FEEDBACKS ON TUNE AND CHROMATICITY

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Abstract

Feedbacks on tune, coupling and chromaticity are becoming an integral part of safe and reliable accelerator operation. Tight tolerances on beam parameters typically constrain the allowed oscillation amplitudes to the micrometre range, leaving only a small margin for the transverse beam and momentum excitations required for tune and chromaticity measurements. This contribution presents an overview of these beam-based feedback systems, their architecture and design choices involved. It discusses performance limitations due to cross-constraints, non-linearities, the coupling between multiple nested loops, and the interdependence of beam parameters.

INTRODUCTION

The control of orbit and energy became de-facto standard as nearly all modern light sources, lepton and hadron colliders alike deploy at least fast orbit and energy feedbacks that minimise transverse beam movements, spurious dispersion by centreing the beam in the quadrupoles and maintain a stable vertical orbit inside the sextupoles that would otherwise give rise to emittance coupling. A summary and overview of beam stability requirements and stabilisation over a large range of accelerators can be found in [1–3].

Recent improvements in hadron colliders have led to significantly increased stored beam energies which require excellent control of particle losses inside a superconducting machine. Thus, most requirements on key beam parameters in superconducting hadron colliders strongly depend on the capability to control particle losses inside the accelerator. In the case of the LHC, the cleaning system has the tightest constraints on the orbit and requires a stability better than $25\,\mu\text{m}$ during nominal operation at the location of the collimators which leaves only a small margin for beam excitation required for the measurement and control of tune and chromaticity [11].

Automated feedback control systems are usually not limited to the control of orbit and energy but are often complemented by tune, coupling and chromaticity feedbacks, which have become an increasingly important part of operation [4–6,9,10]. As for the orbit, the requirements on tune stability in superconducting hadron colliders is primarily given by the ability to control particle loss and the negligible synchrotron radiation damping. The usually large tune footprint makes it necessary to avoid up to the 12th order resonances [12]. The corresponding stability is usually in the order of $\delta Q \approx 10^{-3}$ and is similar to the one of B-factories as well as ramping synchrotron light sources, although the latter is due to performance rather than machine safety reasons. In addition, these machines have often quite demanding tune working points in the vicinity of the half-integer resonance, as these regions are able to accommodate the usually large footprints as well as mitigate the effect of beam-beam interactions and electron clouds that cause the large footprints in the first place (PEP-II [7]: $q_x = 0.505$ (LER) & $q_x = 0.503$ (HER); KEK-b [8]: $q_x = 0.504$ (LER) & $q_x = 0.510$ (HER)).

The decay and snapback phenomena, a particularity of superconducting magnets causes, in case of the LHC, a large chromaticity drift that if uncorrected is expected to exceed more than 100 units within a few hundred seconds after the start of the ramp [12]. Consequently, to guarantee the stability of multiple particles ensemble, the chromaticity should be controlled within $Q' \approx 2\pm1$. The LHC is thus the first accelerator that, in addition to orbit, energy, tune and coupling, may need to rely on an automated control of chromaticity for safe and reliable machine operation.

FEEDBACK CONTROL DESIGN

In case of low-order beam parameters – orbit, tune, coupling, chromaticity and energy – the effect of individual corrector circuits is, for most accelerators, sufficiently linear and can be cast into matrices. The parameter control in space domain establishes corrector circuit strengths that for steady-state perturbations minimises the residual measured parameter deviation. The control in space domain consists essentially of the inversion of the beam response matrices that relate the corrector circuit strength change to the given bean parameter. Singular-Value-Decomposition (SVD) is one of the most popular and widely used inversion algorithms ([1, 2]). Further information can be found in [3, 13, 14].

Time Domain

A simple loop block diagram consisting of a single-input-single-output (SISO) process $G(s)$ and controller $D(s)$ is shown in Figure 1.

