample adds further distortions to the correlation function, due to pions from resonance decays. To reduce these e ects, we introduced the (double) ratio C⁰ of the correlation functions C in the data and in a sample of 7.2 m illion Jetset 7.4 [23]m ultihadronic M onte C arlo (M C) events, without BEC s:

$$C^{0} = \frac{C^{DATA}}{C^{MC}} = \frac{N_{like}^{DATA} = N_{unlike}^{DATA}}{N_{like}^{MC} = N_{unlike}^{MC}} :$$
(4)

The M onte Carlo sam ples are processed through a full simulation of the OPAL detector [24]. The simulation parameters of the generator were tuned in [25].

The dependence of the correlation functions $C^{0}(q_{t};q_{r};q_{D})$ and $C^{0}(Q_{r};Q_{t_{side}};Q_{t_{out}})$ on the pair average 4-m om entum K has been analyzed by selecting pions in di erent intervals of two components of K : the pair rapidity

$$j'_{2} j = \frac{1}{2} \ln \left[\frac{(E_{1} + E_{2}) + (p'_{;1} + p'_{;2})}{(E_{1} + E_{2}) (p'_{;1} + p'_{;2})} \right]^{\#}$$
(5)

and the pair average transverse m om entum with respect to the event thrust direction

$$k_{t} = \frac{1}{2} j(p_{t,1} + p_{t,2}) j:$$
 (6)

The di erential j' j and k_t distributions, $\frac{dn}{dj'j}$ and $\frac{dn}{dk_t}$, of the data are shown in Fig.1. The sam e distributions for Jetset events are also presented in Fig.1: the com parison shows a good agreem ent between data and M onte Carlo events.



Figure 1: (a) H istogram of the di erential distribution in the pair rapidity j' jand (b) in the pair mean transverse momentum k_t of the data (dots) and Jetset events (line). The number of pairs in the M onte C arlo sample has been normalized to the number of pairs in the data sample.

The dependence of C and C⁰ on K has been studied in three bins of f j(0:0 f j< 0:8, 0:8 f j< 1:6,1:6 f j< 2:4) and ve bins of k_t (0:1 $k_t < 0.2 \text{ GeV}, 0:2 k_t < 0:3 \text{ GeV},$ 0:3 $k_t < 0:4 \text{ GeV}, 0:4 k_t < 0:5 \text{ GeV}, 0:5 k_t < 0:6 \text{ GeV}$). In this domain, a total of 47.3 m illion like-charge and 54.7 m illion unlike-charge pairs have been analysed.

3 The experimental correlation functions

Sam ples of two-dimensional projections of the correlation function $C(Q,;Q_{t_{side}};Q_{t_{out}})$ for a single bin of j′ j and k_t are shown in Fig. 2 for the data and the MC Jetset events. For the exam ple shown ², the bin corresponding to pair rapidities and transverse m omenta in the intervals 0.8 j′ j< 1.6 and 0.3 G eV $k_t < 0.4$ G eV was chosen. Sm all (< 0.2 G eV) values of $Q_{t_{out}}$ and of Q_{\cdot} have been required in the $(Q_{\cdot};Q_{t_{side}})$ and in the $(Q_{t_{side}};Q_{t_{out}})$ projections, respectively. Bose-Einstein correlation peaks are visible in the data at low $Q_{\cdot};Q_{t_{side}};Q_{t_{out}}$ but they are not present in the M onte Carlo sam ples. The same two-dimensional projections for the correlation function $C^{0}(Q_{\cdot};Q_{t_{side}};Q_{t_{out}})$ are presented in Fig. 3(a) and (b). A lso shown, in Fig. 3(c), (d) and (e), are the one-dimensional projections for low (< 0.2 G eV) values of the other two variables.

²Files of the three-dimensional correlation functions will be made available in the Durham HEP database.



Figure 2: Two-dimensional projections of the correlation function $C(Q,;Q_{t_{stile}};Q_{t_{out}})$ for 0.8 $j_{t}j < 1.6$ and 0.3 GeV $k_t < 0.4$ GeV for the data ((a) and (b)) and for Jetset MC events ((c) and (d)). $Q_{t_{out}} < 0.2$ GeV in (a) and (c); Q < 0.2 GeV in (b) and (d).

The two-dimensional (q; q_0) and the one-dimensional q_projections of the correlation function C (q_p; q, q_0) in the bin 0.8 jr j< 1.6 and 0.3 GeV $k_t < 0.4$ GeV are shown in Fig. 4, for data and Jetset events. Narrow cuts (< 0.2 GeV) on the other variables have been applied to make the projections. The combination $[(q_t^2 + q_t^2) q_0^2]$ of the three variables is an invariant greater than zero. This condition and the bound on the pair rapidity constrain the correlation function to be di erent from zero only in a limited region of the (q, ;q_0) plane, as can be seen in Fig. 4 (a) and (c). The (q, ;q_0) and (q, ;q_t) projections of the correlation function C $^0(q_t;q_r;q_0)$ are shown in Fig.5 together with the one-dimensional projections, for small (< 0.2 GeV) values of the other variables. BEC enhancements are clearly seen in both the q_t and q, projections, Fig. 5 (c) and (e). Fig. 5 (d), on the other hand, show s that the range available to the variable q_0 is quite restricted, and that no Bose-E instein peak can be observed.



Figure 3: Two-dimensional ((a) and (b)) and one-dimensional ((c), (d) and (e)) projections of the correlation function $C^{0}(Q,;Q_{t_{side}};Q_{t_{out}})$ for 0.8 j' j < 1.6 and 0.3 GeV $k_{t} < 0.4$ GeV. $Q_{t_{out}} < 0.2$ GeV in (a), Q < 0.2 GeV in (b). In (c), (d) and (e) the one-dimensional projections are obtained for low values (< 0.2 GeV) of the remaining two variables.

