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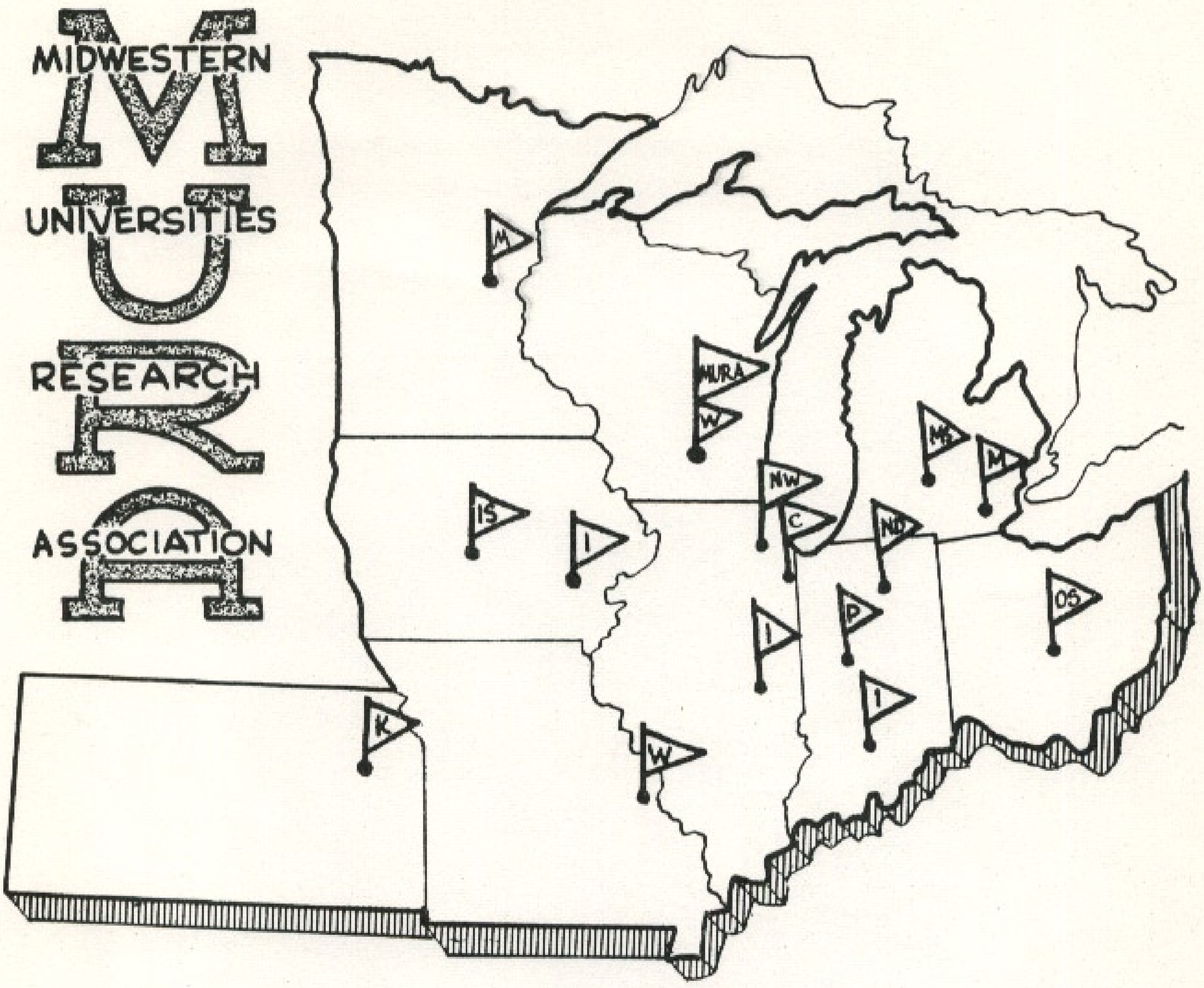

EXPERIMENTS ON RADIO FREQUENCY KNOCKOUT OF

STACKED BEAMS

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EXPERIMENTS ON RADIO FREQUENCY KNOCKOUT OF STACKED BEAMS

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ABSTRACT: A major problem in the building up of intense circulating currents in an FFAG accelerator is the effect on the stacked beam of the radio frequency voltage accelerating additional particles. When this accelerating frequency is in resonance with the betatron oscillations of the stacked beam, the oscillation amplitude may be driven and the beam destroyed. Experiments have been performed with the radial sector FFAG electron model which demonstrate that the quantitative magnitude of the effect is in agreement with calculations.