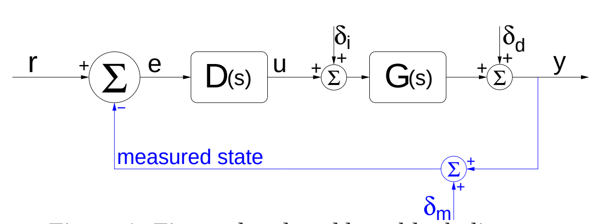

Figure 1: First order closed loop block diagram.

The stability and sensitivity to perturbations and noise is defined by the following functions

$$T(s) := \frac{y}{r} = \frac{D(s)G(s)}{1+D(s)G(s)} \quad (1)$$

$$S_d(s) := \frac{y}{\delta_d} = \frac{1}{1+D(s)G(s)} \quad (2)$$

$$S_i(s) := \frac{y}{\delta_i} = \frac{G(s)}{1+D(s)G(s)} \quad (3)$$

$$S_u(s) := \frac{u}{\delta_d} = \frac{D(s)}{1+D(s)G(s)} \quad (4)$$

where $T(s)$ is the complementary (nominal) transfer function, $S_d(s)$ the *nominal sensitivity* defining the loop disturbance rejection, $S_i(s)$ the *input-disturbance sensitivity* and $S_u(s)$ the *control sensitivity*. The state variables are indicated in Figure 1. The sensitivity to measurement noise is equal to the nominal transfer function T_0.

Classic feedback designs rely on the discussion of denominator zeros in equation 1 and 2 while keeping constraints such as required bandwidth, minimisation of overshoot, limits on the maximum possible excitation signal and robustness with respect to model and measurement errors. For ideal processes, this yields adequate controller designs but often falls short in providing a simple comprehensive method for estimating and modifying the loop sensitivity (robustness) in the presence of process uncertainties, non-linearities and noise.

This paper focuses on Youla's affine parameterisation method for optimal controllers, which is based on the analytic process inversion, first introduced in [15]. For an open-loop stable process $G(s)$, the nominal closed-loop transfer function is stable if and only if $Q(s)$ is an arbitrary stable proper transfer function and $D(s)$ parameterised as:

$$D(s) = \frac{Q(s)}{1-Q(s)G(s)} \quad (5)$$

The stability of the closed loop system follows immediately out of the above definition if inserted into equations 1 to 4. The sensitivity functions in the $Q(s)$ form are given as:

$$T(s) = Q(s)G(s) \quad (6)$$

$$S_d(s) = 1-Q(s)G(s) \quad (7)$$

$$S_i(s) = (1-Q(s)G(s))G(s) \quad (8)$$

$$S_u(s) = Q(s) \quad (9)$$

Assuming $G(s)$ is stable, the only requirement for closed loop stability is for $Q(s)$ to be stable. The strength of this method is the explicit controller design with respect to required closed loop performance, as visible in equation 6, and required stability (equations 7 to 9). Equations 6 and 7 are complementary and illustrate the intrinsic limiting trade-off of feedbacks that either have a good disturbance rejection or are robust with respect to noise. The ultimate limit is thus defined rather by the bandwidth and noise performance of the corrector circuits and beam measurements than by the feedback loop design itself. Systematic and thorough analysis of involved beam instrumentation and corrector circuits are thus essential for achieving best beam parameter stabilisation.

First Order Example

The design formalism can be demonstrated using a simple first order system $G_0(s) = \frac{K_0}{\tau \cdot s+1}$ with open-loop gain K_0 and time constant τ. A common controller design ansatz is to write $Q(s)$ as

$$Q(s) = F_Q(s) \cdot G_0^i(s) \quad (10)$$

with $F_Q(s)$ a trade-off function and $G_0^i(s)$ the pseudo-inverse of the process. Since G_0 does not contain any instable zeros, the pseudo-inverse equals the inverse and is given by $G_0^i(s) := [G_0(s)]^{-1} = \frac{\tau \cdot s+1}{K_0}$. $Q(s)$. In order for $D(s)$ to be biproper, $F_Q(s)$ must have a degree of one and can be written as:

