VI. MICROWAVE ELECTRONICS*

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A. DENSE SPACE-CHARGE THEORY OF GAP INTERACTION

The interaction between electromagnetic waves in dense electron-beam waveguides and electromagnetic fields of gaps has been previously considered (1). We shall now represent the interaction by a linear three-port, and present the associated matrix elements. These results are compared with those obtained under the assumption that the relative space-charge densities (ω_p/ω) in the electron-beam are weak (2). We shall consider only electron beams that are focused by infinite magnetic fields, and consider the presence of only a fast and a slow space-charge wave.

The following matrix will be taken to describe the interaction:

$$\begin{bmatrix} V_2(z,r) \\ I_2(z,r) \\ I_g \end{bmatrix} = \begin{bmatrix} A & B & a \\ C & D & b \\ c & d & Y_{e\ell} \end{bmatrix} \begin{bmatrix} V_1(-\ell,r) \\ I_1(-\ell,r) \\ V_g \end{bmatrix} \tag{1}$$

where $V_1(-\ell,r)$ and $I_1(-\ell,r)$ are the excitations made up of both waves at the input plane, and $V_2(z,r)$ and $I_2(z,r)$ are the kinetic voltage and current at the output plane.

The matrix elements are tabulated in Table VI-1 for the dense and weak space-charge cases. In this table the subscripts "+" and "1" denote the fast space-charge wave; and "-" and "2", the slow space-charge wave. Also,

$$\xi_{1,2} = 1 + \frac{P_{e1,2}}{P_{k1,2}} \tag{2}$$

where P_e is the total electromagnetic power flow (both inside and outside the electron beam) for one wave, and P_k is the kinetic power flow for one wave. It can be shown that for a propagating wave with a pure real longitudinal propagation constant,

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Table VI-1.

Matrix Element	Weak Space Charge $\left(\frac{\omega_p}{\omega} \ll 1\right)$ (One mode)	Dense Space Charge (A fast and a slow space-charge wave)
A	$\frac{1}{2}[\exp[-j\beta_+(\ell+z)]+\exp[-j\beta_-(\ell+z)]]$	$\frac{Z_{01}\exp[-j\beta_1(\ell+z)]+Z_{02}\exp[-j\beta_2(\ell+z)]}{Z_{01}+Z_{02}}$
B	$\frac{Z_0}{2}[\exp[-j\beta_+(\ell+z)]-\exp[-j\beta_-(\ell+z)]]$	$\frac{\exp[-j\beta_1(\ell+z)]-\exp[-j\beta_2(\ell+z)]}{Y_{01}+Y_{02}}$
C	$\frac{Y_0}{2}[\exp[-j\beta_+(\ell+z)]-\exp[-j\beta_-(\ell+z)]]$	$\frac{\exp[-j\beta_1(\ell+z)]-\exp[-j\beta_2(\ell+z)]}{Z_{01}+Z_{02}}$
D	$\frac{1}{2}[\exp[-j\beta_+(\ell+z)]+\exp[-j\beta_-(\ell+z)]]$	$\frac{Y_{01}\exp[-j\beta_1(\ell+z)]+Y_{02}\exp[-j\beta_2(\ell+z)]}{Y_{01}+Y_{02}}$
a	$\frac{F(pr)}{2}[M_+C_+\exp(-j\beta_+z)+M_-C_-\exp(-j\beta_-z)]$	$\frac{M_1C_1F(p_1r)}{2\xi_1}\exp(-j\beta_1z)+\frac{M_2C_2F(p_2r)}{2\xi_2}\exp(-j\beta_2z)$
b	$\frac{Y_0F(pr)}{2}[M_+C_+\exp(-j\beta_+z)-M_-C_-\exp(-j\beta_-z)]$	$\frac{Y_{01}M_1C_1F(p_1r)}{2\xi_1}\exp(-j\beta_1z)-\frac{Y_{02}M_2C_2F(p_2r)}{2\xi_2}\exp(-j\beta_2z)$
c	$\frac{Y_0}{2F(pr)}\left[M_+^*K_+\exp(-j\beta_+\ell)-M_-^*K_-\exp(-j\beta_-\ell)\right]$	$\frac{1}{Z_{01}+Z_{02}}\left[\frac{M_1^*K_1}{F(p_1r)}\exp(-j\beta_1\ell)-\frac{M_2^*K_2}{F(p_2r)}\exp(-j\beta_2\ell)\right]$
d	$\frac{1}{2F(pr)}\left[M_+^*K_+\exp(-j\beta_-\ell)-M_-^*K_-\exp(-j\beta_-\ell)\right]$	$\frac{1}{Y_{01}+Y_{02}}\left[\frac{Y_{01}M_1^*K_1}{F(p_1r)}\exp(-j\beta_1\ell)+\frac{Y_{02}M_2^*K_2}{F(p_2r)}\exp(-j\beta_2\ell)\right]$
$G_{e\ell}$	$\frac{Y_0}{4}\left[\lvert M_+\rvert^2C_+K_+-\lvert M_-\rvert^2C_-K_-\right]$	$\frac{Y_{01}}{4\xi_1}\lvert M_1\rvert^2C_1K_1-\frac{Y_{02}}{4\xi_2}\lvert M_2\rvert^2C_2K_2$
$B_{e\ell}$	$\frac{Y_0}{4}\mathrm{Im}\left[2M_+K_+M_-^*K_-\exp\left(j2\beta_pd\right)\right]-4\omega\left(W_k-W_e^p\right)$	$-4\omega(W_k-W_e+W_m)$