4 Param eterizations of the correlation functions

To extract the spatial and tem poral extensions of the pion source from the experim ental correlation functions, the Bertsch-Pratt (BP) [26]

$$C^{0}(Q, iQ_{t_{side}}; Q_{t_{out}})$$

$$= N (1 + e^{(Q^{2}, R^{2}_{long} + Q^{2}_{t_{side}} R^{2}_{t_{side}} + Q^{2}_{t_{out}} R^{2}_{t_{out}} + 2Q (Q t_{out} R^{2}_{long;t_{out}})})F (Q (Q + Q t_{out}) (7)$$

and the Yano-Koonin (YK) [27]

)

$$C^{0}(q_{t};q_{t};q_{0}) = N (1 + e^{(q_{t}^{2}R_{t}^{2} + {}^{2}(q_{t} vq_{0})^{2}R_{t}^{2} + {}^{2}(q_{0} vq_{t})^{2}R_{0}^{2})})F (q_{t};q_{t};q_{0})$$
(8)

param eterizations were tted to the measured correlation functions in all intervals of $k_{\rm t}$ and γ j.

In both parameterizations, N is a normalization factor while measures the degree of incoherence of the pion sources, and is related to the fraction of pairs that actually interfere.



Figure 4: Two-dimensional (q; q) and one-dimensional q projections of the correlation function C (q; q; q) for data ((a) and (b)) and Jetset events ((c) and (d)). The correlation function was measured in the bin 0.8 f j < 1.6 and 0.3 GeV $k_t < 0.4$ GeV. It was required $q_t < 0.2$ GeV in (a) and (c). In (b) and (d) the one-dimensional projections are obtained for low values (< 0.2 GeV) of the remaining two variables.

The two parameters N and , whose product determ ines the size of the BEC peak, are however signicantly (anti)correlated: this limits the interpretation of $\$ and the comparison of its values between the two parameterizations.

The two functions $F(Q, Q_{t_{side}}; Q_{t_{out}}) = (1 + \log Q + t_{side} Q_{t_{side}} + t_{out} Q_{t_{out}})$ and $F(q_t; q, q_t) = (1 + tq_t + q_t + q_t + 0q_t)$, where i and i are free parameters, were introduced in Eq. (7) and (8) to take into account residual long-range two-particle correlations, due to energy and charge conservation.

The interpretation of the other free parameters in Eq (7), is the following:

 $R_{t_{side}}$ and R_{long} are the transverse and longitudinal source radii in the LCMS, i.e. the longitudinal rest fram e of the pair;



Figure 5: Two-dimensional projections of the correlation function C⁰($q_t;q_r;q_b$): ($q_r;q_b$) for $q_t < 0.2 \text{ GeV}$ in (a) and ($q_r;q_t$) for $q_b < 0.2 \text{ GeV}$ in (b). One-dimensional projections ((c), (d) and (e)) of C⁰($q_t;q_r;q_b$), obtained for low values (< 0.2 GeV) of the remaining two variables. The correlation function has been measured in the bin 0.8 jrj < 1.6 and 0.3 GeV $k_t < 0.4 \text{ GeV}$.

 $R_{t_{out}}$ and the cross-term $R_{long_{t_{out}}}$ are a combination of both the spatial and temporal extentions of the source. Under certain assumptions [13], the difference ($R_{t_{out}}^2 - R_{t_{side}}^2$) is proportional to the duration of the particle emission process, and $R_{long_{t_{out}}}$ to the source velocity with respect to the pair rest fram e [22].

In the YK function Eq. (8), where $= 1 = 1 = \sqrt{1 + v^2}$, the free parameters are interpreted as follows:

v is the longitudinal velocity, in units of c, of the source element in the CMS frame;

 R_0 m easures the time interval, times c, during which particles are emitted, in the rest frame of the emitter (source element). Disculties in achieving reliable results for the time parameter R_0^2 in YK ts have been reported in the literature [28], due to the limited phase-space available in ${}^2(q_0 - vq_1)^2$;

 R_t and R_i are the transverse and longitudinal radii, i.e. the regions of hom ogeneity of the source, in the rest fram e of the em itter.

The parameters R_0 , R_t and R_i do not depend on the frame in which the correlation function has been measured, since they are evaluated in the rest frame of the source element.

The two parameterizations are not independent [13], so that a comparison between the BP and the YK $\,$ ts represents an important test.

5 Results

M in im um 2 ts of the Bertsch-Pratt and the Yano-K oon in parameterizations to the experimental correlation functions were performed using the MINUIT [29] program. The error associated to each entry of the three-dimensional matrices C and C⁰ was computed attributing a Poissonian uncertainty to the number of like and unlike charge pairs in the corresponding bin. The

t range allowed to each variable was set between 40 M eV and 1 G eV. The region below 40 M eV was excluded to avoid problem s of detector resolution and poorly reconstructed or split tracks which m in ic two like charged particle tracks with very low q. In Sections 5.1 and 5.2 the results of the ts are presented. Sources of system atic uncertainties on the t parameters are discussed in Section 5.3. Section 5.4 is devoted to a comparison between the BP and the YK parameterizations.

5.1 Bertsch-Pratt ts

The best-t parameters of the BP function, Eq. (7), are listed in Table 1, and their dependence on j' jand k_t is shown in Fig. 6. Errors in Fig. 6 include both statistical standard deviations as given by the t program ³ and system atic uncertainties (discussed in Section 5.3), added in quadrature. One notes that there is only a minor dependence on the rapidity, but som e parameters depend on k_t . In more detail:

varies between 0.25 and 0.4. The coe cient of correlation between the parameters and N is about 0.35, almost independent of k_t ;

 $R_{t_{side}}^2$, $R_{t_{out}}^2$ and, less marked ly, R_{long}^2 decrease with increasing k_t . The presence of correlations between the particle production points and their momenta is an indication that the pion source is not static, but rather expands during the particle emission process. R_{long}^2 is larger than the corresponding transverse parameter $R_{t_{side}}^2$, in agreement with a pion source which is elongated in the direction of the event thrust axis [10];

the cross-term parameter $R_{long,t_{out}}^2$ is compatible with zero, apart from a few bins at the highest rapidity interval. This result may be explained [13] assuming that the source velocity, measured with respect to the rest frame of the pion pair, is close to zero;

the di erence between the \out" and \side" transverse parameters, $(R_{t_{out}}^2 = R_{t_{side}}^2)$ for j' j < 1.6 is positive at low k_t , then it decreases and becomes negative for $k_t = 0.3 \text{ GeV}$. In the highest rapidity interval, 1.6 j' j < 2.4, $(R_{t_{out}}^2 = R_{t_{side}}^2)$ is compatible with zero, for all k_t . As a consequence, it is not possible to estimate the particle emission time from $(R_{t_{out}}^2 = R_{t_{side}}^2);$

 $^{^{3}}$ The HESSE algorithm in MINUIT calculates the error matrix inverting the matrix of the second derivatives of the t function with respect to the t parameters.