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METHOD

While rf knockout is an established and reliable method of determining betatron oscillation frequencies[1,2] , to date no quantitative measurments had been made on the effect. Terwilliger[3] has calculated the increase in betatron oscillation amplitude of a stacked beam to be expected when an rf voltage is modulated through a knockout frequency knowing the parameters of the rf and of the stacked beam. The radial sector FFAG electron model[2] was used to test this effect. A coasting, circulating beam can be readily established, and the rf system for rf acceleration experiments[4] can be used to modulate through the frequencies which affect radial or vertical betatron oscillations.

First, the rf knockout frequencies at the target radius were determined with the rf probes designed for that purpose.[2] Table I lists the knockout frequencies thus found and the resultant ν_s.

TABLE I

f (Mc)	character	
12.1	vertical	$f_0 = 12.1 + 61.2 = 73.3$
15.4	radial	$f_0 = 15.4 + 58.3 = 73.7$
		$\bar{f_0} = \frac{73.3 + 73.7}{2} = 73.5$
58.3	radial	$\nu_x = \frac{58.3}{73.5} + 2 = 2.79$
61.2	vertical	$\nu_z = \frac{61.2}{73.5} + 1 = 1.83$

[1] Hammer, Pidd, and Terwilliger, R.S.I. 26, 555 (1955)
[2] MURA-219
[3] MURA-KMT-3
[4] MURA-255, MURA-256

The beam was then allowed to coast for two milliseconds at an energy of 300 kv corresponding to a circulating frequency, f_0, of about 72.2 Mc. During this coasting period the frequency modulated rf oscillator was pulsed on for a short time. This oscillator, tied across the accelerating gap as for rf acceleration, was tuned to sweep the range 55-61 Mc. Effects on the coasting beam were clearly apparent when the frequency crossed 55.9 Mc (radial K. O.) and 60.0 Mc (vertical K. O.).

RESULTS: VERTICAL KNOCKOUT

When the rf was turned on at about 58 Mc and the turn off frequency, f_T, varied, it was found that the beam subsequently accelerated by the second betatron pulse was attenuated partly when f_T was 59.8 Mc and was completely destroyed when it was extended to 60.2 Mc.

With the gap voltage reduced to about 5 volts and a rate of frequency modulation of about 3 $\frac{kc}{\mu s}$ the beam was reduced to half intensity by sweeping the rf to a frequency well above 60 Mc. The attenuated beam still appeared at the same time on the second betatron pulse and with the same spread and shape, indicating no observable change in the energy or radial oscillation amplitudes of the surviving beam due to the vertical knockout. There should be no vertical knockout if the fields are perfectly symmetric about the median plane. However, in this accelerator the rf is applied to the top of the gap, which has dimensions not negligible compared to a wavelength. (The 2mm gap is about ten inches long) so that it is unlikely that this symmetry condition is met, and vertical rf fields might well exist on the median plane. Since it has not been possible to determine these vertical effects quantitatively, calculations were only made from the radial experiments.

RADIAL KNOCKOUT

With the rf turnon frequency at $\sim$ 55 Mc, f_T was varied about 56 Mc. The beam was not destroyed completely by the rf but it was attenuated by as much as a factor of three, and the character of beam as observed during the second betatron pulse[4] was altered. The beam shape as detected by the photomultiplier for successively higher values of f_T is illustrated schematically in figure 1.