$$\xi = \left[- \frac{p(\beta_e - \beta)^3}{q^2 \beta_p^2} \frac{\frac{\partial \Delta}{\partial \beta}}{\frac{\partial \Delta}{\partial p}} \right]_{\Delta=0} \qquad q^2 = -\beta^2 + k^2, \quad p^2 = q^2 \left[1 - \frac{\beta_p^2}{(\beta \ \beta_e)^2} \right] \tag{3}$$

where Δ is the system determinantal equation arising from the boundary conditions. It is also possible to express ξ in terms of integrals of the longitudinal electric fields. For a propagating wave with a pure real longitudinal propagation constant (3),

$$\xi - 1 = \frac{P_e}{P_k} = \frac{p^2 \beta (\beta_e - \beta)^3}{q^4 \beta_p^2} \left[1 + \frac{q^2 \int_{A_a} \left| \hat{E}_z^a \right|^2 da_a}{p^2 \int_{A_p} \left| \hat{E}_z^p \right|^2 da_p} \right] \tag{4}$$

In Table VI-1, $F(p_n r)$ denotes the transverse variation of the longitudinal electric field inside the electron beam, which can be determined from the Fourier integral solution for E_z^p (see Bers (1)), and

$$Y_{01,2} = \pm \frac{\sigma \omega \epsilon_o \beta_p^2}{\beta_e - \beta_{1,2}} \tag{5}$$

$$Y_0 = \pm \frac{\sigma \omega \epsilon_o \beta_p^2}{\beta_e - \beta_{\pm}} = \frac{\sigma \omega \epsilon_o \beta_p^2}{\beta_q} \tag{6}$$

In Eq. 5, the upper sign is to be taken with the subscript "1" and the lower with the subscript "2". In the dense space-charge formulation

$$C_{1,2} = \frac{2p_{1,2}}{\left(q_{1,2}^2 - p_{1,2}^2\right) \frac{\partial \Delta}{\partial p} \Big|_{1,2}} \tag{7}$$

and $M_{1,2}$ is found from the relationship defining $M_{\pm}$ by changing the propagation constant from $\beta_{\pm}$ to $\beta_{1,2}$, and

$$K_{1,2} = C_{1,2} \frac{1}{\sigma} \int_{\sigma} [F(p_{1,2} r)]^2 \, da \tag{8}$$

Reference to Table VI-1 shows that the weak space-charge matrix elements can be obtained from the dense space-charge elements when $\left(\frac{\omega_p}{\omega} \ll 1\right)$ by the following procedure:

(i) Set ξ_n equal to 1 (disregard electromagnetic power flow in comparison to kinetic power flow).

(ii) Assume that the transverse wave numbers are the same for the fast and the slow space-charge waves. Then $F(p_1r) \rightarrow F(p_2r) \rightarrow F(pr)$.

(iii) Assume the same β_q for the fast and slow space-charge waves; that is, $|\beta_e-\beta_1| \rightarrow |\beta_e-\beta_2| \rightarrow \beta_q$.

Then $Y_{01} \rightarrow Y_{02} \rightarrow Y_0$.