F igure 6: Best- t param eters of the Bertsch-Pratt param eterization, Eq. (7), to the correlation function C⁰(Q \cdot ;Q $_{t_{side}}$;Q $_{t_{out}}$), as a function of k_t , for di erent intervals of rapidity jY j. The correlation functions were measured in the LCM S fram e. Horizontal bars represent bin widths and vertical bars include both statistical and system atic errors. (a) the normalization factor N; (b) the incoherence parameter ; (c) the cross term $R^2_{long,t_{out}}$; (d) the parameter $R^2_{t_{out}}$; (e) the squared longitudinal correlation length R^2_{long} and (f) the squared transverse correlation length $R^2_{t_{side}}$.

the parameters $_{\rm i}$ are not negligible: the function F (Q , ;Q $_{\rm t_{side}}$;Q $_{\rm t_{out}}$) typically di ers from unity for atm ost 15% to 20% at Q $_{\rm i}$ 1 G eV .

5.2 Yano-Koonin ts

Table 2 and Fig. 7 show the param eters of the YK ts, Eq. (8), in di erent j' jand k_t intervals. Error bars in Fig. 7 include both statistical and system atic uncertainties, added in quadrature. It can be seen that:



Figure 7: Best-t parameters of the Yano-Koonin parameterization, Eq. (8), to the correlation function $C^{0}(q_{t};q_{t};q_{0})$, as a function of k_{t} , for dimensional error intervals of rapidity jY j. The correlation functions were measured in the event centre-of-mass frame. Horizontal bars represent bin widths and vertical bars include both statistical and systematic errors. (a) the normalization factor N; (b) the parameter ; (c) the source velocity v; (d) the time parameter R_{0}^{2} ; (e) the squared longitudinal correlation length R_{t}^{2} and (f) the squared transverse correlation length R_{t}^{2} .

the parameter is almost independent of rapidity and increases with $k_{\rm c}$, reaching values of about 0.5 for the largest $k_{\rm t}$ values. It is however signi cantly anticorrelated with the parameter N , the correlation coe cient increasing in absolute value from about 0:50 at low $k_{\rm t}$ up to 0:80 for $k_{\rm t} >$ 0:4 GeV ;

both R_t^2 and R^2 decrease with increasing k_t and jt j. The longitudinal radii are larger than the transverse radii. This agrees with an expanding, longitudinally elongated source;

 R_0^2 is compatible with zero at high rapidities, and assumes negative values for j' j < 1.6. This excludes an interpretation of R₀=c in terms of the time duration of the particle

em ission process;

those of the parameters $_{i}$ which are not negligible, contribute typically 10% to 15% to the function F ($q_{t};q_{i};q_{b}$) at large q_{i} ;

the source velocity v does not depend on k_{ℓ} , but it is strongly correlated with the pair rapidity.

The dependence of v on $\frac{1}{2}$ jcan also be presented [30,4] in term s of a plot a la G IBS, i.e. the Yano-K oon in rapidity

$$Y_{YK} = \frac{1}{2} \ln \frac{1+v}{1-v}$$
 (9)

as a function of the pair rapidity jY j. Y_{YK} measures the rapidity of the source element with respect to the centre-of-mass frame: a non-expanding source would therefore correspond to $Y_{YK} = 0$ for any jY j. On the other hand, for a boost-invariant source ⁴, the strict correlation $Y_{YK} = jY$ j is expected [13, 27], since only the source elements which move with velocities close to the velocity of the observed particle pair contribute to the correlation function. In Fig. 8 the Yano-K conin rapidity Y_{YK} is shown as a function of the pair rapidity. Since in a given jY j interval the parameter v is alm ost independent of k_t , see Fig. 7(c), each Y_{YK} is computed, according to Eq. (9), using the average value of v over all k_t in that jY jbin. Each jY jhas been computed as the weighted average of the corresponding bin, rather than the centre of the bin. A clear positive correlation between Y_{YK} and jY j is observed, even if $Y_{YK} < jY$ jat the largest pair rapidities. This is in agreement with a pion source which is emitting particles in a nearly boost-invariant way.

To try to understand the YK t results of the parameter R_0^2 , it is useful to analyse the twodimensional projection $(q_i;q_0)$ of the correlation function $C^0(q_t;q_i;q_0)$ after the longitudinal boost to the rest frame of the source element. We then introduce $q_{i}^{\text{boost}} =$ (œ vq_0) and $q_0^{\text{boost}} =$ (q vq), where the best-t parameter v is used to boost the variables. In Fig. 9(a) the two-dimensional $(\dot{p}_{i}^{\text{boost}} j, \dot{p}_{b}^{\text{boost}} j)$ projection of C⁰ is presented. The phase space available to \dot{g}_0^{boost} j is lim ited, when q_t approaches 0, and the one-dimensional \dot{g}_0^{boost} j projection (Fig. 9(b)) is approximately at: it is not possible to distinguish any peak due to Bose-E instein correlations and, for most rapidity and k_t intervals, the tted R_0^2 have negative values. In Fig. 9(b), the solid line shows the one-dimensional $j_0^{\text{boost}} j \text{ projection}$ ($j_c^{\text{boost}} j < 0.2 \text{ GeV}$, $q_t < 0.2 \text{ GeV}$) of the YK t, Eq. (8); the line is an increasing function of q_0^{boost} , because of the negative value of R_0^2 . Sim ilar limitations in the temporal acceptance have been reported in the literature [28]. On the other hand, the $j_{\mu}^{\text{boost}} j \text{ projection for } j_{\mu}^{\text{boost}} j < 0.2 \text{ GeV}$ and $q_{t} < 0.2 \text{ GeV}$, Fig. 9(c), shows a clear BEC peak at sm all jp^{boost} j, reproduced by the one dimensional jp^{boost} j projection of the best-tYK function (solid line).

