



## ENERGY-SAVING BUILDING PROGRAM EVALUATION WITH AN INTEGRATED METHOD UNDER LINGUISTIC ENVIRONMENT

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**Abstract.** In the context of sustainable development, building energy conservation has become the development trend of the construction industry. The selection of energy-saving building program, as a multi-criteria decision-making (MCDM) problem, has a direct influence on the actual energy-saving effect. In this paper, an integrated MCDM method combining the extended best worst method (BWM) and Weighted Aggregated Sum Product Assessment (WASPAS) method is proposed to solve the energy-saving building program selection problem under the linguistic Pythagorean fuzzy environment. The Linguistic Pythagorean fuzzy sets (LPFSs) are used to model the uncertain evaluation information of experts. The extended BWM is developed to determine the weights of criteria, while the extended WASPAS method is proposed to determine the ranking of alternatives. To validate the applicability and reliability of the proposed method, this paper presents a numerical example of the selection problem for energy-saving building programs. Some managerial insights are also given for practitioners to use the proposed method.

**Keywords:** energy-saving building, construction industry, multi-criteria decision making, linguistic Pythagorean fuzzy set, weighted aggregated sum product assessment, best worst method.

### Introduction

The United Nations convened the Conference on Environment and Development with sustainable development as its core content in 1992. Since then, the concept of sustainable development has been widely recognized (Heravi et al., 2017). All countries in the world have realized the importance of sustainable development and made efforts to put it into practice. Sustainable development refers to the coordinated development of economy, society, resources and environmental protection, in which saving resources and energy and protecting natural ecological environment are important contents. Nowadays, the global economy and society are developing fast, and the demand of energy is increasing constantly. Energy production and application and the impact on the environment have become major concerns of the public (Chenari et al., 2016). The construction industry is one of the major energy consumers. According to the statistics, construction accounts for 40% of global energy consumption and the resulting

greenhouse gases are responsible for 30% of global energy-related emissions. Besides, the construction industry consumes more than 10% of water resource and generates more than 30% of waste, which leads to great harm to the environment (Si & Marjanovic-Halburd, 2018). The implementation of energy-saving building is considered to be the most successful carbon dioxide emission reduction and energy-saving strategy (Huang & Wang, 2014). Energy-saving buildings refer to buildings that use energy reasonably and improve energy efficiency without affecting living comfort. Therefore, to realize the sustainable development of the construction industry, it is critical to adopt building energy-saving measures to increase the efficiency of energy use and lower the environmental pollution.

The evaluation of energy-saving building programs can be regarded as an MCDM problem, because it includes multiple criteria in terms of technology, economy, and

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environment, and there may be conflicts between them. MCDM methods embody high efficiency and practicability in dealing with decision-making problems in the field of building sustainable development. The typical MCDM methods include the Elimination and choice expressing reality (ELECTRE) (Yu et al., 2018), Technique for order of preference by similarity to ideal solution (TOPSIS) (Chen & Tsao, 2008), Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) (Sanayei et al., 2010), and Analytic hierarchy process (AHP) (Vaidya & Kumar, 2006). At present, the applications of MCDM methods in the building sustainable development mainly focused on green building technology evaluation (Si et al., 2016), green building material selection (Mathiyazhagan et al., 2019; Khoshnava et al., 2018), and sustainable building program evaluation (Heravi et al., 2017; Chen & Pan, 2016). For example, Si et al. (2016) applied MCDM methods to green technology selection for building renovation, in which the criteria weights and the relative performance scores of green technologies were derived by AHP. Mathiyazhagan et al. (2019) introduced a hybrid MCDM method combining the BWM and TOPSIS for sustainable building material evaluation.

The BWM (Rezaei, 2015) is a new MCDM method to determine the weights of criteria. Compared with AHP, the BWM has simpler operation and higher consistency (Rezaei, 2016). The core step of the BWM is to compare the best and worst criteria with the other criteria respectively to get preference vectors, and then build an optimization model according to the consistency conditions (Mi et al., 2019). If there are  $n$  criteria, the number of pairwise comparisons needed in the BWM is  $2n - 3$ , while in AHP is  $n(n - 1) / 2$ . In addition, the WASPAS (Zavadskas et al., 2012) is a method to integrate the evaluation values of alternatives and then get the final ranking of them. The BWM and WASPAS method are suitable for solving the problem of energy-saving building program selection.

The intuitionistic fuzzy set (IFS) (Atanassov, 1986), which expresses the decision information from membership and non-membership aspects, is a good tool for deal with ambiguity. Yager (2014) extended the IFS theory and defined Pythagorean fuzzy set (PFS), which can better represent the ambiguity. In a PFS, the following conditions must be satisfied that the sum of squares of membership and non-membership degrees is no more than 1. However, in the actual situation, experts may have difficulty in expressing membership and non-membership with exact numbers, while language is a better way for expression. In this regard, the concept of linguistic Pythagorean fuzzy set (LPFS) was introduced (Garg, 2018) to adapt to the complicated decision circumstance in which linguistic terms are used to represent decision information. The use of LPFS can express the uncertainty in the evaluation process of energy-saving building schemes and improve the evaluation accuracy.

Motivated by the above analysis, this paper aims to develop an integrated MCDM method integrating the ex-

tended BWM and WASPAS method to handle the energy-saving building program selection problem in the LPF environment. The highlights of this study lie in:

- 1) The application steps of the BWM method under the LPF environment are proposed. To this point, the criteria weights and the uncertainty in the determination process are considered and the integrity of evaluation information is improved.
- 2) The calculation steps of the WASPAS method using LPFSs as evaluation information are given, which can improve the accuracy of alternatives sequencing.
- 3) We develop an MCDM model based on the extended BWM and WASPAS method under the LPF environment, and then apply it to the evaluation of energy-saving building programs to verify the performance of this method in practice.

The paper is structured as follows: Section 1 reviews the current research status of energy-saving buildings, the basic concepts of LPFS and the steps of the original BWM and WASPAS method. In Section 2, the BWM and WASPAS method are extended into LPF environment, and the procedure of the proposed multi-expert MCDM method is introduced. In Section 3, a case study of energy-saving building program evaluation is given to verify the practicability of the method, and in Section 4 a sensitivity analysis is given. Final section draws some conclusions.

## 1. Preliminaries

In this section, we review the researches on energy-saving buildings, and then briefly introduce the basic concepts of LPFS and the steps of the original BWM and WASPAS method.

### 1.1. Literature review on energy-saving buildings

The core idea of energy-saving buildings is to reduce the building energy use and improve the energy use efficiency. The application of energy-saving technology is conducive to reducing the building energy consumption. Some scholars have conducted the research on the performance evaluation of energy-saving technology, and analyzed its energy-saving effect in the actual application. For instance, Kim (2017) analyzed the performance of different energy-saving technologies in reducing energy cost saving based on the idea of economic analysis. Geng et al. (2015) established an assessment framework based on the fuzzy AHP, and carried out the suitability evaluation of energy-saving technologies. Lu et al. (2018) introduced a pre-evaluation system based on the TOPSIS and SA (simulated annealing) to evaluate energy-saving technologies during the architectural design phase.

As the performance of building materials has a direct impact on building energy efficiency, some researches on the evaluation of building materials were carried out. Bribian et al. (2011) used the life cycle assessment method

to make a comparative analysis of common building materials and ecological materials, and analyzed the impact of building materials on energy and environment, so as to provide a reference basis for selecting building materials. Lee et al. (2011) studied the influence of materials on the building's embodied energy to better control the energy demand of buildings in the whole life cycle.