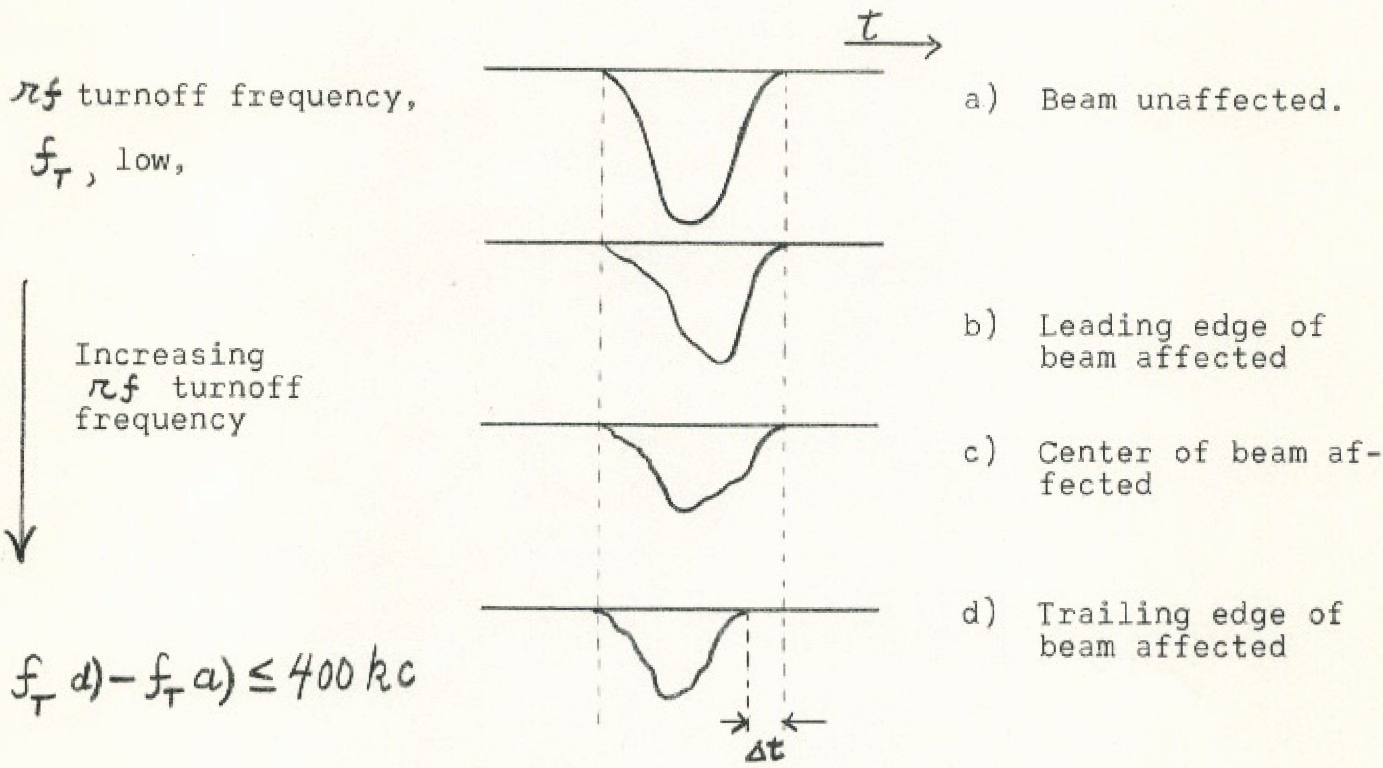

Figure 1.

Since it has been found that the energy spread of the beam is quite small, the time spread of the beam has been interpreted as due to the range of radial betatron oscillations; the earlier particles have the largest amplitudes and the later particles the smallest amplitudes. It is also known that at this tuning of the machine ($\sigma_x > \frac{2}{3}\pi$) ν_x decreases with increasing amplitude. The interpretation of the observed effect then is that at a low f_T only the large amplitude particles are affected, and as f_T is increased, successively smaller amplitudes of oscillation are affected. From the failure of the front edge to move earlier and the attenuation of the beam, there appears to be a limiting radial amplitude. The range of f_T represented by the extreme cases of figure 1 is less than 400 kc, corresponding to a range of ν_x values less than 0.007 or a range of $\frac{\sigma_x}{\pi}$ values less than 0.0018, where $\nu_x = 2.79$ and $\frac{\sigma_x}{\pi} = .697$.