5.3 Systematic e ects

The system atic uncertainties of the t parameters and the stability of the results concerning the dependence of the transverse and longitudinal radii on k_t was studied by considering a number of changes with respect to the reference analysis. The following changes were taken into account:

⁴ A source expands boost-invariantly in the longitudinal direction if the velocity of each element is given by v = z=t, where t and z are, respectively, the time elapsed since the collision and the longitudinal coordinate pf the element, in the centre-of-m ass frame. In that case, particle em ission happens at constant proper times $\frac{t^2}{t^2} = z^2$.



Figure 8: The Yano-Koonin rapidity Y_{YK} plotted versus the pion pair rapidity j' j. Each j' j was computed as the weighted average of the corresponding bin. Y_{YK} values were computed by means of Eq. (9), using the average value of v over all k_t in that j' j bin. Horizontal bars are rm s. deviations from the average. Vertical bars include both statistical and system atic errors. A loo shown is the line $Y_{YK} = j'$ j, corresponding to a source which expands boost-invariantly.

A correction was applied to the correlation functions, based on the G am ow factors &1], in order to take into account nal-state C oulom b interactions between charged pions.

The analysis was repeated with more stringent cuts in the selection: a maximum momentum of 30 GeV instead of 40 GeV and a charge unbalance smaller than 0.25 per event instead of 0.4.

The tswere repeated changing the upper bound of the trange from $1 \, \text{GeV}$ to $0.8 \, \text{GeV}$.

In the cases listed above, we found negligible di erences in the parameters with respect to the reference analysis. The system atic e ect on the correlation function C 0 , due to the M onte C arbo m odelling, was assumed negligible.

The correlation functions were measured in bins of 60 M eV, instead of 40 M eV, to test the stability of the ts. B in widths larger than 60 M eV would prevent a correct reconstruction of the BEC peak, which is about 300 400 M eV wide.

Possible non-G aussian shapes of the correlation functions at low q were tested replacing the G aussian functions in the BP and YK param eterizations with rst order Edgeworth expansions [32] of the G aussian. The 2 /D oF of the two tswere found to be comparable.



Figure 9: (a) The two-dimensional projection $(\dot{p}_{r}^{\text{boost}} j; \dot{p}_{0}^{\text{boost}} j)$, after the longitudinal boost to the source element rest frame, measured for pion pairs in the rapidity interval 0.8 \dot{j} j < 1.6 and with mean transverse momenta in the range 0.3 GeV $k_{t} < 0.4$ GeV. The projection was made requiring $q_{t} < 0.2$ GeV.

(b) The one-dimensional projection in $j_0^{\text{boost}} j$ ($j_0^{\text{boost}} j < 0.2 \text{ GeV}$). The curve is the onedimensional projection of the Yano-Koonin three-dimensional best-t function.

(c) The one-dimensional projection in $\dot{p}_{1}^{\text{boost}}$ j ($\dot{p}_{0}^{\text{boost}}$ j < 0.2 GeV). The curve is the onedimensional projection of the Yano-Koonin three-dimensional best-t function.

System atic errors on the t parameters have been computed adding in quadrature the deviations from the standard t; they are reported in Tables 1 and 2.

A ssum ing sim ple linear dependences of the squared BP and YK longitudinal and transverse radii on k_t , we measured the slopes, $dR_i^2 = dk_t$, by minimum ² ts. Fits were performed on the radii of the reference analysis, with statistical errors only. The systematic errors on the slopes were then estimated comparing the slopes from the reference analysis with the slopes from the systematic checks listed above. Table 3 shows the best-t slopes with errors. In all cases a decrease of the radii with increasing k_t is favoured even if, in one rapidity interval, the longitudinal BP radius is compatible with independence on k_t .

To investigate further the decrease of the radii on k_t , the YK and BP functions were tted to the correlation function C, Eq. (1). Larger (about 30%) squared transverse and longitudinal

radii with respect to the correlation function C⁰ are obtained in this case. However, the slopes of the linear dependences of the squared radii on k_t are the same, within uncertainties, for C and C⁰. A comparison of the YK best-t parameters from minimizing ² values and from maximizing a likelihood function [33] has been done for the correlation function C. The di erences between the parameters tted with the two techniques were negligible.

O nem ore check was done on the YK transverse radius R_t : we computed the one-dimensional projection $C^0(q_t;q;q_0)$ of the three-dimensional correlation function $C^0(q_t;q;q_0)$, by requiring q_0 and $q_0 = 0.08$ GeV, and we tted the function

$$C^{0}(q_{t}) = N (1 + e^{q_{t}^{2}R_{t}^{2}})$$
 (10)

to the projection. We rst checked that the best-tR $_t^2$ is compatible, within errors, to the one we obtain if the right-hand side of Eq. (10) is multiplied by a \long-range" factor (1 + $_tq_t$). Based on the same one-dimensional projection C $^0(q_t;0;0)$, we also measured the transverse correlation length in a t-independent way [34], introducing the parameter R $_t$

$$\vec{R}_{t} = q \frac{1}{2hq_{t}^{2}i} \quad \text{where} \quad hq_{t}^{2}i = \frac{\prod_{k=1}^{N} q_{t}^{2} \left[C^{0}(q_{t};0;0) \ 1 \ hq_{t}\right]}{\left[C^{0}(q_{t};0;0) \ 1 \ hq_{t}\right]}$$
(11)

i.e. the inverse variance of the correlation function for small q_t values ⁵. We found that R_t , computed using Eq. (11), agrees with the best-t R_t from Eq. (10); the slope of the linear decrease is about 20% smaller than the one measured with three-dimensional YK ts, Eq. (8).

The standard analysis was also repeated for a subsample of events classi ed as two-jets by the D urham jet- nding algorithm [35]. The subsample was dened by setting the resolution parameter at $y_{cut} = 0.04$. The dependences of the best-t parameters on jY jand k_t are similar to those found for the inclusive sample of events. In particular, the longitudinal and the transverse radii decrease with increasing k_t . How ever, the radii measured in the case of two-jet events are smaller, by about 10%, than in the inclusive sample [5]. An increase of the \jettyness" of the two-jet subsample, obtained using a smaller y_{cut} ($y_{cut} = 0.02$) in the jet-nding algorithm, does not change signi cantly the behaviour of the parameters.