$$F_Q(s) = \frac{1}{\alpha s+1} \quad (11)$$

Inserting equation 10 into Youla's controller parameterisation equation 5 yields the following controller

$$D(s) = \frac{\tau}{K_0\alpha} + \frac{1}{K_0\alpha s} = K_p + K_i \cdot \frac{1}{s} \quad (12)$$

which shows a a simple PI controller structure with proportional gains K_p and integral gain K_i. Inserting equation 10 into equation 6 yields

$$T_0(s) = F_Q(s) \quad (13)$$

that the closed loop response is essentially determined by the choice of trade-off function $F_Q(s)$ and that the closed loop bandwidth is proportional to the parameter $1/\alpha$. This can be used to tune the closed loop between: high disturbance rejection but high sensitivity to measurement noise (small α) and low noise sensitivity but low disturbance rejection (large α) depending on the operational scenario. The maximum possible closed loop bandwidth is limited by the excitation, as described by equation 9. In case of power converters, for example, the excitation is limited by the maximum available voltage.

Non-linear Feedback Loops

The same method can be extended to open-loop instable and multi-input-multi-output (MIMO) systems [15]. Real life feedbacks may contain significant delays λ (due to e.g. data transmission, data processing etc.) and non-linearities $G_{NL}(s)$, due to e.g. saturation and rate limits of the corrector circuits' power supplies. The modified process can be written, for example as:

$$G(s) = G_0(s) \cdot e^{-\lambda s} G_{NL}(s) \quad (14)$$

Using the same pseudo-inverse $G_0^i(s)$ as for the above example and inserting equation 10 into equation 5 yields a controller parameterisation $D_{NL}(s)$ including a classic Smith-Predictor and anti-windup paths, discussed in more detail in [16, 17]. Inserting equation 10 including the delay and non-linearities into equation 6 yields the following closed loop transfer function:

$$T(s) = F_Q(s) \cdot e^{-\lambda s} G_{NL}(s) \quad (15)$$

Similar to the linear case discussed above, the closed loop is essentially defined by the function $F_Q(s)$ that within limits can be chosen arbitrarily based on the required disturbance rejection and robustness during possibly different operational scenarios (gain-scheduling). Further information and a review on Youla's parameterisation can be found in [17, 18].

APPLICATION TO PHASE-LOCKED-LOOP SYSTEM DESIGN

Beam-based feedbacks are ultimately limited by thermal drifts, noise and systematics of involved devices. A systematic and thorough analysis of involved beam instrumentation and corrector circuits is thus mandatory for achieving best beam parameter stabilisation.

The above principles can readily be applied to the design of a Phase-Locked-Loop (PLL) control system that continuously adjusts phase φ and frequency f_e of its reference exciter to match and thus to track changes of the betatron tunes. A classic application of PLL is to modulate the beam momentum using the RF frequency while tracking the tune. The modulation amplitude can be then be used to derive the linear machine chromaticity while the barycentre is a measure of the unperturbed tunes.

The tune resonance is described by a second order harmonic oscillator and thus the tune resonance is found once the the phase between excited and measured oscillation equals $\pi/2$. An exemplary PLL block diagram is shown in Figure 2.

