# Lifetime, cross-sections and activation

P. Grafström

CERN, Geneva, Switzerland

### Abstract

The concept of cross-section and its relation to the beam lifetime in accelerators is being introduced. Some general properties of cross-sections for different particles and different interaction types are discussed. For some specific cases of elastic and inelastic reactions the conditions for beam losses and the corresponding lifetime is derived. The basics of activation or induced radioactivity in accelerators are also discussed at the end of lecture.

### 1 Introduce the concept of cross-section and its relation to the lifetime of a beam

To start with we shall introduce the concept of cross-section. The concept of cross-section in particle physics or in atomic physics concerns the interaction of elementary particles, nuclei, or atoms with each other. Given a particle approaching another particle, the cross-section for this process is the probability that the two particles interact with each other. This is the most general definition of cross-section. In a simple geometrical interpretation of the cross-section it can be thought of as the area within which a reaction will take place. Thus the units of a cross-section are the units of an area. In the early days of nuclear physics the following definition was introduced

$$1 \text{ barn} = 10^{-24} \text{ cm}^2 . \tag{1}$$

Since that time the cross-section has always been measured in 'barns'. Why this strange name of a unit? The explanation is historical. During the early experiments the physicists discovered that the interactions were far more probable than expected. The nucleus was 'as big as a barn'.

It is easy to deduce the relation between a given cross-section and the lifetime of a beam. We shall do this for a general case but in practice we have, of course, in mind particles in an accelerator interacting with the rest gas in the beam pipe.

Start with a beam of particles hitting a target (see Fig. 1).



Fig. 1: The concept of cross-section

Assume that the beam flux is I particles/s/cm<sup>2</sup> and the target density is n target atoms/cm<sup>3</sup>. If the target length is dx cm, the number of particles interacting in the target and thus disappearing from the beam, will be given by

$$dI = -\ln dx\sigma.$$
 (2)

Here we have introduced the symbol  $\sigma$  for the cross-section. In nearly all cases the symbol  $\sigma$  is used to indicate cross-sections. Given that the beam particles will move with a speed of v m/s the thickness traversed during a time interval dt will be

$$dx = v \, dt \,. \tag{3}$$

Combining Eq. (2) and Eq. (3) we have

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -\mathrm{In}\,v\,\sigma\tag{4}$$

with the solution

$$I = I_0 e^{-\frac{t}{\tau}} \tag{5}$$

where

$$\tau = \frac{1}{n\sigma v}$$

Thus we see that the intensity of the beam will fall off exponentially with a time constant inversely proportional to the cross-section. In the case of an accelerator beam interacting with the rest gas we normally have a mixture of H<sub>2</sub>, CH<sub>2</sub>, CO, CO<sub>2</sub> and other gases. In this case we have to replace  $n\sigma$  in the formula above with  $\sum_{i} n_i \sigma_i$  and then we get

$$\frac{1}{\tau_{\text{total}}} = \frac{1}{\tau_{\text{H}_2}} + \frac{1}{\tau_{\text{CH}_2}} + \frac{1}{\tau_{\text{CO}}} \frac{1}{\tau_{\text{CO}_2}} + \dots$$
(6)

### 2 Classifications of cross-sections

In order to estimate the lifetime in a concrete case we thus need to know the different cross-sections entering in the calculation. What are typical values of cross-sections? Unfortunately there is no simple rule. There are no typical values. The cross-section depends on many factors. It depends of course on the target particle, i.e., the composition of the rest gas but also on the type of particle being accelerated. In addition there is often a strong energy-dependence and a dependence on the type of interaction that is involved. There are no simple guidelines.

To get an idea of the dependence on the target particle we can again rely upon some simple considerations. The typical size of the radius of an atom is of the order of one angstrom or  $10^{-8}$  cm and

thus if we apply the simple geometrical rule  $\sigma \sim \pi R^2$  we get a cross-sections in the order of megabarns. The typical radius of a nucleus is of order 10 fermi or  $10^{-12}$  cm giving cross-sections in the barn region. The proton in itself has radius of about a fermi giving cross-sections in the millibarn range.