5.4 Comparison between BP and YK ts

The following relations should hold between the correlation lengths of the BP and YK functions measured in the LCM S and CM S frames, respectively [13]:

$$R_{t_{side}}^2 = R_t^2$$
(12)

$$R_{long}^{2} = \frac{2}{LCMS} (R_{*}^{2} + \frac{2}{LCMS} R_{0}^{2})$$
(13)

$$(R_{t_{out}}^2 R_{t_{side}}^2) = {}^{2}_{t LCMS} (R_0^2 + {}^{2}_{LCMS} R_{\ell}^2):$$
(14)

In Eq. (13) and (14) $_{\text{LCM S}}$ is the velocity of the source element measured in the LCM S, i.e. with respect to the pair longitudinal rest frame; $_{\text{LCM S}} = 1 = 1 - \frac{2}{1 - \frac{2}{L_{\text{CM S}}}}$. In Eq. (14) $_{\text{t}}^2 = \frac{2k_t}{E_1 + E_2}$, where the brackets stand for the average over all pion pairs in the given $\frac{1}{2}$ jand k_t range. For a boost-invariant source, $_{\text{LCM S}} = 0$ and Eqs. (13) and (14) reduce to:

$$R_{long}^2 = R_{\prime}^2$$
(15)

$$(\mathbf{R}_{t_{out}}^2 \quad \mathbf{R}_{t_{side}}^2) = {}_{t}^2 \mathbf{R}_{0}^2 :$$
(16)

⁵ In the actual estimate of h_{t}^{2} i we have computed $\frac{p q_{t}^{2} [C^{0}(q_{t};0;0) N]}{[C^{0}(q_{t};0;0) N]}$, where N is the normalization parameter of the tEq. (10) and each q_{t} has been taken as the central value of the corresponding 40 M eV bin.



Figure 10: (a) (d) (g) The best-t longitudinal radius R_{long}^2 of the Bertsch-Pratt param eterization (open dots) compared with the Yano-Koonin longitudinal radius R_{τ}^2 (full dots). (b) (e) (h) The BP transverse correlation length $R_{t_{side}}^2$ (open dots) compared with the YK transverse correlation length R_{t}^2 (full dots). (c) (f) (i) The di erence of the BP transverse radii ($R_{t_{out}}^2 = R_{t_{side}}^2$) (open dots) compared with the YK time parameter R_0^2 times $\frac{2}{t}$ (full dots). Errors on the parameters include both statistical and system atic uncertainties, added in quadrature.

In Fig. 10 the best-tBP parameters R_{long}^2 , $R_{t_{side}}^2$ and $(R_{t_{out}}^2 - R_{t_{side}}^2)$ are compared with the YK parameters R_{t}^2 , R_{t}^2 and ${}_{t}^2 R_{0}^2$.

The longitudinal parameter R_{long}^2 is system atically larger than R_{i}^2 in all the rapidity intervals analyzed (Fig. 10(a), (d) and (g)). According to Eq. (13), $R_{long}^2 > R_{i}^2$ corresponds to $_{LCMS}$ greater than zero, in agreement with a pion source whose expansion is not exactly boost-invariant.

The equality of the transverse parameters $R_{t_{side}}^2$ and R_t^2 , Eq. (12), is con rm ed within errors, with possible deviations at low k_t (Fig. 10(b), (e) and (h)).

The negative values of R_0^2 and $(R_{t_{out}}^2 - R_{t_{side}}^2)$ appearing in the two rst rapidity intervals (Fig. 10(c), (f) and (i)) prevent an interpretation in terms of the time duration of the particle emission process. Negative values of R_0^2 have been suggested [36] as possible indicators for opacity of the source, i.e. surface dominated emission. A dependence of $(R_{t_{out}}^2 - R_{t_{side}}^2)$ on k_t similar to the one shown in Fig. 10(c) and (f) has been reported in heavy-ion collision experiments [37].

6 Conclusions

An analysis of B ose-E instein correlations in e^+e^- annihilation events at the Z⁰ peak performed in bins of the average 4-m omentum of the pair, K, has been presented for the rst time. B ased on this, dynamic features of the pion emitting source were investigated. Previous BEC analyses, not dierential in K, were not sensitive to these features.

Using the Yano-Koonin and the Bertsch-Pratt form alisms, the correlation functions were studied in intervals of two components of K: the pion pair rapidity j' jand them can transverse momentum k_t . We found that the transverse and longitudinal radii of the pion sources decrease for increasing k_t , indicating the presence of correlations between the particle production points and their momenta. The Yano-Koonin rapidity scales approximately with the pair rapidity, in agreement with a nearly boost-invariant expansion of the source of pions. Limitations in the available phase space did not allow measurement of the duration of the particle emission process.

Sim ilar results have been observed in m ore complex systems, such as the pion sources created in pp and heavy-ion collisions, which are now complemented with such measurements in the simpler hadronic system formed in e^+e^- annihilations.