Some scholars have studied on the practical performance of energy-saving buildings, mainly taking the specific energy consumption of heating, lighting, refrigeration, ventilation and other aspects as evaluation criteria (Harish & Kumar, 2016). Liu et al. (2018) proposed a new building energy efficiency index which was derived based on outdoor temperature to evaluate actual energy-saving effect of building envelopes. Delgarm et al. (2016) developed an optimization procedure based on the particle swarm optimization, and applied it to building energy performance evaluation and optimization. To satisfy the needs of sustainable development, energy-saving buildings also need to consider social, economic, technological, institutional and environmental sustainability (Danish et al., 2019). In this sense, the sustainability should be fully considered in the design stage of energy-saving buildings.

### 1.2. Linguistic Pythagorean fuzzy set (LPFS)

To better model the uncertainties faced in decision-making process, Yager (2014) proposed the PFS, in which both membership and non-membership degrees are expressed numerically. However, in many practical situations, decision-makers (DMs) may have difficulty in quantifying their preferences with precise values due to the ambiguity of decision information. Using linguistic variables to express membership and non-membership degree can solve this problem. In this regard, Garg (2018) proposed the concept of LPFS. Suppose that  $X$  is a universe set and  $S_{[0,t]}$  is a continuous linguistic term set with  $S_{[0,t]} = \{s_\alpha \mid s_0 \leq s_\alpha \leq s_t, \alpha \in [0,t]\}$ . An LPFS in  $X$  is defined as  $L = \{(x, s_\sigma(x), s_\tau(x)) \mid x \in X\}$ , where  $s_\sigma(x), s_\tau(x) \in S_{[0,t]}$  represent the degrees of linguistic membership and linguistic non-membership of any element  $x$  to set  $L$ , respectively. Garg (2018) called the pair  $(s_\sigma(x), s_\tau(x))$  a linguistic Pythagorean fuzzy number (LPFN), and simply represented it as  $\beta = (s_\sigma, s_\tau)$ . For any  $x \in X$ , it always holds that  $\sigma^2 + \tau^2 \leq 1$ .  $\pi(x) = s_{\sqrt{t^2 - \sigma^2 - \tau^2}}$  represents the degree of linguistic indeterminacy of  $x$  to  $L$ . The functions of calculating the score and accuracy of the LPFN  $\beta$  were respectively defined as (Garg, 2018):

$$S(\beta) = s_{\sqrt{(t^2 + \sigma^2 - \tau^2)/2}}; \tag{1}$$

$$H(\beta) = s_{\sqrt{\sigma^2 + \tau^2}}. \tag{2}$$

According to the above two functions, the comparison rules of two LPFNs  $\beta_1 = (s_{\sigma_1}, s_{\tau_1})$  and  $\beta_2 = (s_{\sigma_2}, s_{\tau_2})$  can be given as below: (1) if  $S(\beta_1) > S(\beta_2)$ , it means that  $\beta_1$  is preferred to  $\beta_2$ , and represented as  $\beta_1 \succ \beta_2$ ; (2) if  $S(\beta_1) = S(\beta_2)$ , then if  $H(\beta_1) > H(\beta_2)$ , it means

that  $\beta_1$  is preferred to  $\beta_2$ , and represented as  $\beta_1 \succ \beta_2$ ; if  $H(\beta_1) = H(\beta_2)$ , then it can be considered that  $\beta_1$  and  $\beta_2$  indicate the same preference, which is represented as  $\beta_1 = \beta_2$ . For example, suppose that there are three LPFNs  $\beta_1 = (s_5, s_1), \beta_2 = (s_6, s_0), \beta_3 = (s_5, s_2) \in S_{[0,6]}$ . We can obtain that  $S(\beta_1) = s_{\sqrt{30}}, S(\beta_2) = s_6, S(\beta_3) = s_{\sqrt{28.5}}$ . According to the above comparison rules, we can conclude that  $\beta_2 \succ \beta_1 \succ \beta_3$ .

For two LPFNs  $\beta_1 = (s_{\sigma_1}, s_{\tau_1})$  and  $\beta_2 = (s_{\sigma_2}, s_{\tau_2})$ , some basic operations were defined as follows (Garg, 2018):

$$\beta_1 \oplus \beta_2 = (s_{\sqrt{\sigma_1^2/t^2 + \sigma_2^2/t^2 - \sigma_1^2\sigma_2^2/t^4}}, s_{t(\tau_1\tau_2/t^2)}); \tag{3}$$

$$\lambda\beta_1 = (s_{\sqrt{1 - (1 - \sigma_1^2/t^2)^\lambda}}, s_{t(\tau_1/t)^\lambda}). \tag{4}$$

To aggregate different LPFNs, some aggregation operators were proposed. Let  $\beta_i = (s_{\sigma_i}, s_{\tau_i}) \in S_{[0,t]} (i = 1, 2, \dots, n)$  and  $w_i$  be the weight of  $\beta_i$  with  $w_i \in [0,1]$  and  $\sum_{i=1}^n w_i = 1$ .

Then, the linguistic Pythagorean fuzzy weighted averaging (LPFWA) operator was defined as (Garg, 2018):

$$LPFWA_w(\beta_1, \beta_2, \dots, \beta_n) = \oplus_{i=1}^n (w_i\beta_i) = (s_{\sqrt{(1 - \prod_{i=1}^n (1 - \sigma_i^2/t^2)^{w_i}}}}, s_{t \prod_{i=1}^n (\tau_i/t)^{w_i}}). \tag{5}$$

The linguistic Pythagorean fuzzy weighted geometric (LPFWG) operator was defined as (Garg, 2018):

$$LPFWG_w(\beta_1, \beta_2, \dots, \beta_n) = \otimes_{i=1}^n (\beta_i)^{w_i} = (s_{t \prod_{i=1}^n (\sigma_i/t)^{w_i}}, s_{\sqrt{(1 - \prod_{i=1}^n (1 - \tau_i^2/t^2)^{w_i}}}}). \tag{6}$$

To convert linguistic terms into numerical values, scholars (Wang et al., 2014; Liao et al., 2019) have studied the numerical conversion scale of linguistic terms. The linguistic scale functions (LSFs), which can use data effectively and express semantics flexibly, and convert linguistic terms into numerical values in different situations. Suppose that  $S = \{s_i \mid i = 0, 1, \dots, t\}$  is a discrete set of linguistic terms with  $t$  being an even number. If  $\theta_i$  is a numerical value between 0 and 1, then an LSF is defined as a mapping  $f: s_i \rightarrow \theta_i$  where  $0 \leq \theta_0 < \theta_1 < \dots < \theta_t (i = 0, 1, \dots, t)$ .  $f$  is a monotonic incremental function.  $\theta_i$  represents the preferences of DMs when they use the linguistic term  $s_i$ . There are three kinds of LSFs. Here we only introduce the first one as an example:

$$f(s_i) = \theta_i = \frac{i}{t}, \text{ where } i = 0, 1, \dots, t. \tag{7}$$

In this way, the linguistic evaluation information can be transformed into evenly divided values. Because of its simplicity and operability, this function is often used in typical decision environments (Wang et al., 2014; Liao et al., 2019).