Similarly, there is a dependence on the incident particle. In accelerators we mainly deal with protons, electrons, or ions. They are fundamentally very different particles. The size and mass are very different but also the compositeness and the forces with which they interact. There are four fundamental forces in nature. The forces of gravity and electromagnetism are familiar in everyday life. Two additional forces are introduced when discussing nuclear phenomena: the strong and weak interaction. The strong interaction is what holds the quarks together to form a proton while the weak force governs beta decay and neutrino interactions with nuclei. The forces which are relevant for us when considering beam gas interactions are the strong force and the electromagnetic force. When two protons encounter each other, they experience both these forces simultaneously. However, the strong force dominates for head-on collisions and the electromagnetic forces dominate for peripheral collisions. Electrons do not feel the strong force at all and thus only the electromagnetic force is relevant.

As mentioned above the cross-section depends also on the energy of the incident particles. Just as an illustration of some typical energy dependence we show the energy dependence in proton–proton interactions in Fig. 2.



# Total cross section

Fig. 2: Example of energy dependence of cross-sections

For future discussions it will be useful to classify the interactions between charged particles and the rest gas atoms. A collision is called *elastic* if the particles do not change identity during the interaction. This is just like the collision of billiard balls. It is possible to have elastic scattering both for the case of electromagnetic interaction and for the case of strong interaction. In the former case both particles must have a charge but in the latter case we can have elastic scattering independent of the charge. Moreover, as you will see, there can be both single and multiple elastic scattering.

All reactions that are not elastic are called *inelastic*. In this case there can be a change of nature of the particles and also new particles can be created. We can further classify the inelastic electromagnetic interactions as follows: bremsstrahlung, ionization, electron capture, and electron loss. In the case of strong interactions we have nuclear reactions, particle break up, and particle creation. In the following we shall look at some of the possible elastic and inelastic reactions in more detail.

# **3** Elastic scattering

### 3.1 Coulomb scattering

Elastic scattering via the electromagnetic force is called Coulomb scattering and sometimes Rutherford scattering. Actually it was Rutherford who realized while observing results from scattering of alpha particles off a thin foil that the atom must have a very tiny nucleus compared to the size of the atom itself. The scattering angle depends on the impact parameter as illustrated in Fig. 3. Small impact parameters give large scattering angles and vice versa.



Fig. 3: Relation between impact parameter and scattering angle

It is rather straightforward to deduce a formula that gives the cross-section per unit solid angle as a function of scattering angle  $\theta$ 

$$\frac{d\sigma}{d\Omega}(\theta) = \left(\frac{ZZ'e^2}{4E}\right)^2 \frac{1}{\left(\sin\theta/2\right)^4}.$$
(7)

Here Z and Z' are the charge of the projectile and the target, respectively, and E is the kinetic energy of the moving particle.

There are deviations from this simple formula as illustrated in Fig. 4.



Fig. 4: Deviations from simple Coulomb scattering

For small scattering angles or large impact parameters, the screening effect of the surrounding electrons starts to play a role. For the very big angles or small impact parameters, the finite size of the nucleus affects the result.

### 3.2 Multiple Coulomb scattering

Many small-angle events will lead to an accumulation effect which will affect the emittance of the beam. The cumulating effect of many small deviations will lead to an emittance growth that in turn affects the lifetime of the beam. The r.m.s. value of the scattering angle can be approximated by a simple formula [1]:

$$\theta_{\text{r.m.s.}} \propto \frac{1}{p} \sqrt{\frac{L}{X_0}}$$
(8)

where  $L/X_0$  is the amount of material seen by the particle expressed in radiation lengths and p is the momentum of the incoming particle. We shall come back to the definition of radiation lengths in a moment.

## 3.3 Elastic scattering via strong interaction

As briefly mentioned before, elastic scattering can also occur via strong interaction. This part of the elastic scattering dominates at large angles as seen in Fig. 5.