Under these conditions the M's and C's of the dense space-charge theory go to those of the weak space-charge theory.

The expressions for $B_{e\ell}$ are rather complicated to evaluate, and at present we have no direct comparison between the two formulations.

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References

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2. A. Bers, Space-charge mode theory of gap interaction, Quarterly Progress Report No. 55, Research Laboratory of Electronics, M. I. T., Oct. 15, 1959, pp. 65-70; Quarterly Progress Report No. 59, Oct. 15, 1960, pp. 68-71.

3. A. Bers, Properties of waves in electron-beam waveguides, Quarterly Progress Report No. 61, Research Laboratory of Electronics, M. I. T., April 15, 1961, pp. 102-105.

B. KINEMATIC GAP THEORY FOR ACCELERATED ELECTRON STREAMS

A small signal kinematic analysis of the interaction of an electron beam and the electric field of a klystron gap has been carried out by Bers (1). This analysis can be extended to include the effects of acceleration of the electron beam.

Following Bers' notation, but not introducing normalized variables, we obtain the following equations.

$$V(\theta, r) = e^{-j\theta'} V(-\infty, r) + e^{-j\theta} \int_{-\infty}^{\theta} \frac{E(\theta, r)}{\beta_e(\theta)} e^{j\theta} d\theta \tag{1}$$

$$I(\theta, r) = e^{-j\theta'} I(-\infty, r) + je^{j\theta} \int_{-\infty}^{\theta} \frac{1}{2} G_o(\theta) V(\theta, r) e^{j\theta} d\theta \tag{2}$$

$$I_g = \int da \int_{-\infty}^{\infty} I(\theta, r) \frac{E(\theta, r)}{V_g \beta_e(\theta)} d\theta \tag{3}$$

From these equations, the matrix coefficients for the equivalent linear three-port can be determined. Of particular interest are the coefficients Y_{13} or M, the voltage coupling coefficient and Y_{33} or $Y_{e\ell}$, the electronic admittance.

$$Y_{13}(r) = \int_{-\infty}^{\infty} \frac{E(\theta, r)}{V_g \beta_e(\theta)} e^{-j\theta} d\theta \tag{4}$$

$$Y_{33} = \int da \int_{-\infty}^{\infty} d\theta \frac{E(\theta, r)}{V_g \beta_e(\theta)} e^{-j\theta} \int_{-\infty}^{n} d_n \frac{1}{2} G_o(n) \int_{-\infty}^{n} d\gamma \frac{E(\gamma, r)}{V_g \beta_e(\gamma)} e^{j\gamma} \tag{5}$$

The real part of Y_{33}, the electronic loading conductance, is given by

$$G_{e\ell} = \int da \left[\int_{-\infty}^{\infty} \frac{E(\theta, r)}{V_g \beta_e(\theta)} e^{-j\theta} d\theta\right] \mathrm{Im}\left[\int_{-\infty}^{\infty} dn \frac{1}{2} G_o(n) \int_{-\infty}^{n} \frac{E(\theta, r)}{V_g \beta_e(\gamma)} e^{j\gamma} d\gamma\right] \tag{6}$$

Consider the special case illustrated in Fig. VI-1. The beam passes through a V-shaped dc potential depression. The gap is gridded and the E-field is independent

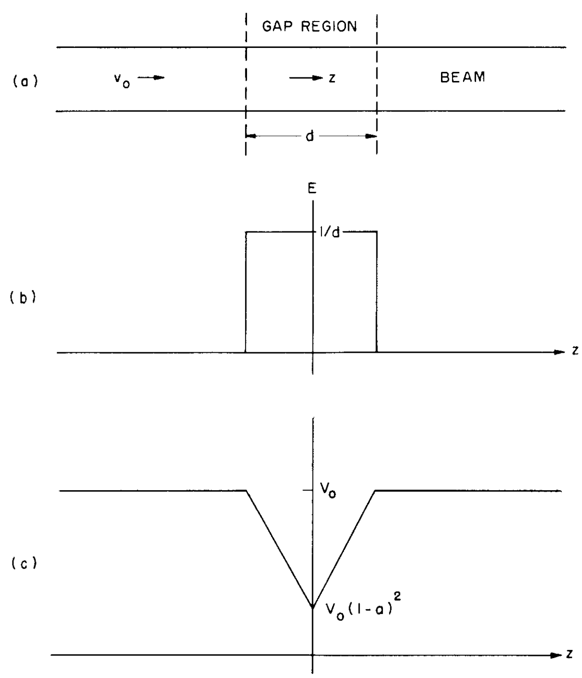

Fig. VI-1. Gridded gap with depressed potential resulting from space charge. (a) Geometry of the problem. (b) Magnitude of gap electric field. (c) Shape of dc potential variation.

of r. Let the velocity at the potential minimum be (1−a) times the velocity at entrance and exit. Then we obtain the following explicit formulas for the voltage coupling coefficient and the electronic loading conductance.