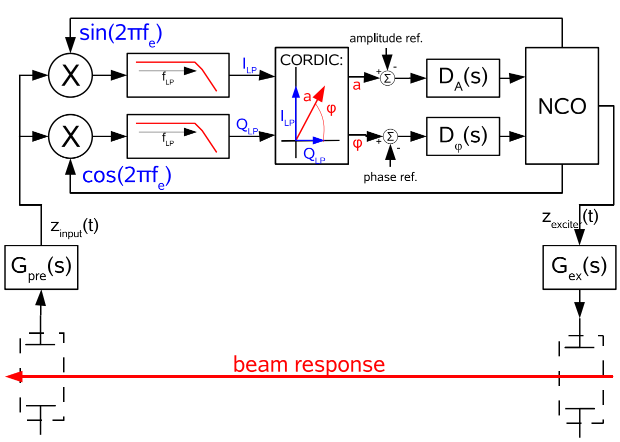

Figure 2: Phase-Locked-Loop Scheme

The system mixes the input with the sine and cosine component of the excitation signal followed by a low-pass filter in order to remove the $2\pi f_e$ frequency component that is created in the process of the mixing. The remaining signals are treated by a rectangular-to-polar (R2P) converter that separates signal phase and amplitude which can further be treated by two independent controller. In comparison to classic analogue phase detectors, this scheme provides a twice as large dynamic range for the phase and a true decoupling from the amplitude which proves to be advantageous in situations when phase and amplitude change at the same time. The actual control input $\Delta\varphi$ is shifted by $pi/2$ to zero in order to obtain a bipolar signal around the tune resonance. Negative phases thus indicate that the excitation frequency is below the tune resonance, and positive phases the opposite. The real implementation requires further compensation for other non-beam related contributions to the measured phase shift such as constant lag due to data processing, cables transmission delays, analogue pre-filters $G_{pre}(s)$ suppressing dominant harmonics (e.g. revolution frequency) and response $G_{ex}(s)$ of the beam exciter itself.

In this representation, the PLL phase control loop dynamics and its design is reduced to a simple first order system with the open loop gain K_0 given by the slope of the phase response at the location of the tune, and the inverse time constant τ by the bandwidth of the low-pass filter. The optimal controller $D_\varphi(s)$ is obtained according to the first order system discussed above.

CROSS-DEPENDABILITY AND CONSTRAINTS

In many accelerators, beam-based feedbacks are usually established and designed one by one. For robust control it is necessary to address possible cross-constraints, cross-talk and coupling between several simultaneous and possibly nested loops already in the design stage.

Betatron-Coupling

In a strict sense, the tune PLL measures eigenmodes rather than the actual tune. In the presence of close tune working points and strong global coupling these eigenmodes may be rotated with respect to the (true) unperturbed tunes. Assuming that the coupling sources are weak and globally distributed, the eigenmodes Q_1 and Q_2 are given by

$$Q_{1,2} = \frac{1}{2}\left(q_x + q_y \pm \sqrt{\Delta^2 + |C^-|^2}\right) \quad (16)$$

with $\Delta = |q_x - q_y|$ being the unperturbed tune-split and C^- the complex coupling parameter. A direct use of the tune eigenmodes may break any tune feedback loop once the eigenmodes are rotated by $\pi/2$ with respect to the real tune. Thus it was recognised that under above conditions a robust tune feedback loop also requires the control of coupling itself [20]. Further details on this scheme and experimental results can be found in [20].

