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A. MEASUREMENT OF THE VELOCITY OF LIGHT

1. RESUMPTION OF MEASUREMENT AND OBJECTIVES

After a three-year interval, during which G. W. Stroke was engaged in theoretical and experimental work involved in the improvement of high-resolution diffraction gratings, we have resumed this measurement which should result in the first measurement of the velocity of light carried out in terms of primary constants. We are first aiming for an accuracy of 10^{-6}, which we estimate to be at least one order of magnitude better than the accuracy of measurements that have been described thus far.

A study of past experiments shows a spread in the centers of gravity of the values of c of almost 10^{-4}, even though the greatest precision given is of the order of 3×10^{-7}. The most recent value for c is now given as c = 299,792.5 ± 0.1 km/sec. It thus becomes clear that the accuracy of c in past measurements was not nearly as great as the precision of the experiments, and one can easily attribute the lack of accuracy to uncertainties in the values of the length standards which have been used heretofore. In fact, any attempt to read more into past discrepancies in various measurements, of different types, carried out at various periods of time (even within the past 50 years) are found to fail because of the inability of checking the length standard used in a particular experiment (or of estimating the absolute error made in its use). (The situation is, of course, somewhat better for the frequency or time standards; here, astronomical records and references to the period of rotation of the earth are of help). It is clear, of course, that we shall make our measurement directly in terms of primary constants (Hg^{198} light source for the length, and Cs^{133} atomic clock for the frequency). Thus we hope to get an accurate value in terms of these standards, provided that any systematic errors are avoided or properly accounted for, as is usually the case in such measurements.

We shall first use our microwave cavity resonance apparatus, which has been described previously,[1] in an attempt to measure c accurately to 10^{-6}. We have also previously reported[2] that variations in the diameter of our cavity are suppressed by a

factor of 10 in the TE_{01n} mode, as a result of a judicious choice of the diameter and lengths for our precision optically ground and polished quartz cavity (which will be silver-plated on the inside). Therefore, if there were only a simple geometrical effect (which is in accord with an estimate of the effect of a small perturbation of a perfect cavity, and in terms of a solution of Maxwell's equations), it would appear that a variation in diameter of 10^{-5} (or approximately 6 optical fringes, or 3 optical wavelengths in the visible domain) in the 6-inch diameter would be tolerable. In reality, the perturbations enter as

$$\int_V (H^2 - E^2)\, dv \tag{1}$$

in a particular perturbed volume region within the cavity; they can thus be estimated somewhat more exactly than they could from geometrical considerations, but the order of magnitude given above is a good one to guide the progress of the experiment. In fact, it immediately appears that the first investigation necessary for the advancement of the experiment is one that is designed to verify the uniformity of the precision cavity that we obtained in 1957 (and which was mechanically measured by the manufacturer to be uniform within approximately 1 optical wavelength).[3] Should the cavity be found to be good within a few optical wavelengths (in accord with the information given by the optician who made it), then a measurement of c that is accurate to 10^{-6} might be obtained rapidly, and undoubtedly with much greater reliability than would usually be associated with correction calculations (even though these may be carried out satisfactorily). The use of a scheme of pneumatic distance-variation detection and the availability of the moving piston (the straightness of motion of which has now been ascertained) are new elements that will permit a considerably more accurate, and much simpler, measurement of the uniformity of the cavity (in diameter and straightness) than the interferometric means that was available at the time of the manufacture of the cylinder. We note also that this new cylinder-measuring scheme might permit a local optical retouching of cylinders, which would not be unusual, neither costly nor long, and would be similar to that used in finishing astronomical mirrors of comparable quality.

Preliminary measurements indicate that the cavity may be sufficiently uniform to permit a measurement of c without the necessity of perturbation calculations, at least for measurements that are accurate to 10^{-6}.

We can report on a number of significant points of progress in this direction. These are discussed in the following sections. All of this work should be referred to previous reports published from 1956 to 1958.

<div align="right">G. W. Stroke</div>

<div align="center">(References on following page)</div>

References

1. G. W. Stroke and J. R. Zacharias, Measurement of the velocity of light,
Quarterly Progress Report No. 61, Research Laboratory of Electronics, M.I.T.,
October 15, 1958, pp. 69-73.

2. G. W. Stroke and C. L. Searle, Measurement of the velocity of light, Quarterly
Progress Report, Research Laboratory of Electronics, M.I.T., July 15, 1956,
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3. J. R. Zacharias and G. W. Stroke, Measurement of the velocity of light,
Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., January 15,
1958, pp. 57-59.