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References

- [1] TASSO Coll., D. A ltho et al., Z. Phys. C 30 (1986) 355;
 AM Y Coll., S.K. Choiet al., Phys. Lett. B 355 (1995) 406.
- [2] ABCDHW Coll., A. Breakstone et al., Phys. Lett. B162 (1985) 400;
 UA1 Coll., C. A lbajar et al., Phys. Lett. B226 (1989) 410.
- [3] W A 25 Coll., D. A llasia et al., Z. Phys. C 37 (1988) 527;
 H1 Coll., C. Adlo et al., Z. Phys. C 75 (1997) 437;
 ZEUS Coll., S. Chekanov et al., Phys. Lett. B 583 (2004) 231.
- [4] E877 Coll., J. Barrette et al., Nucl. Phys. A 610 (1996) 227c;
 NA 49 Coll., H. Appelshauser et al., Eur. Phys. J. C 2 (1998) 661;
 W A 97 Coll., F. Antinori et al., J. Phys. G 27 (2001) 2325;
 PHOBOS Coll., B. B. Back et al., Phys. Rev. C 73 (2006) 031901.
- [5] OPAL Coll., P.D. Acton et al., Phys. Lett. B 267 (1991) 143;OPAL Coll., G. A lexander et al., Z. Phys. C 72 (1996) 389.
- [6] L3 Coll., P. A chard et al., Phys. Lett. B 524 (2002) 55;O PA L Coll., G. A bbiendi et al., Phys. Lett. B 559 (2003) 131.
- [7] OPAL Coll, R. Akers et al., Z. Phys. C67 (1995) 389;
 DELPHIColl, P. Abreu et al., Phys. Lett. B379 (1996) 330;
 OPAL Coll, G. Abbiendi et al., Eur. Phys. J. C21 (2001) 23.
- [8] O PAL Coll., K. Ackersta et al., Eur. Phys. J. C 5 (1998) 239.
- [9] O PAL Coll., G. Abbiendi et al., Phys. Lett. B 523 (2001) 35.
- [10] OPAL Coll, G. Abbiendi et al, Eur. Phys. J. C16 (2000) 423.
- [11] L3 Coll., M. Acciarri et al., Phys. Lett. B 458 (1999) 517;
 DELPHIColl., P. Abreu et al., Phys. Lett. B 471 (2000) 460;
 ALEPH Coll., D. Abbaneo et al., Eur. Phys. J. C 36 (2004) 147.
- [12] L3 Coll., P. A chard et al., Phys. Lett. B 547 (2002) 139; OPAL Coll., G. Abbiendi et al., Eur. Phys. J. C 36 (2004) 297; ALEPH Coll., D. Abbaneo et al., Phys. Lett. B 606 (2005) 265; D ELPH I Coll., J. Abdallah et al., Eur. Phys. J. C 44 (2005) 161.
- [13] U.Heinz, Nucl. Phys. A 610 (1996) 264.
- [14] U.W iedem ann, P. Scotto and U.Heinz, Phys. Rev. C 53 (1996) 918.
- [15] Z.Chajecki (STAR Coll.), Nucl. Phys. A 774 (2006) 599.
- [16] M G Bow ler, Z. Phys. C 29 (1985) 617;
 B.Andersson and W .Hofm ann, Phys. Lett. B169 (1986) 364;
 U.Heinz and B.V. Jacak, Ann. Rev. Nucl. Part. Sci. 49 (1999) 529;
 K.Geiger et al., Phys. Rev. D 61 (2000) 054002.

- [17] G.Alexander, Phys. Lett. B 506 (2001) 45;G.Alexander, Rept. Prog. Phys. 66 (2003) 481.
- [18] T.C sorg and J.Zim anyi, Nucl. Phys. A 517 (1990) 588;
 A.Bialas et al., Phys. Rev. D 62 (2000) 114007;
 A.Bialas et al., A cta Physica Polonica B 32 (2001) 2901.
- [19] OPAL Coll., K. Ahm et et al., Nucl. Instr. and M ethods A 305 (1991) 275.
- [20] P.P. A llport et al., Nucl. Instr. and M ethods A 324 (1993) 34; Nucl. Instr. and M ethods A 346 (1994) 476.
- [21] T.C sorgo and S.Pratt, Proc.W orkshop on Relativistic Heavy Ion Physics at Present and Future Accelerators, eds. T.C sorgo et al., KFK I-1991-28/A, (KFK I, Budapest, 1991), p. 75.
- [22] S.Chapman, P.Scotto and U.Heinz, Phys. Rev. Lett. 74 (1995) 4400.
- [23] T.Sjostrand, Comp.Phys.Comm. 39 (1986) 347; T.Sjostrand and M Bengtsson, Comp.Phys.Comm. 43 (1987) 367; Comp.Phys.Comm. 82 (1994) 74.
- [24] J.Allison et al., Nucl. Instr. and Methods A 317 (1992) 47.
- [25] O PA L Coll., G. A lexander et al., Z. Phys. C 69 (1996) 543.
- [26] S.Pratt, Phys. Rev. D 33 (1986) 1314;G.Bertsch, M.Gong and M.Tohyama, Phys. Rev. C 37 (1988) 1896.
- [27] F.Yano and S.Koonin, Phys. Lett. B78 (1978) 556;
 M J.Podgoretsky, Sov.J.Nucl.Phys. 37 (1983) 272;
 S.Chapman, J.Rayford Nix and U.Heinz, Phys. Rev. C 52 (1995) 2694.
- [28] B. Tom asik and U. Heinz, Acta Physica Slovaca 49 (1999) 251;
 K. Morita et al., Phys. Rev. C 61 (2000) 034904.
- [29] F. James, CERN Program Library Long W riteup D 506, CERN, 1994.
- [30] G IBS Coll., M Kh. Anikina et al., Phys. Lett. B 397 (1997) 30;
 M Kh. Anikina et al., Phys. Atom. Nucl. 67 (2004) 406.
- [31] M.Gyulassy et al., Phys. Rev. C 20 (1979) 2267.
- [32] T.C sorgo and S.Hegyi, Phys. Lett. B489 (2000) 15.
- [33] E-802 Coll, L.Ahle et al., Phys. Rev. C66 (2002) 054906.
- [34] U.A.W iedem ann and U.Heinz, Phys.Rev.C56 (1997) 3265.
- [35] S.Cataniet al, Phys. Lett. B 269 (1991) 432.
- [36] H. Heiselberg and A. P. Vischer, Eur. Phys. J. C1 (1998) 593.
- [37] STAR Coll, C.Adler et al., Phys. Rev. Lett. 87 (2001) 082301; PHENIX Coll, K.Adcox et al., Phys. Rev. Lett. 88 (2002) 192302.

