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Conformal windows of SU(N) gauge theories, higher dimensional representations, and the size of the unparticle world

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We present the conformal windows of SU(N) supersymmetric and nonsupersymmetric gauge theories with vectorlike matter transforming according to higher irreducible representations of the gauge group. We determine the fraction of asymptotically free theories expected to develop an infrared fixed point and find that it does not depend on the specific choice of the representation. This result is exact in supersymmetric theories while it is an approximate one in the nonsupersymmetric case. The analysis allows us to size the unparticle world related to the existence of underlying gauge theories developing an infrared stable fixed point. We find that exactly 50% of the asymptotically free theories can develop an infrared fixed point while for the nonsupersymmetric theories it is circa 25%. When considering multiple representations, only for the nonsupersymmetric case, the conformal regions quickly dominate over the nonconformal ones. For four representations, 70% of the asymptotically free space is filled by the conformal region. According to our theoretical landscape survey the unparticle physics world occupies a sizable amount of the particle world, at least in theory space, and before mixing it (at the operator level) with the nonconformal one.

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I. INTRODUCTION

Recently we have completed the analysis of the phase diagram of asymptotically free nonsupersymmetric gauge theories with two Dirac fermions in a single arbitrary representation of the gauge group as function of the number of flavors and colors [1,2]. The phase diagram is sketched in Fig. 2 with the exceptions of a few isolated higher dimensional representations below nine colors [2]. The analysis exhausts the phase diagram for gauge theories with Dirac fermions in a single generic representation and is based on the ladder approximation presented in [3,4]. Further studies of the nonsupersymmetric conformal window and its properties can be found in [5–10]. The adjoint and the two-index symmetric representations need only a very low number of flavors, almost independent of the number of colors, to be near an infrared fixed point. This fact has led to the construction of the *minimal walking technicolor* theories [1,2,11]. The walking dynamics was first introduced in [12–18]. By walking one refers to the fact that the underlying coupling constant decreases much more slowly with the reference scale than in the case of QCD-like theories. The theoretical estimates for the nonsupersymmetric conformal window need to be tested further. The very low number of flavors needed to reach the conformal window, for certain representations, makes the minimal walking theories amenable to lattice investigations. Recent lattice results [19] show that the theory with two Dirac fermions in the adjoint representation of the SU(2) gauge group possesses dynamics which is different from the one with fermions in the fundamental representation.

Here, we study the conformal window of SU(N) supersymmetric gauge theories with vectorlike matter transforming according to a single but generic irreducible representation of the gauge group. The results are subsequently confronted with the nonsupersymmetric ones. We compute the fraction, for each representation, of asymptotically free theories in the flavor-color space which can develop an infrared fixed point. We find this fraction to be $1/2$ and at the same time to be a universal number independent of the specific representation. Intrigued by this result we compute it in the nonsupersymmetric case as well. Here we find the value 0.25. Although there is some dependence on the representation the differences among the various representations are still small.

Another interesting application of our work is as a study of the theoretical landscape underlying the *unparticle* physics world proposed by Georgi [20,21]. With emphasis on the phenomenological applications, studies of the unparticle physics have recently received much attention [22]. An interesting theoretical and phenomenological study of the *CP* and CPT properties of unparticle physics has been performed in [23].

The theories presented here, belonging to the various conformal regions, are natural candidates for a *particle* theory description of the unparticle world following [23–26].

Our analysis allows us to *size* the unparticle world related to the existence of underlying gauge theories developing an infrared fixed point. As already reported above,

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with only one type of representation, in the supersymmetric case, we find that 50% of the theories can develop an infrared fixed point while for the nonsupersymmetric theories this conformal area is about 25% of that of all total asymptotically free ones.

We expect this fraction to increase when considering multiple representations simultaneously present. In this case the conformal regions will quickly dominate over the nonconformal ones. In order to estimate this amount we considered the case of multiple representations for the nonsupersymmetric case. Here we find that with four different simultaneously present representations, in the nonsupersymmetric case, about 70% of the space is filled by theories which can develop a fixed point. We have investigated gauge theories but it would be interesting to also study quantum gravity theories where the role of the infrared fixed point is replaced by the possible existence of a nontrivial ultraviolet fixed point (asymptotic safety) [27–31].