Fig. 5: Differential elastic cross-section as a function of t

In the figure the differential cross-section is given as a function of the variable *t*. This variable is often used in particle physics and is proportional to the square of the scattering angle. All the effects at large t values are due to the strong interaction. However, as can be seen, the cross-section has dropped considerably and thus is of little importance for beam losses.

#### 3.4 Beam losses from single Coulomb scattering

The important question for us is of course to understand when elastic scattering with the beam-gas atoms implies a loss of beam particles. We shall look at single Coulomb scattering as an illustrative example. It can be shown that giving a circulating beam particle an angular kick  $\theta$  at a point i around the ring will result in an oscillation [2]

$$u(s) = \theta_{i} \sqrt{\beta(s)\beta_{i}} \sin[\varphi(s) - \phi_{i}].$$
(9)

Here u(s) is the amplitude at a point s around the ring, and  $\beta(s)$  and  $\phi(s)$  the beta function and phase at that point. Similarly  $\beta_i$  and  $\varphi_i$  are the beta function and phase at the point i. Let A be the half aperture of the dynamic aperture at the position of minimum aperture and  $\beta_A$  the value of the  $\beta$ function at this point. At the position of minimum aperture the particles will thus be lost if

$$\theta_{i}\sqrt{\beta_{A}\beta_{i}} \ge A.$$
<sup>(10)</sup>

Averaging over the circumference of the machine we get

$$\boldsymbol{\theta}_{\max} = \frac{A}{\sqrt{\boldsymbol{\beta}_A \boldsymbol{\beta}_{\text{average}}}} \,. \tag{11}$$

To get the loss cross-section  $\sigma_{\text{loss}}$  we just need to integrate  $\frac{d\sigma}{d\Omega}(\theta)$  from Eq. (7) from  $\theta_{\text{max}}$  to  $\pi$ 

and we get

$$\sigma_{\rm loss} \propto \frac{Z^{\prime 2}}{E^2} \frac{1}{\theta_{\rm max}^2} = \frac{Z^{\prime 2} \beta_A \beta_{\rm average}}{E^2 A^2} \,. \tag{12}$$

This is the cross-section to be used for lifetime calculation. We see that the loss cross-section is a strong function of Z' of the gas and that for a given aperture the loss cross-section decreases with higher energies and becomes small with small beta values.

#### 4 **Inelastic cross-sections**

#### **Electron beams—bremsstrahlung** 4.1

In general, when a charged particle is accelerated it emits electromagnetic radiation. If a charged particle is accelerated by the field of an atomic nucleus it will emit electromagnetic radiation in the form of photons. This phenomenon is called bremsstrahlung. The origin of the word is German and it simply means 'braking radiation'. The energy that is emitted by the accelerated particle is proportional to the inverse mass squared of the particle and thus bremsstrahlung is very important for light particles like electrons and positrons. The energy loss of the particle is roughly proportional to the energy of the particle and thus bremsstrahlung becomes important at high energies. The constant of proportionality is normally written as  $1/X_0$  where  $X_0$  is called the radiation length and is a characteristic of the material (it depends on the square of the atomic number of the material and the density of the material).

We thus have

$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{E}{X_0} \tag{13}$$

and

$$E_{\rm ave} = E_0 e^{-\frac{x}{X_0}}.$$
 (14)

From this we see that the radiation length can be interpreted as the distance a particle traverses in a medium until its energy has gone down a factor 1/e.

The energy distributions of photons which are emitted in the process follow the so-called Bethe–Heitler spectrum [1]:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon} \propto \frac{4}{3} \frac{1}{X_0} \frac{F(\varepsilon, E)}{\varepsilon}$$
(15)

where  $F(\varepsilon, E)$  is given by

$$F(\varepsilon, E) \cong 1 - \frac{\varepsilon}{E} + \frac{3}{4} \left(\frac{\varepsilon}{E}\right)^2.$$
(16)

Here *E* is the nominal energy of the radiating particle and  $\varepsilon$  the energy of the emitted photons. Observe that  $F(\varepsilon, E)$  is a rather slowly varying function.