0.0 j¥j< 0.8	$0.1 k_t < 0.2 G eV$	$0.2 k_t < 0.3 G eV$	0.3 $k_t < 0.4 \text{ G eV}$	$0.4 k_t < 0.5 G eV$	0.5 $k_t < 0.6 \text{ GeV}$		
N	0 : 974 0 : 003 0 : 057	0:996 0:004 0:042	1:011 0:004 0:040	1:003 0:007 0:040	1:016 0:009 0:052		
	0:286 0:011 0:067	0:364 0:009 0:061	0:429 0:012 0:047	0:398 0:013 0:044	0:337 0:016 0:063		
$R_{t_{out}}^2$ (fm ²)	0 : 60 0 : 07 0 : 18	0:36 0:03 0:10	0:294 0:020 0:079	0 : 174 0 : 013 0 : 050	0:169 0:014 0:051		
$R_{t_{side}}^{2}$ (fm ²)	0:50 0:03 0:14	0:38 0:02 0:11	0:37 0:02 0:11	0:30 0:02 0:10	0:22 0:03 0:10		
R_{long}^{2} (fm ²)	1:09 0:11 0:37	0:72 0:04 0:17	0:82 0:05 0:16	0 : 60 0 : 04 0 : 15	0 : 75 0 : 06 0 : 18		
$R_{long tout}^2$ (fm ²)	0:06 0:08 0:14	0:020 0:036 0:037	0:065 0:028 0:031	0:023 0:022 0:007	0:121 0:024 0:065		
tout (G eV ¹)	0:091 0:004 0:060	0:056 0:004 0:035	0:037 0:004 0:027	0:016 0:005 0:018	0:003 0:006 0:018		
t_{side} (G eV 1)	0:123 0:004 0:071	0:130 0:004 0:061	0:140 0:004 0:071	0:18 0:01 0:10	0:24 0:01 0:13		
$_{ m long}$ (G eV $^{ m 1}$)	0:081 0:004 0:023	0:048 0:004 0:017	0:019 0:005 0:015	0:016 0:007 0:019	0:018 0:008 0:038		
² /D oF	16389=15617	16080=15617	15596=15617	15864=15617	15439=15617		
0.8 jYj< 1.6							
N	0:972 0:003 0:049	0:990 0:004 0:075	1:017 0:005 0:052	1:019 0:007 0:057	1:024 0:010 0:066		
	0:315 0:008 0:070	0:386 0:008 0:064	0 : 393 0 : 011 0 : 053	0:379 0:013 0:055	0:318 0:016 0:062		
$R_{t_{out}}^2$ (fm ²)	0:62 0:04 0:20	0:38 0:02 0:11	0 : 271 0 : 016 0 : 079	0:204 0:014 0:062	0:141 0:015 0:053		
R_{teide}^{2} (fm ²)	0:52 0:03 0:13	0:38 0:02 0:11	0:34 0:02 0:10	0:272 0:021 0:081	0:226 0:026 0:079		
R_{long}^{2} (fm ²)	1:06 0:08 0:35	0:71 0:04 0:16	0:65 0:04 0:15	0 : 64 0 : 05 0 : 16	0:50 0:05 0:12		
$R_{long;t_{out}}^2$ (fm ²)	0:019 0:055 0:076	0:029 0:026 0:036	0:036 0:023 0:025	0:061 0:022 0:045	0:034 0:021 0:042		
tout (GeV ¹)	0:070 0:004 0:049	0:046 0:004 0:035	0:033 0:004 0:033	0:015 0:005 0:027	0:007 0:007 0:009		
$_{t_{side}}$ (G eV 1)	0:106 0:004 0:059	0:104 0:004 0:051	0:131 0:005 0:064	0:161 0:005 0:088	0:23 0:01 0:14		
$_{ m long}$ (G eV $^{ m 1}$)	0:066 0:004 0:026	0:035 0:004 0:026	0:003 0:005 0:036	0:028 0:006 0:047	0:060 0:009 0:072		
² /D oF	15856=15617	15745=15617	15658=15617	15895=15617	15592=15617		
1.6 j¥j< 2.4							
Ν	0:991 0:003 0:082	1:019 0:005 0:069	1:066 0:005 0:078	1:055 0:008 0:074	1:07 0:01 0:10		
	0:261 0:008 0:079	0:307 0:008 0:072	0:299 0:011 0:065	0:264 0:014 0:074	0:24 0:02 0:11		
$R_{t_{out}}^2$ (fm ²)	0:54 0:04 0:19	0:35 0:02 0:11	0:35 0:03 0:10	0:219 0:017 0:064	0:25 0:03 0:11		
$R_{t_{side}}^{2}$ (fm ²)	0 : 53 0 : 03 0 : 15	0:34 0:02 0:10	0 : 279 0 : 023 0 : 077	0:229 0:026 0:072	0:169 0:034 0:085		
R_{long}^{2} (fm ²)	1:13 0:09 0:41	0:61 0:04 0:17	0:78 0:06 0:19	0 : 62 0 : 05 0 : 17	0 : 54 0 : 07 0 : 13		
$R_{long;t_{out}}^2$ (fm ²)	0:05 0:05 0:13	0:012 0:029 0:033	0:137 0:033 0:076	0:148 0:024 0:077	0:09 0:04 0:11		
tout (GeV 1)	0:102 0:004 0:070	0:063 0:004 0:048	0:060 0:005 0:043	0:027 0:006 0:028	0:16 0:01 0:12		
t _{side} (G eV ¹)	0:134 0:004 0:079	0:130 0:004 0:068	0:167 0:005 0:082	0:19 0:01 0:11	0:31 0:01 0:17		
$_{\rm long}$ (G eV 1)	0:045 0:004 0:056	0:003 0:005 0:045	0:046 0:005 0:053	0:078 0:007 0:072	0:15 0:01 0:13		
² /D oF	15966=15617	15866=15617	15735=15617	15235=15617	15279=15617		

Table 1: Results of the Bertsch-Pratt ts, Eq. (7), to the experimental three-dimensional correlation functions C⁰(Q,;Q_{t_side};Q_{tout}) over the range 0.04 Q;Q_{t_side};Q_{tout} 1.0 G eV. The rst errors are statistical and the second system atic. The quality of the ts is indicated by the value of ²/D oF, which ranges from 0.98 to 1.05.