### 1.3. The best worst method (BWM)

The BWM (Rezaei, 2015) is a new MCDM method based on programming model. Scholars have studied the BWM and extended it to uncertain environments to solve practical MCDM problems. The latest review of the BWM can be found in Mi et al. (2019). After the original BWM was proposed, Guo and Zhao (2017) extended it to the fuzzy BWM in which the reference comparisons were expressed in triangular fuzzy numbers. Mou et al. (2016) proposed the intuitionistic fuzzy multiplicative BWM and used it for multi-criteria group decision making. Mou et al. (2017) also investigated the intuitionistic fuzzy BWM combined with graph theory. Liu et al. (2019) developed an extended BWM in interval-valued intuitionistic uncertain linguistic environment and used it to solve sustainable supplier selection problem. Mi and Liao (2019) extended the BWM to deal with preference information expressed in hesitant fuzzy numbers and developed three different models under different situations to compute the criteria weights. Li et al. (2019) introduced a new MCDM approach for the selection problem of best investment company, which combined the BWM and dominance degree to process probabilistic fuzzy information.

In the BWM, the weights of criteria can be derived through pairwise comparisons. And the BWM performs fewer pairwise comparisons and has higher consistency degree than AHP. First, the BWM obtains two preference vectors by determining the best and worst criteria and comparing them with other criteria respectively. On this basis, a model based on preference information consistency is set up to obtain the weights of criteria. The main steps of the BWM are shown below (Rezaei, 2015).

**Step 1.** Construct the set of criteria  $\{c_1, c_2, \dots, c_n\}$  based on the decision objectives.

**Step 2.** Determine the best and worst criteria, expressed as  $c_B$  and  $c_W$ , respectively.

**Step 3.** Implement pairwise comparisons between the best criterion and the other criteria. The preferences of the best criterion on the other criteria are hereinafter simply expressed as “the preferences of best-to-others”, which are represented by numbers between 1 and 9. And the preference vector of best-to-others (BO) is denoted as  $V_B = (v_{B1}, v_{B2}, \dots, v_{Bn})$ .

**Step 4.** Implement pairwise comparisons between the other criteria and the worst criterion. The preferences of the other criteria on the worst criterion are hereinafter simply expressed as “the preferences of others-to-worst”, which are represented by numbers between 1 and 9. And the preference vector of others-to-worst (OW) is denoted as  $V_W = (v_{1W}, v_{2W}, \dots, v_{nW})^T$ .

**Step 5.** Establish an optimization model according to the consistency conditions. When the preference information is completely consistent, the conditions of  $w_B / w_j = v_{Bj}$  and  $w_j / w_W = v_{jW}$  are satisfied. To get the optimal weights, the following model is built, which aims to minimize the maximum values of  $|w_B / w_j - v_{Bj}|$  and  $|w_j / w_W - v_{jW}|$  for all criteria.

### Model 1

$$\begin{aligned} & \min \xi \\ & \text{s.t.} \\ & \left| w_B / w_j - v_{Bj} \right| \leq \xi, j = 1, 2, \dots, n \\ & \left| w_j / w_W - v_{jW} \right| \leq \xi, j = 1, 2, \dots, n, \\ & \sum_j w_j = 1 \\ & w_j \geq 0, j = 1, 2, \dots, n \end{aligned}$$

$$\text{where } \xi = \max_j \left\{ \left| w_B / w_j - v_{Bj} \right|, \left| w_j / w_W - v_{jW} \right| \right\}.$$

Solving Model 1, the optimal weights  $(w_1^*, w_2^*, \dots, w_n^*)$  and the maximum absolute difference  $\xi^*$  are obtained.

Based on the ideas of Rezaei (2016), if we minimize the maximum of  $\left\{ \left| w_B - v_{Bj} w_j \right|, \left| w_j - v_{jW} w_W \right| \right\}$  instead of the maximum  $\left| w_B / w_j - v_{Bj} \right|$  and  $\left| w_j / w_W - v_{jW} \right|$ , Model 1 will be transformed into a linear optimization model as shown below:

### Model 2

$$\begin{aligned} & \min \xi^L \\ & \text{s.t.} \\ & \left| w_B - v_{Bj} w_j \right| \leq \xi^L, j = 1, 2, \dots, n \\ & \left| w_j - v_{jW} w_W \right| \leq \xi^L, j = 1, 2, \dots, n, \\ & \sum_j w_j = 1 \\ & w_j \geq 0, j = 1, 2, \dots, n \end{aligned}$$

$$\text{where } \xi^L = \max_j \left\{ \left| w_B - v_{Bj} w_j \right|, \left| w_j - v_{jW} w_W \right| \right\}.$$

There is a unique solution of Model 2, that is the weights of criteria  $(w_1^*, w_2^*, \dots, w_n^*)$  and  $\xi^{L*}$ .  $\xi^{L*}$  can be regarded as the consistency indicator directly, and the value of  $\xi^{L*}$  is closer to 0, it represents a higher level of consistency. In this study, Model 2 is selected to calculate the weights of criteria.

### 1.4. The Weighted Aggregated Sum Product Assessment (WASPAS) method

The WASPAS method is an MCDM method developed in 2012. By combining the weighted sum model (WSM) and weighted product model (WPM), it can significantly improve the ranking accuracy and thus has been applied in many fields for solving actual MCDM problems. Turskis et al. (2015) combined the WASPAS and AHP with fuzzy values, and presented a hybrid model to solve construction site selection problem. Keshavarz Ghorabae et al. (2016) introduced an integrated MCDM approach based on the WASPAS method for supplier selection problem with interval type-2 fuzzy set (IT2FS). Devenci et al. (2018) introduced a hybrid MCDM method with IT2FS to solve the problem of car sharing station selection, which integrated the WASPAS with the TOPSIS. Mishra and Rani (2018) developed an MCDM approach by extending the

WASPAS method into interval-valued intuitionistic fuzzy environment, and used it into a case of flood control management. Ren et al. (2019) developed a SWARA-WASPAS approach with hesitant fuzzy linguistic information and applied it for the site selection of electric vehicle charging station.

For an MCDM problem, suppose that  $x_{ij}$  represents the performance value of the alternative  $A_i$  with regard to the criterion  $c_j$ , and  $w_j$  represents the relative importance of the criterion  $c_j$ . The steps for evaluating alternatives using the WASPAS method are as follows (Zavadskas et al., 2012):

**Step 1.** Normalize performance values in the following way:

for beneficial criteria, the normalized values  $\bar{x}_{ij}$  are computed by

$$\bar{x}_{ij} = \frac{x_{ij}}{\max_i x_{ij}} \quad (8)$$

for non-beneficial criteria, then,

$$\bar{x}_{ij} = \frac{\min_i x_{ij}}{x_{ij}}. \quad (9)$$

**Step 2.** Compute the values of WSM,  $Q_i^1 = \sum_{j=1}^m w_j \bar{x}_{ij}$  and WPM,  $Q_i^2 = \prod_{j=1}^m (\bar{x}_{ij})^{w_j}$ , for each alternative.

**Step 3.** Compute the aggregated measure  $Q_i = \lambda Q_i^1 + (1-\lambda)Q_i^2$  of the WASPAS method for each alternative where  $\lambda \in [0,1]$  is the parameter of the WASPAS method. When  $\lambda = 1$ , the WASPAS method turns into the WSM, and  $\lambda = 0$  leads to the WPM.