In the case of bremsstrahlung it will be the energy acceptance of the accelerator that will determine the losses. Assume that all energies from the nominal *E* down to  $E - \varepsilon_m$  are accepted. This means that energy losses bigger than  $\varepsilon_m$  lead to particle losses.

To get  $\sigma_{\text{loss}}$  we need to integrate  $d\sigma/d\varepsilon$  of Eq. (15) from  $\varepsilon_{\text{m}}$  to E and we get

$$\sigma_{\rm loss}^{\rm brems} \propto \frac{4}{3X_0} \left( \ln \frac{E}{\varepsilon_{\rm m}} - \frac{5}{8} \right) \quad (\varepsilon_{\rm m} \text{ is small relative to } E).$$
 (17)

We see that  $\sigma_{\text{loss}}$  has a strong dependence on the atomic number of the residual gas via the  $1/X_0$  factor but a rather weak dependence on the maximum energy acceptance. It should be pointed out that this is a rather simplified treatment excluding the effect from electrons of the atom and screening effects.

### 4.2 Ion beams

Ion beams are in general more complicated than electron or proton beams. There are two more degrees of freedom to take into account. There is the charge of the nucleus being accelerated and there is the total charged state of the atom (i.e., number of electrons attached minus the charge of the nucleus). At low energies, the interaction with the rest gas atoms will lead to either a capture of additional electrons or stripping (loss) of existing electrons. Both the capture cross-section and the loss cross-section are functions of the charge of the nucleus, the total charge state of the atom, the energy of the ion being accelerated, and the charge of the rest gas nucleus.

At low energies we can compare in a simple and intuitive picture the velocity of the accelerated ion with the velocity of its outermost electrons. If the velocity of the ion is close to the velocity of the outermost electrons, the capture cross-section will be approximately balanced with the loss cross-section. Electron capture will dominate if the velocity of the ion is significantly lower than the electron velocity. On the contrary, for higher energies with velocities of the ion significantly higher than the electron velocity, the electron loss will dominate. The notion of equilibrium charge is important in this context. The equilibrium charge ( $\overline{q}$ ) is reached after many collisions in a given gas and it is approximately the charge state for which the orbit velocity of the outermost electrons is equal to the ion velocity. If the charge of the ion is not too far away from the equilibrium charge, there are some simplified scaling rules for the cross-section. Below we give an example of a simple scaling rules [3],

$$\sigma_{\rm c}(q) = \sigma_{\rm c}(\overline{q}) \left(\frac{q}{\overline{q}}\right)^a \tag{18}$$

$$\sigma_{\rm l}(q) = \sigma_{\rm c}(\overline{q}) \left(\frac{q}{\overline{q}}\right)^b \tag{19}$$

where  $a \approx 4$  and  $b \approx -2.3$  for charges lower than the equilibrium charge state and  $a \approx 2$  and  $b \approx -4$  for higher charge states. Observe that other scaling rules also exist.

At very high and relativistic energies like RHIC (100 GeV/A) and LHC (2.76 TeV/A) we deal with bare ions with high atomic number and there are different mechanisms coming into play. For the peripheral collisions with large impact parameter the electromagnetic cross-section is large and of the order of several hundred kbarn. Because of the high atomic number of the ion there is a strong electromagnetic field and  $e^+e^-$  production dominates. This reaction in itself is a rather harmless inelastic reaction because there is no significant change of momentum of the ion. However, in some cases, the produced electron is captured by the ion and then the charge state of the ion changes and it is of course lost from the beam. In this high-energy regime there is also the mechanism of electromagnetic dissociation of the nucleus. The photon exchange between the bare nucleus and the rest gas nucleus leads to a break-up of the ion which is then again lost from the beam. These processes also occur between the ions in the two colliding beams. The predicted values of the cross-section for those processes at LHC and for Pb–Pb collisions are given in Table 1 [4].