0.0 j¥j< 0.8	0.1 k _t < 0.2 G eV	0.2 $k_t < 0.3 \text{ GeV}$	0.3 $k_t < 0.4 \text{ GeV}$	0.4 $k_t < 0.5 \text{ GeV}$	$0.5 k_t < 0.6 G eV$		
Ν	0:993 0:003 0:010	1:004 0:003 0:008	0:985 0:004 0:017	0:946 0:005 0:041	0:85 0:01 0:14		
	0:266 0:012 0:067	0:358 0:009 0:056	0:441 0:012 0:035	0:440 0:013 0:023	0:482 0:017 0:069		
V	0:10 0:19 0:32	0:288 0:042 0:018	0:320 0:030 0:013	0:249 0:034 0:010	0:211 0:031 0:031		
R_0^2 (fm 2)	0:52 0:20 0:16	0 : 184 0 : 044 0 : 075	0:226 0:022 0:088	0:203 0:013 0:081	0:110 0:009 0:053		
R_t^2 (fm 2)	0:75 0:04 0:17	0:50 0:02 0:12	0:41 0:02 0:10	0:313 0:011 0:081	0:193 0:007 0:048		
R^2 , (fm ²)	1:15 0:15 0:34	0:74 0:04 0:13	0:66 0:03 0:11	0:46 0:02 0:10	0:31 0:02 0:11		
₀ (G eV ¹)	0:045 0:007 0:069	0:014 0:006 0:052	0:008 0:007 0:019	0:061 0:008 0:062	0:156 0:011 0:091		
$_{ m t}$ (G eV $^{ m 1}$)	0:099 0:004 0:029	0:089 0:005 0:023	0:065 0:006 0:030	0:066 0:008 0:036	0:054 0:013 0:019		
· (G eV ¹)	0:038 0:002 0:098	0:014 0:002 0:090	0:005 0:003 0:051	0:001 0:003 0:025	0:002 0:005 0:012		
² /D oF	13583=11677	16008=14375	17555=16338	18166=17554	18702=18232		
0.8 j¥j< 1.6							
Ν	0:948 0:005 0:021	0 : 977 0 : 006 0 : 006	0:964 0:009 0:010	0:914 0:012 0:051	0:859 0:022 0:072		
	0:324 0:008 0:065	0:380 0:010 0:057	0 : 425 0 : 013 0 : 035	0:464 0:019 0:023	0:464 0:034 0:028		
V	0:754 0:022 0:061	0:782 0:014 0:036	0:742 0:017 0:027	0:777 0:014 0:031	0:743 0:023 0:040		
R_0^2 (fm 2)	0:187 0:054 0:076	0:104 0:024 0:051	0:114 0:016 0:054	0:102 0:013 0:051	0:106 0:011 0:052		
R_t^2 (fm ²)	0:56 0:02 0:15	0:39 0:01 0:11	0:32 0:01 0:10	0 : 235 0 : 010 0 : 071	0:164 0:009 0:049		
R^2 , (fm ²)	0:83 0:06 0:27	0:58 0:03 0:13	0:46 0:03 0:10	0:43 0:03 0:12	0:294 0:032 0:085		
₀ (G eV ¹)	0:07 0:01 0:10	0:00 0:01 0:12	0:071 0:010 0:053	0:106 0:012 0:076	0:125 0:019 0:084		
$_{ m t}$ (G eV $^{ m 1}$)	0:068 0:007 0:041	0:075 0:008 0:033	0:069 0:011 0:043	0:046 0:015 0:036	0:079 0:027 0:079		
· (G eV ¹)	0:099 0:010 0:078	0:022 0:008 0:099	0:032 0:008 0:062	0:058 0:010 0:047	0:077 0:014 0:060		
² /D oF	8624=7139	9778=8788	11004=9870	11365=10518	11603=10885		
1.6 jYj< 2.4							
Ν	0:899 0:016 0:054	0 : 963 0 : 020 0 : 064	0:902 0:021 0:040	0:888 0:028 0:052	0:48 0:01 0:39		
	0:342 0:019 0:078	0:354 0:022 0:072	0:454 0:012 0:041	0:438 0:038 0:030	1:26 0:04 0:63		
V	0:893 0:012 0:044	0 : 931 0 : 008 0 : 047	0:927 0:009 0:033	0:912 0:012 0:042	0:93 0:04 0:11		
R_0^2 (fm ²)	0:031 0:043 0:029	0:006 0:032 0:022	0:015 0:026 0:025	0:020 0:030 0:026	0:034 0:024 0:019		
R_{t}^{2} (fm ²)	0:39 0:02 0:12	0 : 291 0 : 017 0 : 087	0:172 0:011 0:052	0:159 0:014 0:048	0:071 0:005 0:021		
R^{2} (fm ²)	0:42 0:05 0:16	0:42 0:05 0:15	0:34 0:03 0:10	0:38 0:06 0:10	0:12 0:03 0:11		
₀ (G eV ¹)	0:03 0:04 0:12	0:060 0:029 0:080	0:014 0:033 0:021	0:196 0:031 0:075	0:00 0:03 0:16		
t (G eV ¹)	0:033 0:023 0:082	0:087 0:024 0:093	0:030 0:009 0:050	0:070 0:032 0:055	0:216 0:031 0:071		
· (G eV ¹)	0:001 0:036 0:066	0:049 0:028 0:081	0:025 0:030 0:054	0:162 0:029 0:065	0:02 0:03 0:16		
² /D oF	4168=3804	5110=4648	5952=5169	5775=5490	5876=5642		

Table 2: Results of the Yano-Koonin ts, Eq. (8), to the experimental three-dimensional correlation functions $C^{0}(q_{\rm L};q_{\rm r};q_{\rm D})$ over the range 0.04 $q_{\rm L};q_{\rm r};q_{\rm D}$ 1.0 GeV. The rst errors are statistical and the second systematic. The quality of the ts is indicated by the value of 2 /D oF, which ranges from 1.03 to 1.20.

	BP radii						YK radii						
		$dR_{long}^2 = dk_t$ (fm $^2/G eV$)		$dR_{t_{side}}^2 = dk_t$ (fm 2 /G eV)		$dR^2 = dk_t$ (fm $^2/G eV$)		$dR_t^2 = dk_t$ (fm 2 /G eV)					
jij< 0.8		0 : 46	0:20	0:35	0 : 59	0 : 08	0:19	1:60	0:13	0:38	1:14	0 : 05	0:23
0.8	j⁄rj< 1.6	0:91	0:18	0:30	0 : 66	0 : 08	0 : 15	1:04	0:12	0:23	0:84	0:04	0:15
1.6	j⁄rj< 2.4	0 : 64	0:21	0:36	0 : 80	0:09	0:28	0:82	0:13	0 : 17	0 : 70	0:04	0:20

Table 3: Slopes of the linear ts to the dependence of the longitudinal and transverse squared radii of the BP and YK param eterizations on k_t . Input to the ts are the measured values of R_{long}^2 , $R_{t_{side}}^2$, R_t^2 and R_t^2 , reported in Tables 1 and 2. The rst errors are statistical and the second system atic.