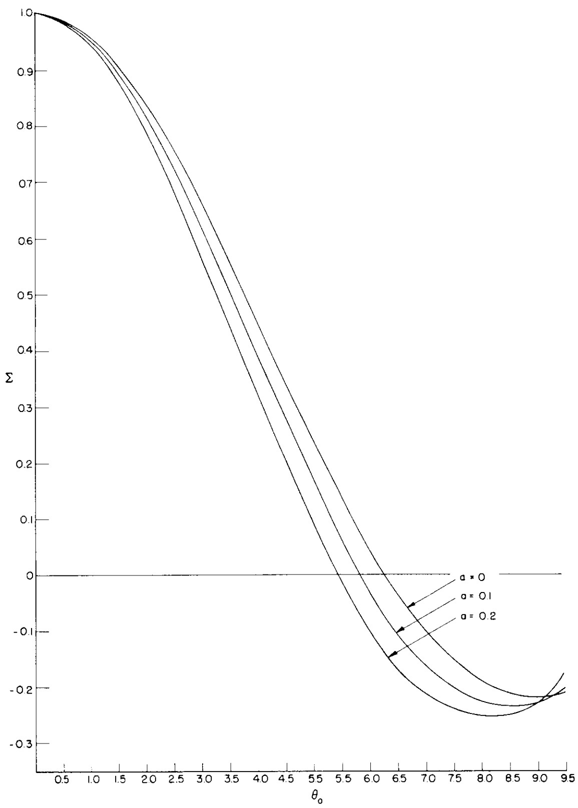

Fig. VI-2. Voltage coupling coefficient versus transit angle.

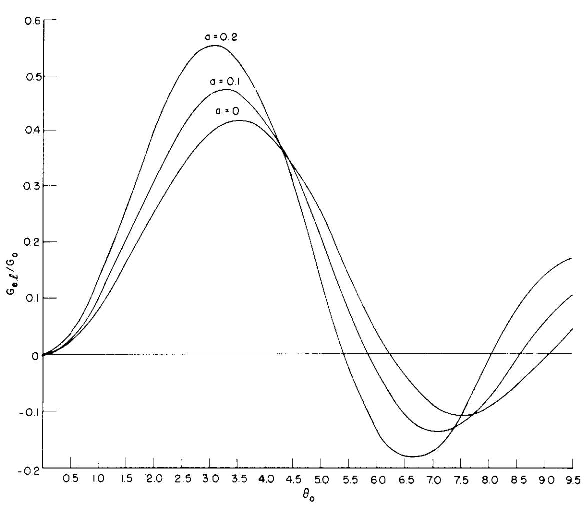

Fig. VI-3. Electronic loading conductance versus transit angle.

$$M = \frac{1}{\frac{1-a}{2}} \frac{2}{\theta}\left[\sin\frac{\theta}{2} - \frac{2a}{\theta}\left(1 - \cos\frac{\theta}{2}\right)\right] \tag{7}$$

$$G_{e\ell} = G_o \frac{2}{\theta}\left[\frac{1}{\frac{1-a}{2}}\right]^2 \left[\frac{1}{1-a}\right]\left[\sin\frac{\theta}{2} - \frac{\theta}{2}\cos\frac{\theta}{2}\right]\left[\sin\frac{\theta}{2} - \frac{2a}{\theta}\left(1 - \cos\frac{\theta}{2}\right)\right] \tag{8}$$

In Figs. VI-2 and VI-3, M and $G_{e\ell}$ are plotted against θ_o, the transit angle in the absence of the depression, for various values of a; θ and θ_o are connected by the relation $\theta = \frac{\theta_o}{(1-a)/2}$.

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References

1. A. Bers, Klystron gap theory, Quarterly Progress Report, Research Laboratory of Electronics, M. I. T., July 15, 1958, p. 49.