XIV. NEUROPHYSIOLOGY*

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A. INFLUENCE ON CONDUCTANCE BETWEEN TWO PARALLEL PLATES OF A CYLINDER RESTING ON ONE PLATE

Measurements carried out on the dielectric properties of squid axons at ultrahigh frequencies yielded results that are not sufficiently conclusive to report. However, a boundary-value problem that occurred during this research has been solved by approximation; its solution may be of general interest and is therefore reported here.

We have measured the complex admittance between 300 mc and 1000 mc of a parallel-plate condensor with a squid giant axon resting on one of the plates. The space between the plates was filled with mineral oil of dielectric constant 2.5. The axon, presumably, can be considered as a homogeneous cylinder, since the membrane is "invisible" at the frequencies used.

To our knowledge, there is no literature concerning the influence of a cylinder on the current between two parallel plates, with the cylinder resting on one plate. We note that the low-frequency problem of a conducting cylinder in a conducting medium is equivalent to the high-frequency problem of a dielectric cylinder in a dielectric medium, and that σ can be replaced by $j\omega\epsilon$. Lord Rayleigh (1) calculated the influence of a cylinder on a regular array of cylinders placed symmetrically between two plates, and his results have been simplified and modified for application to practical systems (see, for example, Cole (2)), including random arrays of large numbers of cylinders (3).

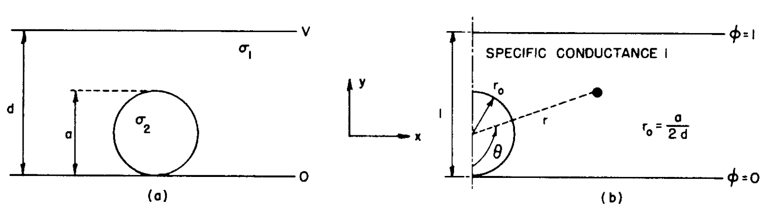

Fig. XIV-1. (a) Boundary-value problem of cylinder between two plates. (b) Reduced problem.

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The problem can be stated as follows (see Fig. XIV-1a). Consider two parallel plates at a distance d, extending infinitely in the z-direction (perpendicular to the plane of the drawing), and to a distance that is large with respect to d, in the x-direction. Let the plates have a potential difference V, let the material between the plates have a specific conductance σ_1, and let the current between the plates, per unit length in the z-direction, be i_o. Now place a homogeneous cylinder with specific conductance σ_2 on one of the plates. If the current per unit length is i, what then is $i - i_o$ as a function of a, d, σ_1, σ_2, and V? The problem is simplified (Fig. XIV-1b) by expressing the potential ϕ in units of V, making the distance between plates 1, the radius of the cylinder $r_o = a/2d$, and expressing the current per unit length in units of $\sigma_1 \cdot V$. The reduced specific conductance of the medium is 1 and that of the cylinder is $\sigma = \sigma_2/\sigma_1$. We shall also use the reduced specific resistance $\rho = \sigma_1/\sigma_2$.

Solution by analytical methods seemed infeasible, and we used a model measurement combined with an analytical approximation method. Equipotential lines were plotted for $\sigma = 0$ and $\sigma = \infty$, and for a/d between 0.5 and 0.9, with the aid of Teledeltos paper. From this plot, $(i-i_o)/\sigma_1 V$ was determined and plotted as a function of a/d. (See Fig. XIV-2 for $\sigma = 0$, and Fig. XIV-3 for $\rho = 0$ or $\sigma = \infty$.) It is worth noting that the approximation

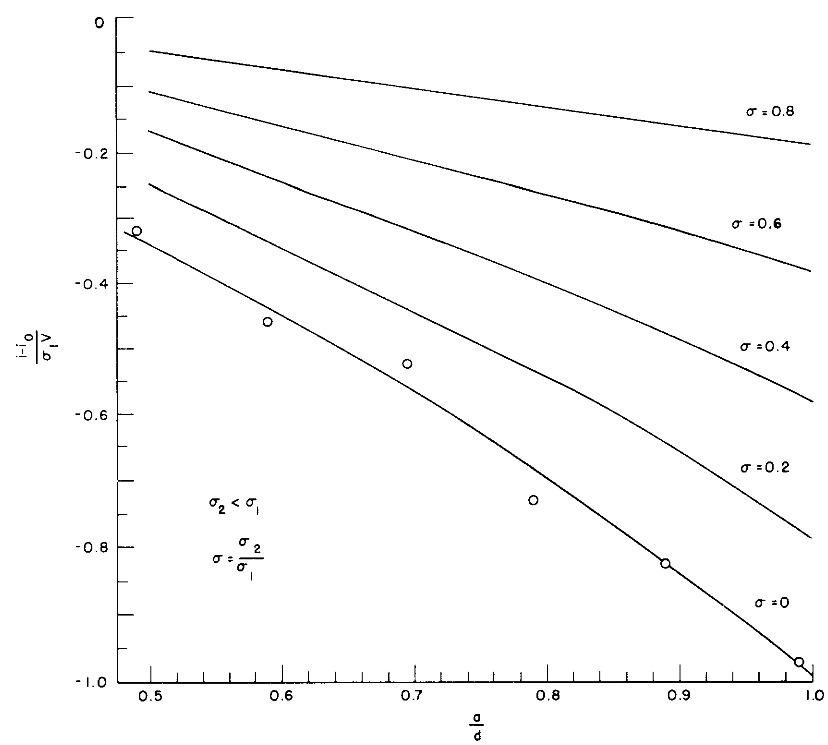

Fig. XIV-2. Solutions for $\sigma_2 < \sigma_1$. The $\sigma = 0$ values are measured by means of a model; the other values are analytically interpolated.