2. STRAIGHTNESS OF TRAVEL OF A MOVING PISTON IN THE 10^{-7} RADIAN DOMAIN

The new alignment interferometer which has been developed by one of us[1] for the
rapid measurement of the straightness of travel of a moving carriage with a sensitivity
of the order of 10^{-7} radian (1/50 second of arc) has been applied to the measurement
of the straightness of travel of the moving piston that serves to obtain the resonance in
the cavity. The piston is moved by a Pyrex cylinder, 2 inches in diameter and 20 inches
long. It is guided by two sets of three brass cylinders (1/4 inch in diameter), tangential
to the Pyrex cylinder and set 6 inches apart. We have found that this arrangement moves
the piston parallel to itself (in the direction of the Pyrex cylinder axis) within ±1 fringe
(10^{-5} inch) measured across a 6-inch lever arm (equal to the cavity diameter), or approx-
imately ±0.3 second of arc. Recall that the 2-inch Pyrex cylinder was optically polished
to be straight, after having been ground on the cylindrical grinder in the machine shop
of the Research Laboratory of Electronics. This work had been carried out, however,
at a time when no straightness measuring tool (such as the new alignment interferometer)
was available. This cylinder straightness is good enough to permit measurement of the
uniformity of the quartz cavity with the help of pneumatic detection, which will be dis-
cussed in the next section. However, we may have to further improve the straightness
of the piston motion for the actual resonance measurement, with the help of servo control
of the brass-cylinder position, or alternatively use two alignment interferometers to help
in calculating the effects of small tilts in the measurement.

A schematic diagram of the interferometer is shown in Fig. VII-1. The interfero-
meter consists of a folded Michelson-Twyman-Green arrangement in which the two mir-
rors, (5) and (6) in Fig. VII-1, may form part of the same "moving mirror," which, when
it advances in the y-direction without any rotation about the z-axis, results in an increase
of the same amount in the path lengths SAD and SBCD', and therefore in no additional
path difference of fringe shift in the observing region R. In fact, it can easily be
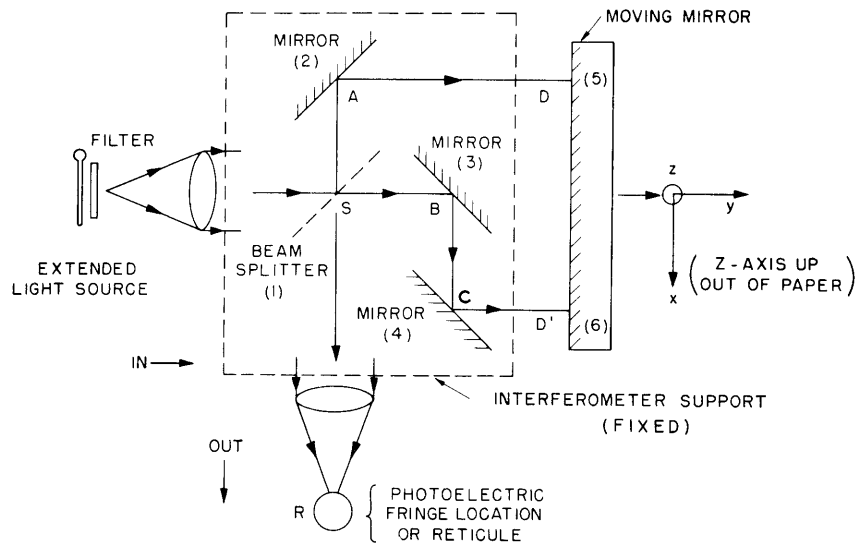seen that a fringe shift will be detected at R only if there is any rotation of the

Fig. VII-1. Alignment interferometer.

"moving mirror" about the z-axis, which gives the desired sensitivity. It is also clear that the interferometric sensitivity is obtained without any of the usual interference fringe "counting" involved in interferometric control interferometers; this feature permits measurement at very high piston speeds if they are necessary. A full description of the interferometer has been given in a paper[1] that will appear in the Journal of the Optical Society of America.

G. W. Stroke, A. Funkhouser

References

1. G. W. Stroke, An alignement interferometer for precision straightness measurements (to be published in J. Opt. Soc. Am.).

3. CAVITY DIAMETER MEASUREMENT

We have previously come to the conclusion that a measurement of the absolute diameter (in optical wavelengths) of the silver-coated cavity could best be obtained by setting a Fabry-Pérot interferometer with its plates tangential to the cavity, with the help of a pneumatic null-reading method such as that described in the next section. The separation of the FP plates can be measured by mere inspection of the coincidence of fringes of different wavelengths originating from different light sources. In future reports we shall give more details about a program prepared for this purpose, during the past quarterly period, on the IBM 709 computer. This program is designed to permit

carrying out such an "absolute" measurement after only a comparatively crude mechanical length measurement to 1/1000 inch. In the past, it was necessary to carry out rather precise mechanical length measurements to better than 1/10,000 inch when using visible wavelengths of 2×10^{-5} inch. The new element here is that we have looked for coincidences of <u>several</u> <u>wavelengths</u> at a particular FP plate separation. This permits us to "slice up" space, as it were, into regions that are rather well separated (by distances of approximately 1/1000 inch).