According to our theoretical landscape survey the unparticle world, before coupling it to the standard model, is at least as common as the particle one.

II. CONFORMAL WINDOW FOR SUPERSYMMETRIC GAUGE THEORIES WITH MATTER IN HIGHER DIMENSIONAL REPRESENTATIONS

The gauge sector of a supersymmetric SU(N) gauge theory consists of a supersymmetric field strength belonging to the adjoint representation of the gauge group. The supersymmetric field strength describes the gluon and the gluino. The matter sector is taken to be vectorial and to consist of N_f chiral superfields Φ in the representation r of the gauge group and N_f chiral superfields $\tilde{\Phi}$ in the conjugate representation $\bar{r}$ of the gauge group. The chiral superfield Φ (or $\tilde{\Phi}$) contains a Weyl fermion and a complex scalar boson.

The generators T_r^a, $a = 1 \ldots N^2 - 1$ of the gauge group in the representation r are normalized according to $\mathrm{Tr}[T_r^a T_r^b] = T(r)\delta^{ab}$ while the quadratic Casimir $C_2(r)$ is given by $T_r^a T_r^a = C_2(r)I$. The trace normalization factor $T(r)$ and the quadratic Casimir are connected via $C_2(r)d(r) = T(r)d(G)$ where $d(r)$ is the dimension of the representation r. The adjoint representation is denoted by G. With this notation we summarize the symmetries of the theory in Table I. The first SU(N) is the gauge group. The two Abelian symmetries are anomaly free with the first one being the baryon number and the second one an R symmetry. Note that the global symmetry is enhanced from $\mathrm{SU}(\mathrm{N}_f) \times \mathrm{SU}(\mathrm{N}_f) \times \mathrm{U}(1)_B$ to $\mathrm{SU}(2\mathrm{N}_f)$ when the representation for the matter field is (pseudo)real.

The exact beta function of supersymmetric QCD was first found in [32,33] and further investigated in [34,35]. For a given representation it takes the form

TABLE I. Summary of the local and global symmetries and charge assignments of the generic $\mathcal{N} = 1$ gauge theory with matter in a given representation r of the gauge group.

	[SU(N)]	$\mathrm{SU}(\mathrm{N}_f)$	$\mathrm{SU}(\mathrm{N}_f)$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
Φ	r	N_f	1	1	$\frac{2T(r)N_f - C_2(G)}{2T(r)N_f}$
$\tilde{\Phi}$	$\bar{r}$	1	$\bar{N}_f$	-1	$\frac{2T(r)N_f - C_2(G)}{2T(r)N_f}$

$$\beta(g) = -\frac{g^3}{16\pi^2}\frac{\beta_0 + 2T(r)N_f\gamma(g^2)}{1 - \frac{g^2}{8\pi^2}C_2(G)}, \tag{1}$$

$$\gamma(g^2) = -\frac{g^2}{4\pi^2}C_2(r) + O(g^4), \tag{2}$$

where g is the gauge coupling, $\gamma(g^2) = -d\ln Z(\mu)/d\ln\mu$ is the anomalous dimension of the matter superfield and $\beta_0 = 3C_2(G) - 2T(r)N_f$ is the first beta function coefficient.

For a given representation the loss of asymptotic freedom manifest itself as a change of sign in the first coefficient of the beta function. The number of flavors N_f^{I} for which this occurs is

$$N_f^{\mathrm{I}} = \frac{3}{2}\frac{C_2(G)}{T(r)}. \tag{3}$$

Note that compared to the nonsupersymmetric case this value is lowered due to the additional screening of the scalars and the gluinos. In fact the coefficient $\frac{3}{2}$ should be replaced by $\frac{11}{4}$ in the nonsupersymmetric case [2].