**Step 4.** Rank the alternatives in decreasing order of  $Q_i$ .

## 2. An integrated method with the extended BWM and WASPAS method under the LPF context

In many practical situations, decision-making information has strong uncertainty. In order to adapt to more complex decision-making environment, in this section, we extend the original BWM and WASPAS method into the LPF environment, and propose an integrated MCDM method. The LPF-BWM is investigated to compute the weights of criteria, while the LPF-WASPAS method is proposed to obtain the ranking of alternatives.

### 2.1. The LPF-BWM

In the LPF-BWM, experts are supposed to use LPFNs to express preference information for criteria. Firstly, each expert  $e_k$  ( $k=1,2,\dots,p$ ) selects the best and worst criteria from a set of criteria  $\{c_1, c_2, \dots, c_j, \dots, c_n\}$ , which are represented by  $\tilde{v}_B^k$  and  $\tilde{v}_W^k$ , respectively.

Then, each expert needs to provide their preferences of best-to-others through making pairwise comparisons. The preferences are expressed in LPFNs, and the LPF-BO vector is obtained as:

$$\tilde{V}_B^k = (\tilde{v}_{B1}^k, \tilde{v}_{B2}^k, \dots, \tilde{v}_{Bn}^k), \quad (10)$$

where  $\tilde{v}_{Bj}^k$  is an LPFN, which refers to the linguistic fuzzy preference of the best criterion  $c_B^k$  on criterion  $c_j$  determined by expert  $e_k$ , and  $\tilde{v}_{BB}^k = (s_4, s_4)$ . Similarly, each expert provides the preferences of worst-to-others using corresponding LPFNs, and obtains the LPF-OW vector as:

$$\tilde{V}_W^k = (\tilde{v}_{1W}^k, \tilde{v}_{2W}^k, \dots, \tilde{v}_{nW}^k)^T, \quad (11)$$

where  $\tilde{v}_{jW}^k$  refers to the linguistic fuzzy preference of criterion  $c_j$  on the worst criterion  $c_W^k$  determined by expert  $e_k$ , and  $\tilde{v}_{WW}^k = (s_4, s_4)$ .

Next, the preference vectors are transformed using the score function and LSFs. Since the score function can indicate the extent to which the alternative meets the requirements of decision-maker (Chen & Tan, 1994), the score function given as Eqn (1) can represent the preference degrees of experts in pairwise comparisons. Therefore, the LPF-BO vector and LPF-OW vector are transformed into the following forms, respectively:

$$S(\tilde{V}_B^k) = (S(\tilde{v}_{B1}^k), S(\tilde{v}_{B2}^k), \dots, S(\tilde{v}_{Bn}^k)); \quad (12)$$

$$S(\tilde{V}_W^k) = (S(\tilde{v}_{1W}^k), S(\tilde{v}_{2W}^k), \dots, S(\tilde{v}_{nW}^k))^T. \quad (13)$$

Afterwards, the LSF given as Eqn (7) is used to further convert the linguistic terms into numerical values. Then, the final LPF-BO vector and LPF-OW vector can be respectively deduced as:

$$f(S(\tilde{V}_B^k)) = (f(S(\tilde{v}_{B1}^k)), f(S(\tilde{v}_{B2}^k)), \dots, f(S(\tilde{v}_{Bn}^k))); \quad (14)$$

$$f(S(\tilde{V}_W^k)) = (f(S(\tilde{v}_{1W}^k)), f(S(\tilde{v}_{2W}^k)), \dots, f(S(\tilde{v}_{nW}^k)))^T. \quad (15)$$

The relationship between preference degrees and the weights of criteria can be expressed as:

$$f(S(\tilde{v}_{Bj}^k)) = \frac{w_B}{w_B + w_j}, f(S(\tilde{v}_{jW}^k)) = \frac{w_j}{w_j + w_W}. \quad (16)$$

Finally, for each expert, the following model is established:

### Model 3

$$\begin{aligned} & \min \zeta^k \\ & \text{s.t.} \\ & \left| w_B^k - f(S(\tilde{v}_{Bj}^k))(w_B^k + w_j^k) \right| \leq \zeta^k \\ & \left| w_j^k - f(S(\tilde{v}_{jW}^k))(w_j^k + w_W^k) \right| \leq \zeta^k \cdot \\ & \sum_j w_j^k = 1 \\ & w_j^k \geq 0, j=1,2,\dots,n, k=1,2,\dots,p \end{aligned}$$

Solving Model 3, the individual optimal weights  $(w_1^{k*}, w_2^{k*}, \dots, w_n^{k*})$  corresponding to each expert and the consistency indicator  $\zeta^{k*}$  are obtained. Then, we can compute the final collective weight of criterion  $c_j$  by taking the average value as follows:

$$w_j = \sum_{k=1}^p w_j^{k*} / p, \text{ for } j=1,2,\dots,n. \quad (17)$$

## 2.2. The LPF-WASPAS method

Experts use LPFNs to evaluate the alternatives, which can better express uncertainty and improve the accuracy of decision-making results. The steps of the LPF-WASPAS method are clarified as follows.

Firstly, experts are required to evaluate the alternatives  $A_i (i=1,2,\dots,m)$  according to the criteria set  $\{c_1, c_2, \dots, c_j, \dots, c_n\}$ , and then the LPF decision matrices are obtained as:

$$\tilde{D}_k = \begin{pmatrix} \tilde{x}_{11}^k & \cdots & \tilde{x}_{1j}^k & \cdots & \tilde{x}_{1n}^k \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{x}_{i1}^k & \cdots & \tilde{x}_{ij}^k & \cdots & \tilde{x}_{in}^k \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1}^k & \cdots & \tilde{x}_{mj}^k & \cdots & \tilde{x}_{mn}^k \end{pmatrix}, \text{ for } k=1,2,\dots,p,$$

where  $\tilde{x}_{ij}^k = (s_{\sigma_{ij}}, s_{\tau_{ij}})$  is a LPFN, indicating the performance value of the alternative  $A_i$  on the criterion  $c_j$  provided by expert  $e_k$ .

Then, we standardize the LPFNs in  $\tilde{D}_k$  and get the normalized LPF decision matrices  $\bar{\tilde{D}}_k$  as:

$$\bar{\tilde{D}}_k = \begin{pmatrix} \bar{\tilde{x}}_{11}^k & \cdots & \bar{\tilde{x}}_{1j}^k & \cdots & \bar{\tilde{x}}_{1n}^k \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{\tilde{x}}_{i1}^k & \cdots & \bar{\tilde{x}}_{ij}^k & \cdots & \bar{\tilde{x}}_{in}^k \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{\tilde{x}}_{m1}^k & \cdots & \bar{\tilde{x}}_{mj}^k & \cdots & \bar{\tilde{x}}_{mn}^k \end{pmatrix}, \text{ for } k=1,2,\dots,p,$$

where:

$$\bar{\tilde{x}}_{ij}^k = (s_{\sigma_{ij}}, s_{\tau_{ij}}), \text{ for beneficial criteria;} \quad (18)$$

$$\bar{\tilde{x}}_{ij}^k = (s_{\tau_{ij}}, s_{\sigma_{ij}}), \text{ for non-beneficial criteria.} \quad (19)$$