Table 1: Cross-sections for Pb–Pb collisions at the LHC [4]

Cross-sections for Pb–Pb collisions	Symbol	(barn)
Hadronic	$\sigma_{ m h}$	8
EM dissociation	$\sigma_{ m emd}$	225
e <sup>-</sup> capture	$\sigma_{ m ec}$	204

In addition to the cross-section for  $e^+e^-$  production followed by electron capture and the cross-section for electromagnetic dissociation, the strong hadronic cross-section is given. Observe that due to the high charge of the ions the hadronic cross-section is smaller than the electromagnetic.

The large cross-section seen in Table 1 means that we get a short lifetime from the beam-beam interaction and normally not from the beam-gas interaction. Beam lifetimes at LHC and RHIC from beam-beam interactions are in the range of hours for high atomic numbers of the colliding ions and vary very quickly with the atomic number. Some examples of how the cross-section varies with the atomic number of the ion species colliding are given in Table 2 [4].

Ion Species	$\sigma_{ m h}$ (barn)	$\sigma_{ m emd}( m barn)$	$\sigma_{ m ec}({ m barn})$	$\sigma_{ m total}$ (barn)
Pb	8	225	204	437
Sn	5.5	44.5	18.5	68.5
Kr	4.5	15.5	3.0	23.0
Ar	3.1	1.7	0.04	4.84
0	1.5	0.13	$1.6 \times 10^{-4}$	1.63

Table 2: Cross-sections for different ion species colliding at the LHC [4]

### 4.3 Proton beams

The proton is about 2000 times heavier than the electron and this implies that the bremsstrahlung of protons is heavily reduced as compared to electrons. Compared to heavy ions, the charge of only one unit for the protons also implies lower rates for electromagnetic reactions like electron capture. As a consequence, for proton beams, the strong interaction is of importance. There are basically two types of reactions involving the strong interaction; diffractive and non diffractive interactions. Diffractive interactions imply that no quantum numbers are exchanged between the incoming proton and the rest gas nucleus. However, in contrast with elastic scattering, some of the kinetic energy of the incoming proton is converted to mass and several new particles are created. The non-diffractive interactions are more violent. Here the quarks in the proton and the quarks of the rest gas nucleus collide head on and both the proton and the rest gas nucleus are typically destroyed. From the point of view of beam losses in an accelerator both the diffractive and non-diffractive reactions in general mean total loss of the proton.

The proton-proton cross-section depends strongly on energy as can be seen from Fig. 2. In general, at high energies, the proton-nucleus cross-section has the same energy dependence as the proton-proton cross-section and as a rule of thumb the proton-nucleus cross-section can be deduced from the proton-proton cross-section according to

$$\sigma_{\rm pA} = \sigma_{\rm pp} A^{0.7} \ . \tag{20}$$

Table 3 gives an example of the expected proton–nucleus cross-section at LHC for some nuclei corresponding to frequently present gases [5].

**Table 3:** Expected proton–nucleus cross-section at LHC for some gases [5]

Gas	$H_2$	Не	$\mathrm{CH}_4$	H <sub>2</sub> O	СО	$CO_2$
Cross-section (mb)	94	130	568	554	840	1300

## 4.4 Ionization of the rest gas

When a charged particle traverses a medium it suffers repeated collisions with the electrons of the atoms. The electron may be kicked out of its orbit and we have what is called ionization energy loss. The cross-section is large but the energy losses are very small and thus the beam particles are hardly affected. The energy loss is given by the well-known Bethe–Block formula [1]:

$$\frac{dE}{dX} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left( \frac{1}{2} \ln \frac{2m_e^2 c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} - \beta^2 - \frac{\delta}{2} \right).$$
(21)

Here K is a constant,  $\beta$  and  $\gamma$  the usual relativistic variables describing the accelerated particle  $(\beta = v/c \text{ and } \gamma = 1/\sqrt{1-\beta^2})$ , z is the charge of the incoming particle, Z/A is the charge mass ratio of the rest gas atom, I is the ionization potential,  $T_{\text{max}}$  is the maximum kinetic energy which can be imparted to a free electron in a single collision, and  $\delta$  is a relativistic correction factor. The energy dependence is seen from this formula. At low energies the  $1/\beta^2$  term dominates and at higher energies the dependence on the logarithmic term takes over. This is seen in Fig. 6.