It might be possible that an infrared fixed point exists since for a certain number of flavors and colors the one-loop coefficient of the beta function is negative while the two-loop coefficient is positive [36]. This situation appears as soon as the two-loop coefficient changes sign. For a given representation this occurs when

$$N_f^{\mathrm{III}} = \frac{C_2(G)}{T(r)}\frac{3C_2(G)}{2C_2(G) + 4C_2(r)}. \tag{4}$$

Note that N_f^{III} does not coincide, in general, with the true critical value of flavors above which a nonperturbartive infrared fixed point is generated. The latter will be determined below and will be referred to as N_f^{II}.

To show the existence of a nontrivial infrared fixed point we will consider the large N limit holding $\frac{N_f}{N_f^{\mathrm{I}}} = 1 - \epsilon$, $\epsilon \ll 1$ and Ng^2 fixed. In case of the fundamental representation it is also important to take the large N_f limit in order to have $\frac{N_f}{N_f^{\mathrm{I}}}$ fixed because the trace normalization is a constant. This is in contrast to the two-indexed representations for which the trace normalization factors grow as N. The fixed point is now given by $C_2(r)g_*^2 = -4\pi^2\epsilon + O(\epsilon^2)$ with

$C_2(r)$ growing as N both for the fundamental and two-indexed representations. The argument above cannot be applied to the case of matter in representations with more than two indices since all these theories are not asymptotically free at large number of colors. In the following we will only consider either the fundamental or the two-indexed representations.

Since a fixed point exists, at least at large N, we follow Seiberg [37] and derive some exact results about the theory. The strategy is to first obtain an exact expression for the dimension D of some spinless operator in terms of the number of colors and flavors. We will then use a property of conformal field theory stating that spinless operators (except for the identity) have $D \geq 1$ in order not to have negative norm states in the theory [38–40]. When this bound is saturated it gives us a relation between the number of colors and flavors at which our conformal description breaks down.

There are two ways to obtain the dimension of chiral operators in the theory. First we note that the superconformal algebra includes an R symmetry and find the following relation between the corresponding R charge and dimension D of the operators $D \geq |R|$. The bound is saturated for chiral operators $D = \frac{3}{2}R$ and for antichiral operators $D = -\frac{3}{2}R$. Since this R symmetry must be anomaly free and commute with the flavor symmetries, it must be the one assigned in Table I. For the spinless chiral operator $\Phi\tilde{\Phi}$ we therefore arrive at

$$D(\Phi\tilde{\Phi}) = \frac{3}{2}R(\Phi\tilde{\Phi}) = 3\frac{2T(r)N_f - C_2(G)}{2T(r)N_f}. \qquad (5)$$

Perhaps an easier way to obtain $D(\Phi\tilde{\Phi})$ is to note that at the zero of the beta function we have $\gamma = \frac{2T(r)N_f - 3C_2(G)}{2T(r)N_f}$. Hence from $D(\Phi\tilde{\Phi}) = \gamma + 2$ we end up with Eq. (5).

As discussed above our conformal description of the theory requires $D(\Phi\tilde{\Phi}) \geq 1$ with the bound being saturated by free fields. Hence the critical number of flavors above which the theory exists in a conformal phase is therefore

$$N_f^{\rm II} = \frac{3}{4}\frac{C_2(G)}{T(r)}. \qquad (6)$$

In Fig. 1 we plot the phase diagram for the supersymmetric gauge theories with matter in one of the three two-indexed representations—adjoint, two-index symmetric and two-index antisymmetric—as well as the fundamental representation. These are the representations remaining asymptotically free for any number of colors for a sufficiently low number of flavors.

In Table II, for the reader's convenience, we list the explicit group factors for the representations used here. A complete list of all of the group factors for any representation and the way to compute them is available in Table II of [2] and the associated appendix [41].