Next, according to the relative importance of experts  $\lambda_k (k=1,2,\dots,p)$ , we utilize the LPFWA operator to integrate all the experts' evaluation information into an aggregated decision matrix  $\tilde{D}$  as:

$$\tilde{D} = \begin{pmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1j} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{x}_{i1} & \cdots & \tilde{x}_{ij} & \cdots & \tilde{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mj} & \cdots & \tilde{x}_{mn} \end{pmatrix},$$

where

$$\tilde{x}_{ij} = LPFWA_{\lambda}(\tilde{x}_{ij}^1, \tilde{x}_{ij}^2, \dots, \tilde{x}_{ij}^p) = \bigoplus_{k=1}^p (\lambda_k \tilde{x}_{ij}^k) = (s \sqrt[1 - \prod_{k=1}^p (1 - \sigma_{ij}^2 / t^2)^{\lambda_k}], s \prod_{k=1}^p (\tau_{ij} / t)^{\lambda_k}). \quad (20)$$

Afterwards, combining the weights of criteria,  $\{w_1, w_2, \dots, w_n\}$ , we can compute the values of the WSM ( $Q_1^i$ ) and the WPM ( $Q_2^i$ ) for each alternative by Eqn (21) and Eqn (22), respectively:

$$Q_1^i = LPFWA_w(\tilde{x}_{i1}, \tilde{x}_{i2}, \dots, \tilde{x}_{in}) = (s \sqrt[1 - \prod_{j=1}^n (1 - \sigma_{ij}^2 / t^2)^{w_j}], s \prod_{j=1}^n (\tau_{ij} / t)^{w_j}); \quad (21)$$

$$Q_2^i = LPFWG_w(\tilde{x}_{i1}, \tilde{x}_{i2}, \dots, \tilde{x}_{in}) = (s \prod_{j=1}^n (\sigma_{ij} / t)^{w_j}, s \sqrt[1 - \prod_{j=1}^n (1 - \tau_{ij}^2 / t^2)^{w_j}]). \quad (22)$$

Finally, we compute the evaluation value  $Q_i$  of each alternative and rank the alternatives:

$$Q_i = \lambda Q_1^i + (1 - \lambda) Q_2^i. \quad (23)$$

## 2.3. Procedure of the LPF-BWM-WASPAS method

This subsection presents an integrated multi-experts MCDM method combing the LPF-BWM and the LPF-WASPAS method. To better illustrate the application process of the proposed method, the main steps are briefly shown as below.

**Step 1.** Set up a decision-making committee composed of multiple experts  $\{e_1, e_2, \dots, e_k, \dots, e_p\}$ . Define the objectives of the decision-making problem and determine the criteria set  $\{c_1, c_2, \dots, c_j, \dots, c_n\}$  and alternative set  $\{A_1, A_2, \dots, A_i, \dots, A_m\}$ .

**Step 2.** Identify the best criterion  $c_B^k$  and the worst criterion  $c_W^k$ , and obtain the LPF-BO and LPF-OW vectors of each experts by making pairwise comparisons.

**Step 3.** Convert the LPFNs in the preference vectors to crisp numbers using the score function and the LSF, and obtain the individual optimal weights  $(w_1^{k*}, w_2^{k*}, \dots, w_n^{k*})$  corresponding to each expert and the consistency indicator  $\zeta^{k*}$  by Model 3. If the acceptable consistency is met, the next step is taken. Otherwise, a reevaluation step is needed.

**Step 4.** Compute the final collective weights of criteria by Eqn (17).

**Step 5.** The LPF decision matrix of each expert is obtained and standardized. Then, according to the weights of experts, obtain the aggregated LPF decision matrix by Eqn (20).

**Step 6.** Compute the aggregated measure of the LPF-WASPAS method for each alternative by Eqns (21)–(23), and get the whole ranking of alternatives.

In the above steps, Steps 2–4 belong to the LPF-BWM, which is used for computing the weights of criteria. In Step 3, experts use LPFNs to express their preferences. Then, the linguistic preference information is converted to numerical values based on the score function and the LSF. Steps 4–6 belong to the LPF-WASPAS method, which are the process of determining the ranking of alternatives. Using the above procedure, we can solve multi-experts MCDM problems under the LPF environment.

### 3. Case study: an evaluation of energy-saving building programs

In this section, we present a case study on the evaluation of energy-saving building programs, and the LPF-BWM-WASPAS method is utilized to solve this case. Then, a sensitivity analysis is further implemented to illustrate the reliability and practicability of this method.

#### 3.1. Case description

With the rapid economic and social development, the construction industry is expanding rapidly, and the demand for construction is increasing. Building energy consumption is also showing an increasing trend. In China, energy consumption in construction industry accounted for a large proportion of all industries, reaching 20% in 2015, with China’s further urbanization and industrialization, China’s building energy consumption will continue to increase (Wang et al., 2019). Because of the huge energy consumption in the construction industry, building energy conservation has become a trend of the construction industry. China has already begun to promote energy-saving buildings.

Company B is a real estate enterprise and one of the top 10 real estate development enterprises in China. Its main business includes real estate development, commercial operation, and intelligent service. The Company B intends to develop a new project in Chengdu, China, to build a residential community. Because Chengdu is very hot in summer, cold and humid in winter, and the temperature changes dramatically, and thus residents use air conditioning appliances frequently. In response to the strategy of sustainable development, the energy-saving building is considered in this project. To maximize the benefits and meet the requirements of building energy conservation, the Company B requires the design unit to provide different alternatives. Among all the alternatives, the optimum program which meets the requirements of economy, energy consumption and usability is selected as the final design program. Now there are three alternatives  $A_i (i = 1, 2, 3)$  to choose from, and the Company B invites four experts  $e_k (k = 1, 2, 3, 4)$  from construction-related specialties to form a decision-making committee to evaluate the alternatives. Due to the different working years and professional titles of each expert, the relative importance of the experts is given as  $\{0.3, 0.2, 0.4, 0.1\}$ . According to the meaning of energy-saving buildings and the objectives of this project, the evaluation criteria are determined from three dimensions: economy, usability and environment. The specific criteria are presented in Table 1.

Among these criteria, the construction cost ( $c_1$ ) refers the direct cost of the project, and the internal rate of return ( $c_2$ ) reflects the profitability and risk resistance of the project. The functional rationality ( $c_3$ ) refers to the degree to which the building meets the living needs of residents and whether the functional zoning is reasonable or not. The maintainability ( $c_4$ ) is the ease of carry-

Table 1. Evaluation criteria of energy-saving building programs

Dimension	Criterion	Type of the criterion
Economy	Construction cost ( $c_1$ )	Non-beneficial
	Internal rate of return ( $c_2$ )	Beneficial
Usability	Functional rationality ( $c_3$ )	Beneficial
	Maintainability ( $c_4$ )	Beneficial
	Comfort ( $c_5$ )	Beneficial
	Safety ( $c_6$ )	Beneficial
Environment	Sustainability ( $c_7$ )	Beneficial
	Performance of building materials ( $c_8$ )	Beneficial
	Application of energy-saving technology ( $c_9$ )	Beneficial
	Building energy consumption level ( $c_{10}$ )	Non-beneficial

ing out the later maintenance work of the building. The comfort ( $c_5$ ) refers to the satisfaction of living. The safety ( $c_6$ ) includes structural safety and environmental safety. The sustainability ( $c_7$ ) is the application degree of sustainable concept in architectural design. The performance of building materials ( $c_8$ ) is the basic performance of building materials involved in design programs. The application of energy-saving technology ( $c_9$ ) refers to the suitability of energy-saving technologies used in the program, and the estimated energy-saving effect. The building energy consumption level ( $c_{10}$ ) refers to the estimated annual building energy consumption of the program.