**Fig. 6:** dE/dX as a function of energy

What is important for us is the  $z^2$  term. The energy loss depends on the square of the charge of the accelerated particle. Even if the energy losses from ionization in general are small, we can have some more substantial energy losses for bare ions with high atomic number. As the cross-section for ionization is large there might thus be some beam losses associated with ionization for ions with high atomic number but ionization is in general not an important beam loss factor.

## 5 Activation

With activation or induced radioactivity we mean the process of making a material radioactive by bombardment with particles or radiation. It basically means the transformation of a stable nucleus of an atom to one or several unstable nuclei.

There are several origins of activation in accelerators. The dominating source is objects that are directly hit by the primary beam, like dumps, septa, collimators or other beam obstacles. Another source is related to localized beam losses associated with some malfunctioning piece of equipment. Bad vacuum is in general not the main source of activation. Observe that the problem of activation is significantly less severe in electron machines as compared to hadron machines and this is of course due to the different interactions and the different processes that are involved.

There are basically two distinct phenomena that are involved in activation. There is the reaction that creates the unstable nuclei and there is the radioactive decay of the unstable nuclei created. The various nuclear decay modes are well known. The most frequent decay modes are alpha and beta decays, gamma transitions, and spontaneous fission. In the first two cases the nucleus emits an alpha particle and an electron, respectively. The latter cases deal with photon emission and spontaneous break-up of the nucleus. In the following we shall no longer discuss the decay modes but give a couple of examples of reactions that create unstable nuclei. A typical example is neutron capture. Here all cross-sections are well known but they may vary from the kbarn range to the mbarn range (see Fig. 7 [6]).



Fig. 7: Neutron cross-section as a function of energy

The activation depends very strongly on the material in the case of neutron capture. As an example we can compare the neutron capture cross-section for  $Ag^{109}$  which is 20 kbarn with that of Pb<sup>208</sup>, being only 3 barns.

The activation by high-energy hadrons has different properties. The origin is so-called 'stars' or violent inelastic reactions. Here there are a large number of possible final states and not all reaction cross-sections are well known. Also the activation depends more weakly on the material. An example of the material dependence is given in Fig. 8 [7].



Fig. 8: Activation dose rate as a function of atomic number

As an example of considerations relevant for choice of material of a beam pipe we shall take the beam pipe of the ATLAS experiment. Observe that here the dominating source of activation will be the beam-beam interactions at the intersection point and not the beam-gas interaction. Looking again at Fig. 8 we see that a beryllium or a carbon beam pipe would hardly get activated at all compared to other materials. However, there are several other inconveniences associated with such materials and thus it is also interesting to compare the difference between an aluminium pipe and a steel pipe. In comparing induced activity from different materials one has to consider both the exposure time and the cool-off time. An example for aluminium and steel is given in Table 4 [7].

Cooling time/running time	5000 days	1000 days	100 days	30 days
1 day	9	13	23	23
5 days	9	15	76	180
30 days	4	7	22	39

Table 4: The ratio of the dose rates from a steel and an aluminium beam pipe

We see that depending on the conditions there can easily be a factor 10 gained by using aluminium instead of steel.

The ATLAS beam pipe was in the end constructed with a beryllium pipe close to the intersection point and stainless-steel pipe further away. The choice of beryllium was dictated by the necessity of minimizing multiple Coulomb scattering in the walls of the beam pipe and not by activation considerations. However, the stainless-steel pipes might be exchanged for aluminium pipes at a later stage because of the favourable activation properties.

# Acknowledgement

I want to thank the organizer for a very pleasantly organized school and I also thank Daniel Brandt for having invited me to give this lecture.

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