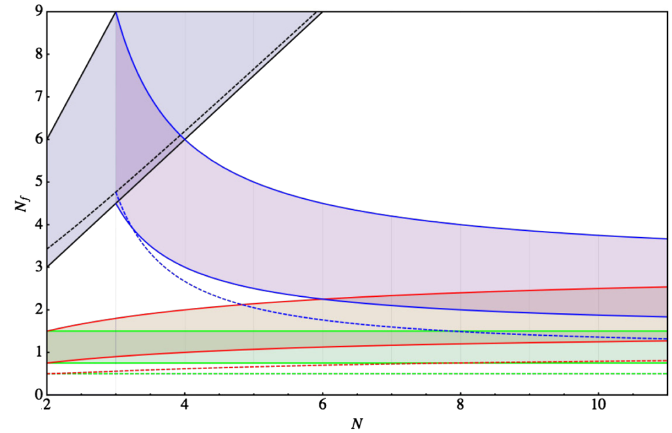

FIG. 1 (color online). Phase diagram for supersymmetric theories with fermions in the (i) fundamental representation (blue), (ii) two-index antisymmetric representation (purple), (iii) two-index symmetric representation (red), (iv) adjoint representation (green) as a function of the number of flavors and the number of colors. The shaded areas depict the corresponding conformal windows. Above the upper solid curve the theories are no longer asymptotically free. In between the upper and the lower solid curves the theories develop an infrared fixed point. The dashed curve represents the change of sign in the second coefficient of the beta function.

The supersymmetric conformal window displays many qualitative features in common with the nonsupersymmetric one which is shown in Fig. 2. Note how consistently the various representations merge into each other when, for a specific value of N, they are actually the same representation.

The nonsupersymmetric window is only an estimate which makes use of the ladder approximation. We observe that in the case of the fundamental representation the supersymmetric conformal window extends below the curve defined as where the two-loop beta function coefficient changes sign. This does not happen for the adjoint and two-index symmetric and antisymmetric representation for any N larger than 4. In the nonsupersymmetric case the curve $N_f^{\rm II}$ stays well above $N_f^{\rm III}$ for any N and any representation in the ladder approximation.

TABLE II. Relevant group factors for the representations used throughout this paper. However, a complete list of all the group factors for any representation and the way to compute them is available in Table II and the appendix of [2].

r	$T(r)$	$C_2(r)$	$d(r)$
$\Box$	$\frac{1}{2}$	$\frac{N^2-1}{2N}$	N
G	N	N	N^2-1
$\Box\Box$	$\frac{N+2}{2}$	$\frac{(N-1)(N+2)}{N}$	$\frac{N(N+1)}{2}$
$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\frac{N-2}{2}$	$\frac{(N+1)(N-2)}{N}$	$\frac{N(N-1)}{2}$

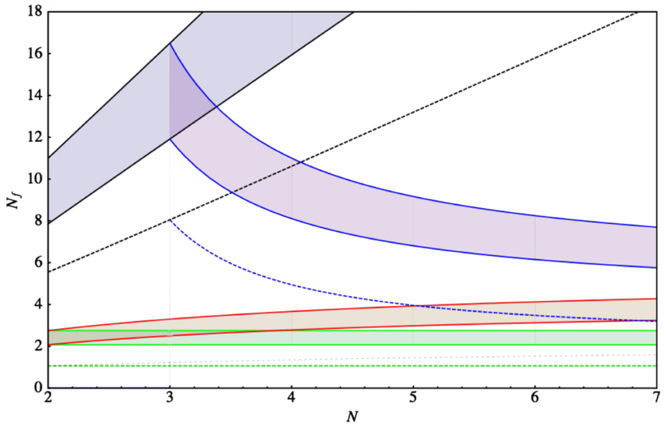

FIG. 2 (color online). Phase diagram for nonsupersymmetric theories with fermions in the (i) fundamental representation (blue), (ii) two-index antisymmetric representation (purple), (iii) two-index symmetric representation (red), (iv) adjoint representation (green) as a function of the number of flavors and the number of colors. The shaded areas depict the corresponding conformal windows. Above the upper solid curve the theories are no longer asymptotically free. In between the upper and the lower solid curves the theories are expected to develop an infrared fixed point. The dashed curve represents the change of sign in the second coefficient of the beta function. Diagram appeared first in [2].