#### 3.2. Select the optimal program by the LPF-BWM-WASPAS method

To select the optimal energy-saving program, the alternatives are evaluated by the LPF-BWM-WASPAS method. The specific procedure is shown below.

In the evaluation process, experts provide their preferences for alternatives and criteria with the following linguistic term set  $S$ :

$$S = \left\{ \begin{array}{l} s_0 = \text{extremely bad}, s_1 = \text{very bad}, s_2 = \text{bad}, \\ s_3 = \text{medium bad}, s_4 = \text{medium}, s_5 = \text{medium good}, \\ s_6 = \text{good}, s_7 = \text{very good}, s_8 = \text{extremely good} \end{array} \right\}.$$

**Step 1.** As shown above, the decision-making committee, criteria and alternatives are determined.

**Step 2.** Each expert selects the best and worst criteria based on his own experience and opinions, and uses LPFNs to determine the preferences in pairwise comparisons. The linguistic fuzzy preferences provided by each expert are collected in Tables 2 and 3.

**Step 3.** Using the score function of LPFNs and the LSF of linguistic terms given as Eqn (7), the preferences of experts are converted into crisp numbers, as shown in Tables 4 and 5.

Table 2. Linguistic fuzzy preferences of best-to-others

Expert	Best criterion	Other criteria									
		$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$
$e_1$	$c_{10}$	$(s_7, s_3)$	$(s_6, s_3)$	$(s_8, s_4)$	$(s_8, s_0)$	$(s_7, s_5)$	$(s_6, s_4)$	$(s_4, s_2)$	$(s_5, s_4)$	$(s_6, s_5)$	$(s_4, s_4)$
$e_2$	$c_7$	$(s_8, s_0)$	$(s_6, s_3)$	$(s_7, s_3)$	$(s_6, s_2)$	$(s_6, s_3)$	$(s_5, s_4)$	$(s_4, s_4)$	$(s_5, s_3)$	$(s_5, s_4)$	$(s_6, s_5)$
$e_3$	$c_9$	$(s_7, s_2)$	$(s_6, s_3)$	$(s_8, s_0)$	$(s_7, s_2)$	$(s_5, s_3)$	$(s_6, s_1)$	$(s_4, s_2)$	$(s_5, s_3)$	$(s_4, s_4)$	$(s_5, s_4)$
$e_4$	$c_{10}$	$(s_7, s_2)$	$(s_6, s_1)$	$(s_5, s_1)$	$(s_6, s_2)$	$(s_8, s_0)$	$(s_5, s_4)$	$(s_4, s_2)$	$(s_5, s_3)$	$(s_5, s_4)$	$(s_4, s_4)$

Table 3. Linguistic fuzzy preferences of others-to-worst

Expert	Worst criterion	Other criteria									
		$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$
$e_1$	$c_4$	$(s_5, s_3)$	$(s_7, s_5)$	$(s_5, s_4)$	$(s_4, s_4)$	$(s_5, s_3)$	$(s_5, s_2)$	$(s_7, s_3)$	$(s_6, s_2)$	$(s_6, s_1)$	$(s_8, s_0)$
$e_2$	$c_1$	$(s_4, s_4)$	$(s_6, s_3)$	$(s_5, s_3)$	$(s_7, s_5)$	$(s_6, s_4)$	$(s_5, s_3)$	$(s_8, s_0)$	$(s_8, s_4)$	$(s_7, s_2)$	$(s_6, s_1)$
$e_3$	$c_3$	$(s_5, s_4)$	$(s_5, s_1)$	$(s_4, s_4)$	$(s_5, s_2)$	$(s_6, s_4)$	$(s_7, s_2)$	$(s_6, s_1)$	$(s_8, s_4)$	$(s_8, s_0)$	$(s_7, s_4)$
$e_4$	$c_5$	$(s_5, s_3)$	$(s_5, s_1)$	$(s_4, s_1)$	$(s_6, s_4)$	$(s_4, s_4)$	$(s_7, s_2)$	$(s_8, s_3)$	$(s_6, s_3)$	$(s_6, s_2)$	$(s_8, s_0)$

Table 4. Numerical preferences of best-to-others

Experts	Best criterion	Other criteria									
		$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$
$e_1$	$c_{10}$	0.9014	0.8432	0.9354	1	0.8292	0.8101	0.7706	0.7552	0.7655	0.7071
$e_2$	$c_7$	1	0.8432	0.9014	0.8660	0.8432	0.7552	0.7071	0.7906	0.7552	0.7655
$e_3$	$c_9$	0.9228	0.8432	1	0.9228	0.7906	0.8795	0.7706	0.7906	0.7071	0.7552
$e_4$	$c_{10}$	0.9228	0.8795	0.8292	0.8660	1	0.7552	0.7706	0.7906	0.7552	0.7071

Then, according to Model 3, the optimization models with respect to each expert are set up to compute the individual weights of criteria. Taking expert  $e_1$  as an example, the following model is established:

**Model 4**

$$\min \zeta^1$$

s.t.

$$\begin{aligned} &|w_{10}^1 - 0.9014(w_{10}^1 + w_1^1)| \leq \zeta^1, |w_{10}^1 - 0.8432(w_{10}^1 + w_2^1)| \leq \zeta^1 \\ &|w_{10}^1 - 0.9354(w_{10}^1 + w_3^1)| \leq \zeta^1, |w_{10}^1 - (w_{10}^1 + w_4^1)| \leq \zeta^1 \\ &|w_{10}^1 - 0.8292(w_{10}^1 + w_5^1)| \leq \zeta^1, |w_{10}^1 - 0.8101(w_{10}^1 + w_6^1)| \leq \zeta^1 \\ &|w_{10}^1 - 0.7706(w_{10}^1 + w_7^1)| \leq \zeta^1, |w_{10}^1 - 0.7552(w_{10}^1 + w_8^1)| \leq \zeta^1 \\ &|w_{10}^1 - 0.7655(w_{10}^1 + w_9^1)| \leq \zeta^1, |w_1^1 - 0.7906(w_1^1 + w_4^1)| \leq \zeta^1 \\ &|w_2^1 - 0.8292(w_2^1 + w_4^1)| \leq \zeta^1, |w_3^1 - 0.7552(w_3^1 + w_4^1)| \leq \zeta^1 \\ &|w_5^1 - 0.7906(w_5^1 + w_4^1)| \leq \zeta^1, |w_6^1 - 0.8149(w_6^1 + w_4^1)| \leq \zeta^1 \\ &|w_7^1 - 0.9014(w_7^1 + w_4^1)| \leq \zeta^1, |w_8^1 - 0.8660(w_8^1 + w_4^1)| \leq \zeta^1 \\ &|w_9^1 - 0.8795(w_9^1 + w_1^1)| \leq \zeta^1 \end{aligned}$$

$$\sum_{j=1}^{10} w_j^1 = 1, w_j^1 \geq 0.$$

Similarly, we can build optimization models in terms of the other experts. Solving these models by LINGO software, we can get the individual weights of criteria and the corresponding consistency indicator values (see Table 6).