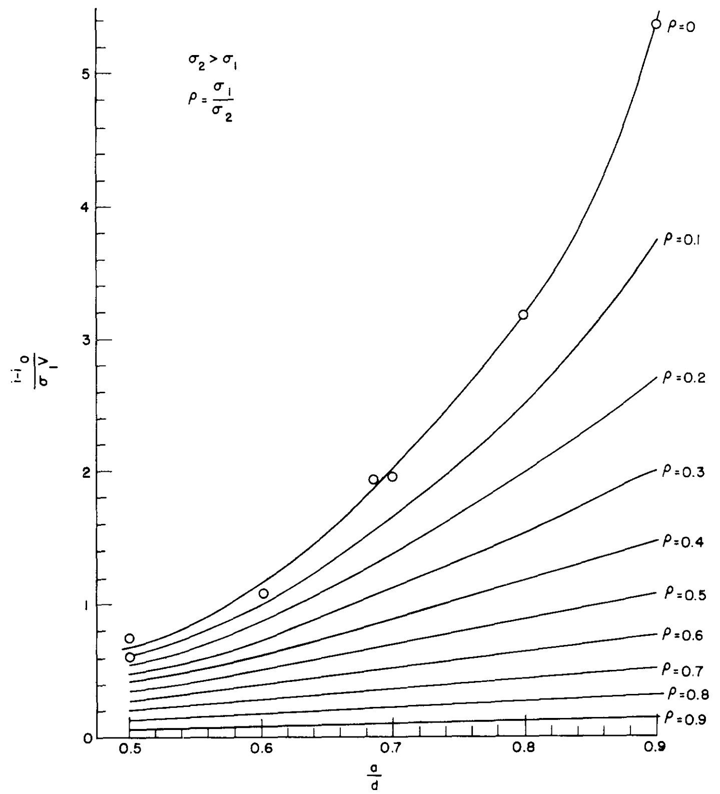

Fig. XIV-3. Solutions for $\sigma_2 > \sigma_1$.

method of solving boundary-value problems by manual construction of sets of equipotential and current lines in patterns consisting of squares leads to results of comparable accuracy.

Solutions for intermediate values of σ were approximated by fitting linear combinations of the potentials for $\sigma = 0$ and $\sigma = 1$ into the boundary conditions. Let the potential distribution outside the cylinder be ϕ^o for $\sigma = 0$ and ϕ^1 for $\sigma = 1$. The potential distribution for an arbitrary σ between 0 and 1 is now set as

$$\phi_1 = \lambda\phi^1 + (1-\lambda)\ \phi^o \tag{1}$$

inside the cylinder, and as

$$\phi_2 = A_o + A_1 r \cos\theta + A_2 r^2 \cos 2\theta + \dots \tag{2}$$

inside the cylinder. Equation 1 automatically satisfies the boundary conditions at the electrodes. At the surface of the cylinder the conditions are:

(a) ϕ is continuous, or $\phi_1(r_o) = \phi_2(r_o)$.

(b) $$\left(\frac{\partial\phi_1}{\partial r}\right)_{r=r_o} = \sigma \cdot \left(\frac{\partial\phi_2}{\partial r}\right)_{r=r_o}.$$

Condition (b) yields the relation $A_1 = -\lambda/\sigma$. Higher terms cannot be fitted. By Fourier analysis of the experimentally determined $\phi^o(r, \theta)$ with respect to θ, another relation between A_1 and λ is found. From the value of λ thus determined the current per unit length is calculated:

$$(i-i_o)_\sigma = (1-\lambda)(i-i_o)_{\sigma=0}$$

The results are given in Fig. XIV-2.

For $\sigma > 1$ $(0<\rho<1)$, an analogous procedure is followed. Since the constants A_i in Eq. 2 vanish for $\sigma = \infty$, another description has to be used at the cylinder surface:

$$\phi = A_o + \left(B_1 r + C_1 \cdot r^{-1}\right) \cos\theta + \dots$$

which is valid outside the cylinder. Conditions (a) and (b) yield

$$C_1 = r_o^2 \cdot \frac{\rho - 1}{\rho + 1} \cdot B_1$$

Fourier analysis is now applied to

$$\int \left(\frac{\partial\phi^\infty}{\partial r}\right)_{r=r_o} d\theta$$

which is experimentally determined from the current lines for $\sigma = \infty$. The best linear combination of ϕ^1 and ϕ^∞ is then determined. The results are given in Fig. XIV-3.

The inaccuracy of these results is estimated at 5 per cent for $\sigma_2 < \sigma_1$ and at 10 per cent for $\sigma_2 > \sigma_1$.

H. J. C. Berendsen

References

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