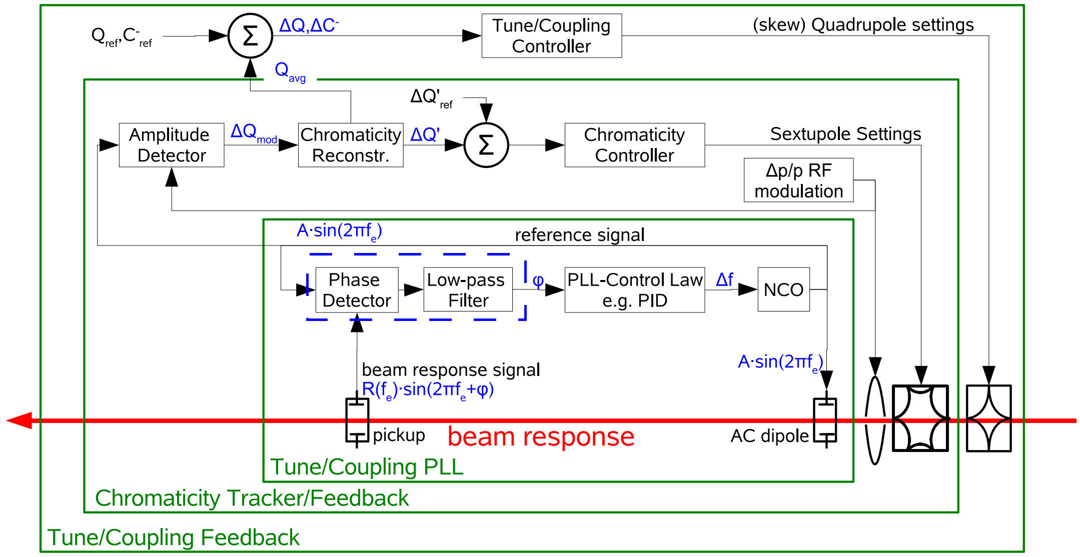

Figure 3: Nested loop scheme required for a coherent control of tune, coupling and chromaticity.

Inter-Loop Dependencies

The LHC, for example, requires a simultaneous control of orbit, tune, coupling, chromaticity and energy. The tightest constraints derives from the LHC collimation system that limits the possible relative momentum modulation, due to aperture constraints and dispersion, to a few $\frac{\Delta p}{p} \approx 10^{-5}$ only. With the requested nominal chromaticity resolution and stability of one unit, the tune changes are minuscule. In order to reflect these constraints in the overall loop design, the following nested control scheme for chromaticity, tune and coupling, shown in Figure 3, is foreseen for the LHC. The tune PLL is the inner-most loop measuring the global tunes and coupling parameters. The loop is first nested within the loop that measures and controls the chromaticity and is then surrounded by the feedback loop controlling the global tunes and coupling. The given hierarchy is based on the fact that the decoupling is obtained by choosing gradually reduced bandwidths for the tune PLL ($f_{bw} \approx 8\,\text{Hz}$), chromaticity ($f_{bw} \approx 1\,\text{Hz}$) and tune feedback ($f_{bw} < 1\,\text{Hz}$). This nesting hierarchy is required particularly to eliminate the cross-talk between tune and chromaticity feedback, as the tune feedback would otherwise minimise the momentum-driven modulation as well as tune modulation and thus compromise the chromaticity measurement.

In addition, cross-talk is introduced between the chromaticity and orbit/energy feedback through the dispersion orbit that is driven by the momentum modulation required by the chromaticity feedback. In order to minimise this cross-dependence, the foreseen LHC orbit feedback filters and separates the dispersion orbit from the measured closed orbit prior to performing any orbit correction.

The required effective correction rate (1 unit per second) and resolution (1 unit) makes the LHC chromaticity feedback one of the most demanding beam-based feedbacks, requires further research and development.

Dependence on Tune Width

As described above, most classic tune PLL implementations assume a constant open loop gain K_0 that depends on the angle of the phase slope at the location of the tune resonance ([4,5,9,10]). Due to varying chromaticity, amplitude detuning, beam-beam, electron cloud, impedance and other effects, the tune width, thus the phase slope and K_0 may change. In this case the optimal controller parameter become functions of, for example, chromaticity itself. Using linear control design only, this cross-dependence implies either a controller design that is optimal for large chromaticities, which becomes sensitive to noise and instable for low values of chromaticity, or a controller design that is optimal for small chromaticities but lags behind the real tune for large values of chromaticity [21][1].