**Step 4.** Using Eqn (17), the final collective weights of criteria are obtained as  $\{0.0403, 0.0735, 0.0469, 0.0494, 0.0634, 0.0913, 0.1666, 0.1003, 0.1335, 0.2348\}$ .

**Step 5.** Each expert uses LPFNs to provide the performance values of all the alternatives with regard to every criterion, and the decision matrixes are shown below:

$$\tilde{D}_1 = \begin{pmatrix} (s_5, s_3) & (s_4, s_2) & (s_6, s_5) & (s_3, s_4) & (s_4, s_3) & (s_5, s_1) \\ (s_6, s_4) & (s_4, s_3) & (s_3, s_2) & (s_5, s_4) & (s_6, s_1) & (s_4, s_3) \\ (s_4, s_3) & (s_3, s_2) & (s_7, s_5) & (s_5, s_3) & (s_4, s_2) & (s_5, s_4) \\ (s_6, s_3) & (s_4, s_2) & (s_3, s_2) & (s_6, s_4) \\ (s_3, s_1) & (s_6, s_3) & (s_6, s_5) & (s_4, s_3) \\ (s_5, s_3) & (s_6, s_4) & (s_4, s_2) & (s_3, s_4) \end{pmatrix};$$

$$\tilde{D}_2 = \begin{pmatrix} (s_4, s_5) & (s_5, s_2) & (s_2, s_3) & (s_6, s_1) & (s_4, s_3) & (s_7, s_4) \\ (s_6, s_4) & (s_7, s_3) & (s_3, s_5) & (s_4, s_4) & (s_5, s_2) & (s_4, s_6) \\ (s_2, s_4) & (s_6, s_3) & (s_7, s_2) & (s_5, s_3) & (s_6, s_2) & (s_6, s_7) \\ (s_4, s_7) & (s_6, s_1) & (s_4, s_2) & (s_6, s_2) \\ (s_6, s_2) & (s_7, s_5) & (s_5, s_7) & (s_2, s_5) \\ (s_7, s_5) & (s_5, s_3) & (s_2, s_5) & (s_4, s_7) \end{pmatrix};$$

$$\tilde{D}_3 = \begin{pmatrix} (s_3, s_4) & (s_4, s_6) & (s_5, s_3) & (s_6, s_4) & (s_5, s_1) & (s_7, s_4) \\ (s_4, s_4) & (s_2, s_4) & (s_6, s_4) & (s_2, s_7) & (s_4, s_4) & (s_5, s_7) \\ (s_3, s_6) & (s_7, s_2) & (s_6, s_4) & (s_3, s_6) & (s_2, s_5) & (s_3, s_5) \\ (s_2, s_7) & (s_3, s_5) & (s_7, s_4) & (s_7, s_5) \\ (s_7, s_1) & (s_3, s_2) & (s_5, s_3) & (s_1, s_7) \\ (s_1, s_7) & (s_5, s_2) & (s_6, s_3) & (s_7, s_2) \end{pmatrix};$$



Table 5. Numerical preferences of others-to-worst

Experts	Worst criterion	Other criteria									
		$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$
$e_1$	$c_4$	0.7906	0.8292	0.7552	0.7071	0.7906	0.8149	0.9014	0.8660	0.8795	1
$e_2$	$c_1$	0.7071	0.8432	0.7906	0.8292	0.8101	0.7906	1	0.9354	0.9228	0.8795
$e_3$	$c_3$	0.7552	0.8292	0.7071	0.8149	0.8101	0.9228	0.8795	0.9354	1	0.8705
$e_4$	$c_5$	0.7906	0.8292	0.7856	0.8101	0.7071	0.9228	0.9642	0.8432	0.8660	1

Table 6. The individual optimal weights of criteria

Experts	Criteria weights										Consistency indicator
	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$	
$e_1$	0.0475	0.0754	0.0328	0.0076	0.0649	0.0744	0.1162	0.1057	0.1184	0.3571	0.0076
$e_2$	0.0105	0.0720	0.0467	0.0616	0.0720	0.0898	0.3200	0.0980	0.1176	0.1118	0.0105
$e_3$	0.0660	0.0905	0.0459	0.0660	0.1094	0.0787	0.1172	0.1093	0.1935	0.1235	0.0459
$e_4$	0.0371	0.0560	0.0624	0.0623	0.0075	0.1223	0.1129	0.0882	0.1045	0.3467	0.0075

$$\tilde{D}_4 = \begin{pmatrix} (s_5, s_7) & (s_5, s_1) & (s_7, s_3) & (s_4, s_4) & (s_3, s_6) & (s_6, s_1) \\ (s_6, s_3) & (s_4, s_1) & (s_2, s_6) & (s_6, s_3) & (s_7, s_2) & (s_3, s_6) \\ (s_5, s_2) & (s_6, s_1) & (s_2, s_3) & (s_6, s_3) & (s_7, s_4) & (s_2, s_6) \\ (s_5, s_2) & (s_2, s_4) & (s_4, s_3) & (s_2, s_7) \\ (s_5, s_2) & (s_3, s_4) & (s_3, s_7) & (s_5, s_1) \\ (s_6, s_5) & (s_3, s_2) & (s_6, s_2) & (s_7, s_5) \end{pmatrix}.$$

According to Eqns (18) and (19), the decision matrices are standardized. Then, we can get the following aggregated decision matrix by Eqn (20):

$$\tilde{D} = \begin{pmatrix} (s_{4.6152}, s_{3.8981}) & (s_{4.3473}, s_{2.8958}) & (s_{5.3871}, s_{3.4968}) \\ (s_{3.9169}, s_{5.1017}) & (s_{4.7341}, s_{3.0157}) & (s_{4.6529}, s_{3.5379}) \\ (s_{6.4197}, s_{3.1737}) & (s_{5.1683}, s_{2.0237}) & (s_{4.7029}, s_{3.6177}) \\ (s_{5.2740}, s_{3.0314}) & (s_{4.3841}, s_{2.0719}) & (s_{6.5371}, s_{2.2974}) \\ (s_{4.1564}, s_{4.8616}) & (s_{5.3971}, s_{2.1435}) & (s_{4.3841}, s_{5.1835}) \\ (s_{6.5796}, s_{3.9585}) & (s_{6.4764}, s_{3.0925}) & (s_{6.6123}, s_{5.0938}) \\ (s_{4.5024}, s_{4.7896}) & (s_{4.1793}, s_{2.6922}) & (s_{5.7097}, s_{2.7482}) \\ (s_{6.0323}, s_{1.2311}) & (s_{5.4187}, s_{2.9077}) & (s_{5.2398}, s_{4.5089}) \\ (s_{6.1446}, s_{4.9076}) & (s_{6.0452}, s_{2.6703}) & (s_{6.1914}, s_{2.8252}) \\ (s_{4.7835}, s_{5.7177}) \\ (s_{5.7487}, s_{2.0451}) \\ (s_{6.3823}, s_{4.8540}) \end{pmatrix}.$$

**Step 6.** Using Eqns (21)–(23), we can compute the measures of the LPF-WASPAS method of these three alternatives. When  $\lambda = 0.5$ , the values of  $Q_i$  and the corresponding scores are listed in Table 7.