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4. PNEUMATIC DETECTION OF SMALL MOTIONS IN THE MICROINCH DOMAIN

Pneumatic measurements of the variation of the distance of a plate from a nozzle with sensitivities of displacement of a liquid column in a manometer up to 1 inch per 10^{-5} inch of plate displacement have been used for more than 20 years, but remarkably little theory has been published. In fact, we have now been able to show that the physical description of the situation can be rather simple and, as a result of the simple description, significant improvements in the present remarkable sensitivity and accuracy have become possible. While past arrangements used only a single nozzle, we are using two nozzles set at approximately the same gap, as shown in Fig. VII-2. Also, by using an almost horizontal, only slightly inclined (10 per cent slope) manometer tube (draft gauge), we have been able to obtain sensitivities of up to 10 inches per 10^{-5}-inch gap variation (Δh), as shown in Fig. VII-3.

With the help of a series of experiments, we have been able to satisfy ourselves that with the nozzle dimensions shown in Fig. VII-4 and pressures at the nozzle of from 5 cm to 20 cm of mercury above atmospheric, the flow through the gap was indeed laminar, as is desirable. This is the case with the gaps from 2/1000 inch to 4/1000 inch used in this series. Bernoulli's equation in the form

$$\frac{v^2}{2g} + \frac{p}{\rho g} = \text{constant} \tag{2}$$

appears to apply to this situation (v is the velocity of flow through a cylindrical cross section of length h and diameter O.D. as in Fig. VII-3a, p is the pressure at the nozzle as in Fig. VII-3b, ρ is the density of the gas, and g is the acceleration of gravity). We obtain

$$2\frac{dv}{v} = -\frac{dp}{p} \tag{3}$$
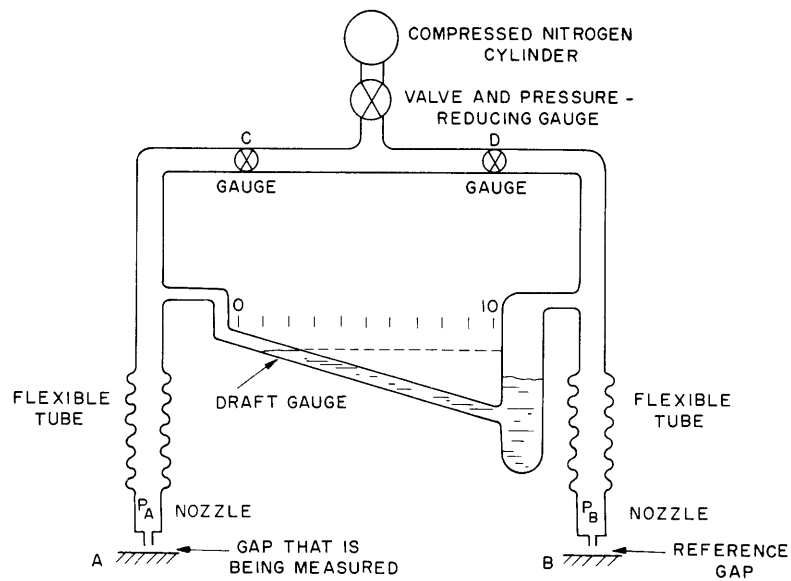
and, since dv/v = -dh/h, we also have

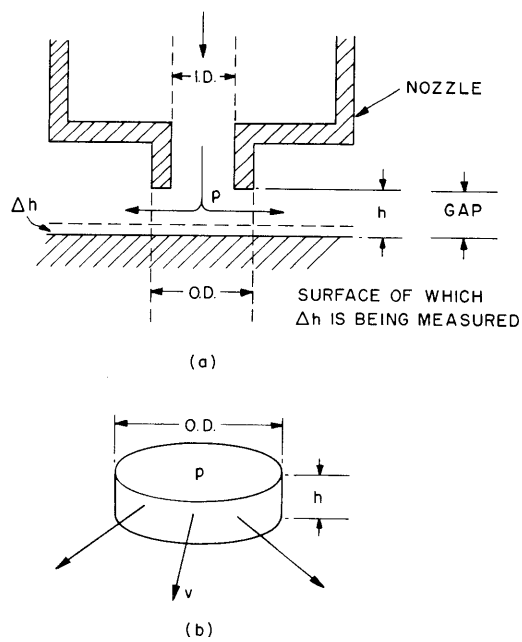Fig. VII-2. Differential arrangement for pneumatic measurement of small surface displacements.