III. SIZING THE UNPARTICLE WORLD

Georgi has recently proposed to couple a conformal sector to the standard model [20]. We find it interesting to provide a measure of how large, in theory space, the fraction of the unparticle world is. We assume, following Georgi, the unparticle sector to be described, at the underlying level, by asymptotically free gauge theories developing an infrared fixed point (FP). A reasonable measure is then, for a given representation, the ratio of the conformal window to the total window of asymptotically free gauge theories

$$R_{\rm FP} = \frac{\int_{N_{\rm min}}^{\infty} N_f^{\rm I} dN - \int_{N_{\rm min}}^{\infty} N_f^{\rm II} dN}{\int_{N_{\rm min}}^{\infty} N_f^{\rm I} dN}, \qquad (7)$$

where $N_{\rm min}$ is the lowest value of number of colors permitted in the given representation for which the above ratio is computed. Similarly we define for the nonconformal region, but still asymptotically free, the following area ratio:

$$R_{\rm NFP} = \frac{\int_{N_{\rm min}}^{\infty} N_f^{\rm II} dN}{\int_{N_{\rm min}}^{\infty} N_f^{\rm I} dN}. \qquad (8)$$

We now estimate the above fractions within the $\mathcal{N} = 1$ phase diagram as well as for the nonsupersymmetric one. Note that we have already taken the upper limit of integration to be infinity, which effectively reduces the set of representations we are going to consider to those with at most two indices.

A. The supersymmetric case

A straightforward evaluation for the supersymmetric case yields

$$R_{\rm FP} = \frac{\int_{N_{\rm min}}^{\infty} \frac{3}{2} \frac{C_2(G)}{T(r)} dN - \int_{N_{\rm min}}^{\infty} \frac{3}{4} \frac{C_2(G)}{T(r)} dN}{\int_{N_{\rm min}}^{\infty} \frac{3}{2} \frac{C_2(G)}{T(r)} dN} = \frac{1}{2}. \qquad (9)$$

Surprisingly the result is independent on the chosen representation and, of course, $R_{\rm NFP} = 1 - 1/2 = 1/2$. The universality of this ratio is impressive.

B. The nonsupersymmetric case

We now determine $R_{\rm FP}$ in the case of nonsupersymmetric gauge theories with only fermionic matter. This task requires the knowledge of $N_f^{\rm I}$ and $N_f^{\rm II}$ for the nonsupersymmetric theories studied in [2] which we report here

$$N_f^{\rm I} = \frac{11}{4} \frac{C_2(G)}{T(r)}, \qquad N_f^{\rm II} = \frac{17C_2(G) + 66C_2(r)}{10C_2(G) + 30C_2(r)} \frac{C_2(G)}{T(r)}. \qquad (10)$$

We now list the ratios for the fundamental (F) and the two-indexed representations, i.e. Adj (G), two-index symmetric (S), and two-index antisymmetric (A):

$$R_{\rm FP}[F] = \tfrac{3}{11} \simeq 0.27, \qquad R_{\rm FP}[G] = R_{\rm FP}[A] = R_{FP}[S] = \tfrac{27}{110} \simeq 0.24. \qquad (11)$$

Remarkably in the nonsupersymmetric case as well the fraction of the conformal window for the representations which are asymptotically free for any number of colors is very close to each other. Circa 25% of the nonsupersymmetric asymptotically free gauge theories with fermions in a given representation is expected to develop an infrared fixed point. This can be compared with the *exact* 50% in case of $\mathcal{N} = 1$ supersymmetric vectorlike theories. We note that in the nonsupersymmetric case, except for the adjoint representation, the values of the ratios are determined by the large N part of the integration.