A quantitity which can be determined is the time shift, Δt of the tail of the pulse, corresponding to the amplitude increase of the smallest amplitude particles. Table II lists measurements of Δt for different rf conditions and the corresponding radial amplitude increase Δx. Δx is found from Δt knowing the volts per turn from the second betatron at the beam time, V_β, the target frequency of the machine, f_o, and the rate of energy increase in the machine with radius, dE/dr. Thus:

$$\Delta x = \frac{V_\beta f_o}{dE/dr} \Delta t .$$

TABLE II

V_{rf} (volts)	$\frac{df}{dt}$ $\left(\frac{\text{cycles}}{\text{second}}\right)$	Δt (μs)	Δx (mm) (crude estimate)
24	9.3×10^9	15	1.4
14.5	8×10^9	11	1.0
7.5	3.8×10^9	7.5	0.7

These Δx determinations are quite crude (to a factor of two or so) since they involve interpreting a time shift and a shape change as some equivalent Δx. Calculations were made from the first data where the greatest effect was seen.

Calculating the increase in amplitude expected from the sweep of an rf pulse through a betatron resonance, Terwilliger shows[3] that the amplitude kicks on successive revolutions add up to form a Cornu spiral. From the rf voltage and the machine parameters the amplitude increment,

a_x, due to one passage across the gap is found. Then from the rate of frequency modulation (df/dt) the center-to-center length of the Cornu spiral is determined, expressed as an equivalent number (n) of in-phase kicks. From reference 3,

$$\Delta x = n\,\overline{a}_x ,$$

$$\overline{a}_x = \frac{Vc^2}{2Ev^2}\sqrt{\left(\frac{r}{k+1}\right)^2 + (\rho\beta\tan\phi)^2} \qquad {}^5$$

and $n = \dfrac{f_0}{\sqrt{df/dt}}$

where: $\overline{a}_x = \frac{1}{2} a_x$, where a_x is amplitude generated from peak rf voltage.

V = rf voltage = 24 volts

E = total energy of electron = 800 kv

v = electron velocity = .78 c

r = orbit radius from machine center = 48 cm

ρ = orbit radius of curvature = 19.5 cm

β = transfer matrix parameter[6] = .72

ϕ = angle between normal to gap and equilibrium orbit $\cong$ 20°.

For the case of interest, $n \cong 800$, $\overline{a}_x \cong 3 \times 10^{-4}$ cm and $\Delta x \cong$ 2.4 mm, as compared to the value from table II of 1.4 mm. Considering the approximate nature of the experiment, the agreement is satisfactory. Nonlinearities of the oscillations have the effect of altering the effective df/dt since the particle betatron oscillation frequency changes as its amplitude grows. In the present case this effect would reduce the calculated Δx.

[5] This expression differs from that of reference 3 in that $\rho\beta$ is substituted for the more approximate $\frac{2r}{\nu_x}$ (see, for example, MURA-LWJ-13)

[6] MURA-203

RADIAL AMPLITUDE LIMITS

It has been noted that there appears to be a limiting amplitude of radial oscillation which may be contained in the machine , and that the frequency shift between the smallest amplitudes present and this limiting amplitude is small (far too small to detune the oscillations to a known resonance, such as $\sigma_x = \frac{2}{3}\pi$). A tempting explanation for the radial amplitude limit is the coupling between radial and vertical oscillations. If an "energy" is constantly being exchanged between radial and vertical motion, then the largest radial oscillation in the machine may "correspond" to a vertical oscillation which just clears the vacuum tank. To see if this is actually the case, an "L" shaped probe[6] was used to limit the vertical aperture at one azimuth, and the detector placed to see x-rays from it. The beam was seen to strike this probe when the radial knockout frequency was crossed, and in fact from this probe the frequency determined to be the knockout frequency corresponded to that of figure 1b, e.g. corresponding to the largest amplitude particles. It thus seems that the usable radial aperture is determined by the vacuum tank vertical height. The machine tune in this case is close to a difference "resonance" ($\nu_x - \nu_z = 1$), which may enhance the effect.