(a)

(b)

Fig. VII-3. (a) Nozzle geometry.
(b) Flow geometry.

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$$\frac{dp}{p} = 2\frac{dh}{h} \tag{4}$$

where dp/p is the variation that is being measured by the manometer (draft gauge). With a 4×10^{-3}-inch gap and p = 10 cm of mercury (approximately 1/8 atmosphere) gauge, a $\Delta h = 10^{-5}$ inch results in a dp/p = 1/200 and Δp = 1/1600 atmosphere, with a gauge inclined at 10 per cent (giving a water-level rise of 4070 inches per atmosphere) we should expect column displacement of approximately 2 1/2 inches, which is in almost perfect agreement with the experimental value illustrated in Fig. VII-4. Furthermore, as can be seen for Eqs. 3 and 4, the sensitivity $\Delta p/p = 2\Delta h/h$ is independent of the density of the gas, provided, of course, that the flow velocity (Reynolds number) is small enough to have laminar flow. This density independence is also very well borne
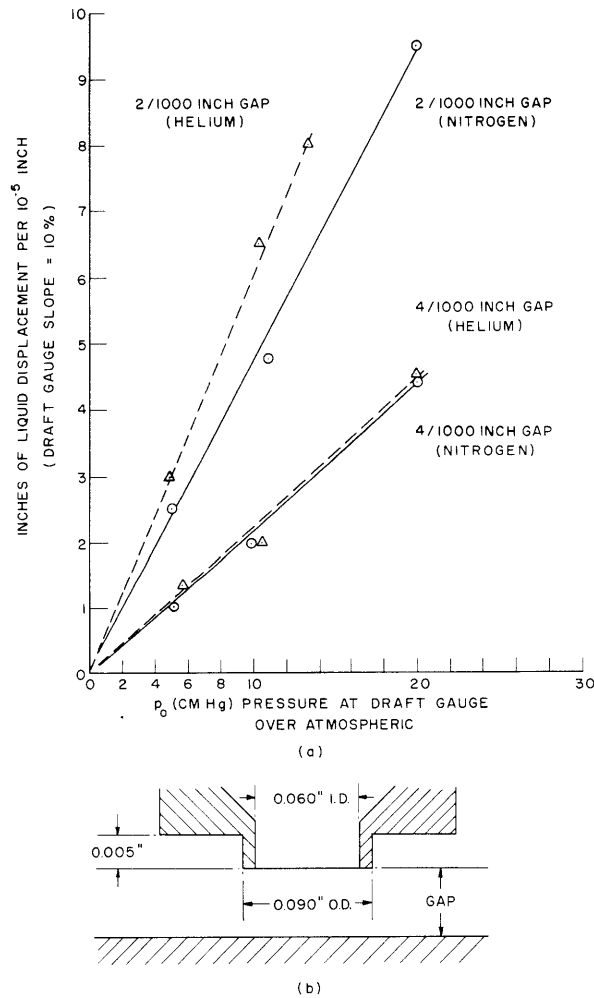

Fig. VII-4. Experimental values of liquid displacement per 10^{-5} inch of displacement of surface A in Fig. VII-2.

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out by the comparison of the measurements of helium and nitrogen (for which the approximate specific gravities are 18×10^{-5} and 13×10^{-4}, respectively). All calibrations were carried out by observing the interference fringe displacement when the gap was varied (1 fringe $\approx 10^{-5}$ inch in the visible domain). We are now applying the scheme of pneumatic detection to the measurement of the straightness of the cylinder, as well as to the measurement of its diameter, as we have mentioned. We shall report in more detail on the corrections that need to be observed when comparing the gap either between the nozzle and a cylinder or between the nozzle and a flat surface. It is clear from the physical description given above that the same pressure reading will be obtained with a flat plate in a position where it cuts a segment out of the cylinder, rather than in a tangential position. Errors up to 15×10^{-5} inch could arise if this correction were neglected in our experiment. This correction can be made negligibly small with a rectangular rather than a circular nozzle cross section. We plan to use such a nozzle for the absolute diameter measurement that has been described. We recall[1] that an absolute diameter measurement is not essential in our velocity of light experiment (except possibly for skin-depth corrections). We can indeed eliminate a diameter measurement by simply measuring the guide wavelength at two frequencies both of which are locked to the atomic clock and can therefore be made to have an exactly known ratio.

G. W. Stroke, J. G. King, D. C. Papa, A. Funkhouser

References

1. J. R. Zacharias, G. R. Harrison, G. W. Stroke, S. J. Mason, and C. W. Searle, Proposed measurement of the velocity of light in terms of two primary standards, cesium (Cs^{133}) and mercury (Hg^{198}), Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., January 15, 1956, pp. 59-68.