IV. MULTIPLE REPRESENTATIONS, CONFORMAL REGION, AND SIZE OF THE UNPARTICLE WORLD

A generic gauge theory will, in general, have matter transforming according to distinct representations of the gauge group. Hence we now begin our analysis of the conformal region for a generic SU(N) gauge theory with $N_f(r_i)$ vectorlike matter fields transforming according to the representation r_i with $i = 1, \ldots, k$. We shall consider the nonsupersymmetric case here.

The generalization to k different representations for the expression determining the region in flavor space above which asymptotic freedom is lost is simply

$$\sum_{i=1}^{k} \frac{4}{11} T(r_i) N_f(r_i) = C_2(G). \qquad (12)$$

We suggest as an estimate of the region above which the theories develop an infrared fixed point the following expression:

$$\sum_{i=1}^{k} a(r_i) T(r_i) N_f(r_i) = C_2(G), \quad \text{with} \quad a(r_i) = \frac{10C_2(G) + 30C_2(r_i)}{17C_2(G) + 66C_2(r_i)}, \qquad (13)$$

which, of course, reproduces the ladder approximation results when reducing to a single representation. Here the coefficients $a(r_i)$ depend on the representation as well as the number of colors.

Because of the expressions above the volume, in flavor and color space, occupied by a generic SU(N) gauge theory can be defined as

$$V[N_{\min}, N_{\max}] = \int_{N_{\min}}^{N_{\max}} dN \prod_{i=1}^{k} \int_0^{(C_2(G) - \sum_{j=2}^{i} \eta(r_j)T(r_j)N_f(r_j))/(\eta(r_{i+1})T(r_{i+1}))} N_f(r_{i+1}), \qquad (14)$$

with the function $\eta(r_i)$ reducing to the number $4/11$ when the region to be evaluated is associated with the asymptotically free one and to $a(r_i)$ when the region is the one below which one does not expect the occurrence of an infrared fixed point. The notation is such that $T(r_{k+1}) \equiv T(r_1)$, $N_f(r_{k+1}) \equiv N_f(r_1)$ and the sum $\sum_{j=2}^{i} \eta(r_j)T(r_j)N_f(r_j)$ in the upper limit of the flavor integration vanishes for $i = 1$. We have defined the volume within a fixed range of number of colors $N_{\min}$ and $N_{\max}$.

The volume occupied by the asymptotically free theories is

$$V_{AF}[N_{\min}, N_{\max}] = \left(\frac{11}{4}\right)^k \int_{N_{\min}}^{N_{\max}} \frac{C_2^k(G)}{k! \prod_{i=1}^{k} T(r_i)} dN, \qquad (15)$$

while the volume associated to the fraction of asymptotically free theories not developing a fixed point is

$$V_{\mathrm{NFP}}[N_{\min}, N_{\max}] = \int_{N_{\min}}^{N_{\max}} dN \prod_{i=1}^{k} \int_0^{(C_2(G) - \sum_{j=2}^{i} a(r_j)T(r_j)N_f(r_j))/(a(r_{i+1})T(r_{i+1}))} N_f(r_{i+1}). \qquad (16)$$

Upon integration in flavor space this reads

$$V_{\mathrm{NFP}}[N_{\min}, N_{\max}] = \int_{N_{\min}}^{N_{\max}} \frac{C_2^k(G)}{k! \prod_{i=1}^{k} a(r_i)T(r_i)} dN. \qquad (17)$$

Hence the fraction of the conformal region to the region occupied by the asymptotically free theories is, for a given number of representations k,

$$R_{\mathrm{FP}} = \frac{V_{AF}[N_{\min}, N_{\max}] - V_{\mathrm{NFP}}[N_{\min}, N_{\max}]}{V_{AF}[N_{\min}, N_{\max}]}. \qquad (18)$$