Table 7. The measures of the LPF-WASPAS method with  $\lambda = 0.5$

	$A_1$	$A_2$	$A_3$
$Q_i^1$	$(s_{5.0798}, s_{3.5723})$	$(s_{5.3597}, s_{2.7007})$	$(s_{6.2093}, s_{3.7433})$
$Q_i^2$	$(s_{4.9042}, s_{4.1960})$	$(s_{5.2293}, s_{3.4427})$	$(s_{6.1336}, s_{4.1313})$
$Q_i$	$(s_{4.9937}, s_{3.8716})$	$(s_{5.2955}, s_{3.0492})$	$(s_{6.1719}, s_{3.9325})$
$S(Q_i)$	6.0806	6.4321	6.5813

It can be concluded from Table 7 that the priority of the three alternatives is  $A_3 \succ A_2 \succ A_1$ , so alternative  $A_3$  is the optimal energy-saving building program.

#### 4. Discussion

##### 4.1. Sensitivity analysis

When the WASPAS method is used to sort alternatives, the selection of  $\lambda$  is subjective. According to specific problems, experts may choose the WSM model or WPM model. In this regard, they can adjust the coefficient subjectively. In this subsection, a sensitivity analysis is implemented to verify the stability of the proposed method by changing the value of parameter  $\lambda$ . When  $\lambda$  takes the values from 0 to 1, the score values of  $Q_i$  are presented in Table 8 and demonstrated by Figure 1.

According to Table 8 and Figure 1, we can conclude that the ranking of the three alternatives is always  $A_3 \succ A_2 \succ A_1$ . The ranking of alternatives does not change with the value of  $\lambda$ , which indicates that this method has good stability.

##### 4.2. Managerial insights of the proposed method

From the above case study we can conclude that practitioners can use this model to solve MCDM problems in many other fields. In the complex decision-making environment, when uncertainties need to be handled, this method shows advantages. Applicable aspects include green supplier selection, construction site selection, investment project evaluation, performance evaluation, facility location and other issues. In specific applications, practitioners can select appropriate criteria according to their needs, evaluate the performance values of alternatives with respect to the criteria, and get the utility values of the alternatives through the weights of criteria and the performance values of alternatives under each criterion.

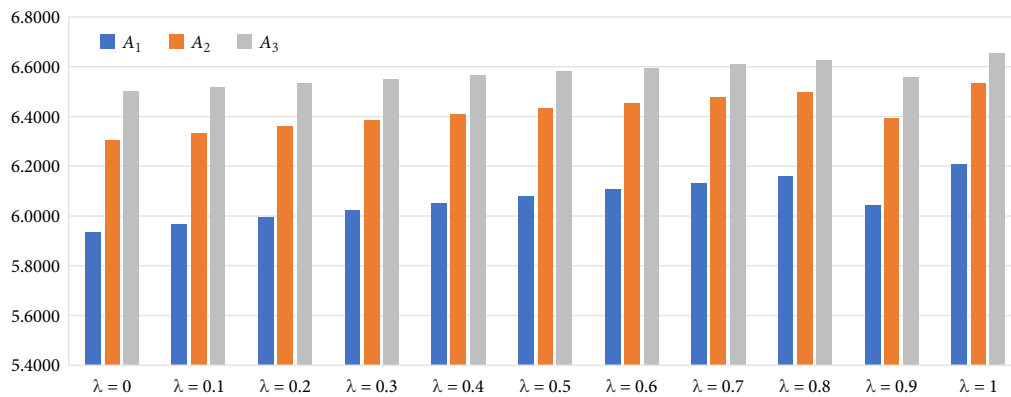


Figure 1. Sensitivity analysis results for different values of λ

Table 8. The score values of  $Q_i$  for different values of λ

Alternatives	λ = 0	λ = 0.1	λ = 0.2	λ = 0.3	λ = 0.4	λ = 0.5	λ = 0.6	λ = 0.7	λ = 0.8	λ = 0.9	λ = 1
A <sub>1</sub>	5.9348	5.9658	5.9958	6.0249	6.0532	6.0806	6.1073	6.1332	6.1583	6.0422	6.2066
A <sub>2</sub>	6.3045	6.3323	6.3589	6.3843	6.4087	6.4321	6.4546	6.4761	6.4968	6.3926	6.5358
A <sub>3</sub>	6.5021	6.5185	6.5346	6.5505	6.5661	6.5813	6.5963	6.6111	6.6255	6.5571	6.6537

When the LPF-BWM method is used to determine the weights of criteria, the linguistic terms can better express the preference information of experts for criteria, and establish a consistency model through pairwise comparisons to get the reasonable weights of criteria. The LPF-WASPAS method integrates two models at the same time, which can improve the decision-making accuracy when choosing the optimal alternative. In addition, this method involves complex operation process. The development of auxiliary calculation program can be considered to make the decision-making process easier.

**Conclusions**

The implementation of energy-saving building is an important measure to improve living comfort, promote environmental protection and reduce pollutant emission. It is also an important part of the sustainable development strategy. Because the energy-saving building program directly affects the actual energy-saving effect, the decision-making at the design stage is of vital importance. Decision-making of energy-saving programs is an MCDM problem, which faces uncertainties in the decision-making process. As for uncertainty modeling, compared with the IFS, the PFS provides more freedom and flexibility for DMs. Furthermore, the LPFS allows experts to use linguistic terms to express membership and non-membership degree, which conforms to decision-making habits of decision-makers and is easier to understand. The BWM and WASPAS method are efficient MCDM methods, which have been widely used for solving practical decision-making problems. When considering the weights of criteria, the BWM needs fewer pairwise comparisons and higher consistency level than that of the AHP, and thus it has great advantages in dealing with decision-making problems that needs to

consider a large number of criteria at the same time. The WASPAS method integrates the WSM and WPM models to make the decision results more accurate.

Therefore, this paper extended the BWM and WASPAS method to the LPF environment, and proposed a new multi-expert MCDM method in which the LPF-BWM was developed to derive the weights of criteria, while the LPF-WASPAS was developed to obtain the ranking of all alternatives. To prove the practicability of the proposed method, this paper applied it to the evaluation of energy-saving building programs. Then, a sensitivity analysis was implemented through changing the values of parameter in the WASPAS method to verify the stability of the proposed method. The result showed that this method has good applicability and stability.

The limitations of this study and future research directions are as follows:

- 1) The criteria are determined by experts without considering the needs of consumers. In the further study, it is a good idea to link design needs with customer needs.
- 2) The psychological and behavioral factors of DMs are not considered in the selection process. In the further research, it is necessary and meaningful to apply prospect theory and regret theory to solve the practical problem of architectural scheme selection.
- 3) Besides, the research on LPFSs focused more on theoretical and qualitative researches, and the LPFS has not been widely used for practical decision-making problems. In future research, more MCDM methods, such as the double normalization-based multiple aggregation method (Liao & Wu, 2020), can be extended to the LPF environment, so as to compare with our results and apply them to more fields to solve practical problems.

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## Author contributions

Min Huang, Xinli Zhang and Huchang Liao proposed the original idea and conceived the study. Min Huang and Huchang Liao were responsible for developing the method, collecting and analyzing the data. Ruxue Ren and Huchang Liao were responsible for data interpretation. Min Huang and Xinli Zhang wrote the first draft of the article. Ruxue Ren, Huchang Liao, Edmundas Kazimieras Zavadskas and Jurgita Antuchevičienė revised the paper.

## Disclosure statement

The authors have no competing financial, professional, or personal interests from other parties that are related to this paper.

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