We now proceed and evaluate R_{FP} in order to size the nonsupersymmetric unparticle world associated with these theories. The results are summarized in Table III. We consider characteristic examples for the representations. For $k = 1$ we use the fundamental F and the adjoint G representation. For $k = 2$ we present the case featuring F and G as well as the one featuring G and the symmetric representation S. For $k = 3$ we present F-G-S and G-A-S, where A is the two-index antisymmetric representation. Finally for $k = 4$, the four representations involved are F, G, S, and A. We observe the near universality of the ratios found for each k. We have explicitly checked that substituting any two-indexed representations with each other does not change the result. To be specific, there is a small difference whenever confronting the above ratio, for a given k, when a two-indexed representation is substituted with the fundamental one. It is, however, interesting that in the ladder approximation one observes an approximately

TABLE III. The size of the nonsupersymmetric unparticle world (i.e. the fraction of the conformal region to the asymptotically free region) when matter is in k distinct representations of the gauge group. We have chosen some characteristic examples for the representations. For $k = 1$ we have considered the fundamental F and the adjoint G representation. For $k = 2$ we present the case featuring F and G as well as the one featuring G and the symmetric representation S. For $k = 3$ we present the F-G-S case and the G-A-S case where A is the two-index antisymmetric representation. Finally for $k = 4$ the four representations used are F, G, S, and A. We observe the near universality of the ratios found for each k. We have explicitly checked that if we use any other two-indexed representation in the table above the results remain unchanged.

k	1		2		3		4
Rep.	F	G	F-G	G-S	F-G-S	G-A-S	F-G-A-S
R_{FP}	0.27	0.24	0.45	0.43	0.59	0.57	0.69

universal behavior for R_{FP}. We have only listed the results for all of the representations which can remain asymptotically free for large N. These are the fundamental and the two-indexed representations. In this case one can take N_{max} to infinity.

The analysis of the phase diagram presented here with mixed representations is of immediate use for various phenomenological studies. It allows, for example, the study and construction of explicit *split technicolor* theories introduced in [42]. These are walking technicolor theories having matter in different representations of the gauge group. Hence we further enlarge the parameter space of theories (see [2]) which can be used to break the electroweak theory dynamically.

To be explicit, consider the SU(2) technicolor gauge theory with two fundamental Dirac flavors and three adjoint Weyl fermions. According to our new Eq. (13) this theory is just below the conformal window. We split the matter spectrum in a way that the two Dirac fields are a doublet with respect to weak interactions while the adjoint fields are singlets. The electroweak symmetry is spontaneously broken via the Dirac fermions condensate. Being a walking theory the S parameter is well approximated by the value $S \simeq 1/(3\pi)$ [43,44]. This theory is then compatible with precision data. Before the analysis on the conformal regions presented first in this paper we could not have provided such an example.

We expect similar results in the case of supersymmetric theories. In the supersymmetry case, however, in evaluating the conformal regions one has to pay special attention to the fact that when multiple representations are present the R-anomaly-free charge for the different chiral multiplets is no longer uniquely determined via the single anomaly-free condition, but one has to resort to extra conditions. One can use, for example, the recently important fact discovered by Intriligator and Wecht [45] that the exact superconformal$R$ symmetry maximizes the central charge of the four-dimenstional SCFT [46–48] which has already led to interesting applications [49–54].

V. CONCLUSIONS

We have constructed the conformal window for arbitrary representations of the gauge group for $\mathcal{N} = 1$ supersymmetric gauge theories and compared it with the one for nonsupersymmetric theories.

We have also proposed a new formula according to which we can estimate the conformal region of nonsupersymmetric gauge theories when multiple matter fields transforming with respect to different representations of the underlying gauge group are part of the dynamics.

We have then defined a measure in theory space allowing us to size the fraction of asymptotically free gauge theories developing an infrared fixed point. We have discovered that this fraction depends uniquely on the representation contributing to the dynamics but not on the specific choice. This is an exact result in supersymmetric theories while it is an approximate one in the nonsupersymmetric case.

According to our findings the four-dimensional unparticle world occupies a sizable amount of the particle world, at least in theory space, and before mixing it (at the operator level) with the nonconformal one. Our results can also be used to further enlarge the number of walking theories which can be used to break the electroweak theory.

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