Non-Tune Resonances

Another effect that can break and compromise the proper function of the PLL loop is in the presence of strong resonances other than the tune such as synchrotron sidebands or synchro-betatron resonances. These add π phase advance transitions that due to the phase detector's 2π wrapping property creates several zero phase locations to which the tune PLL can lock onto. Using the Hilbert transform on can show for a minimum-phase system that the PLL essentially locks on the largest peak within the bandwidth. A narrowband excitation signal close to, for example, a synchrotron sideband will thus always cause a lock onto the same resonance. The spurious tune locking can be mitigated by increasing the excitation bandwidth prior to reaching the desired lock condition. This can be done using a chirped excitation and setting the initial PLL working point close to the tune, by adding additional exciter ([10]) or by largely

[1]The effect can be mitigated by stabilising the variation in the first place, which in a way resembles a 'chicken-egg' situation: one needs to stabilise the chromaticity for a stable tune width and thus PLL, while the PLL is needed to measure the chromaticity in the first place.

increasing the closed loop bandwidth to create additional jitter on the excitation frequency. The additional jitter increases the effective sampling range ideally to cover more than one synchrotron resonance. Once the loop is locked, the bandwidth could be reduced in order to improve the tracking stability and to reduce the bleed-through of phase contribution of neighbouring resonances.

LOCKING ON COUPLED BUNCH MODES

A typical cross-dependency is intrinsic to the stability requirements on orbit and tune: though tight constraints on orbit excursion to micrometre level are beneficial to minimise feed-down effects and beam life-time, it also imposes constraints on other feedbacks such as tune and chromaticity, the measurements of which rely on transverse excitations and momentum modulation. For the LHC the tune and coupling PLL operates with transverse excitation levels below $1\ \mu\mathrm{m}$ within the noise level and thus further minimises the cross between PLL and fast transverse damper. However, since the PLL operates within the 'noise floor' of the transverse damper feedback system, it does not benefit from the suppression of coupled bunch modes, which introduce additional resonance lines in the tune spectrum that the PLL can potentially lock onto. This issue is usually addressed by so-called *pilot* or *sacrificial* bunches usually in the beginning of a bunch train, which are excluded by the transverse feedback and explicitly selected for PLL operation.

CONCLUSIONS

Youla's affine parameterisation provides a simple yet powerful design tool for optimal adaptive non-linear control of these feedbacks. Its strength is the explicit controller representation that enables an un-obscured feedback design with respect to closed loop robustness (noise insensitivity) and steering precision.

Feedbacks are commonly deployed as an ensemble and it is thus necessary to reflect this in the early design stage in order to minimise cross-dependences and coupling of multiple simultaneous nested loops and thus cross-talk in between them. Possible decoupling techniques involve orthogonalisation of the feedback parameter space, separation in amplitude or by choosing different bandwidths for each individual feedback loop.

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REFERENCES

[1] PSI, Proceeding of "3rd International Workshop on Beam Orbit stabilization", 2004, http://iwbs2004.web.psi.ch/

[2] M. Böge, "Achieving Sub-micron Stability in Light Sources", EPAC'04, Lucerne, Switzerland, 2004

[3] R.J. Steinhagen, "Real-Time Feedback on Beam Paramters", APAC'07, Indore, India, 2007

[4] O. Berrig et al., "The Q-Loop: A Function Driven Feedback System for the Betatron Tunes During the LEP Energy Ramp",CERN SL-98-039, 1998

[5] A.Fisher et al., "Tune Feedback in PEP-II",SLAC-PUB-10230, 2003

[6] A. Camiel et. al., "Tune Feedback System at ELETTRA", EPAC'94, London, England, 1994

[7] J. Seemann et al., "Achieving a Luminosity of 1034/cm2/s in the PEP-II B-factory", EPAC'06, Edinburgh, Scotland, 2006

[8] "KEKB 2005 Annual Report", http://www-kekb.kek.jp/Publication/KEK_Annual_Report/AR2006.pdf

[9] P. Cameron et al., "Measuring Chromaticity Along the Ramp Using the PLL Tune-Meter in RHIC", EPAC'02, Paris, 2002

[10] C.Y. Tan, "The Tevatron Tune Tracker PLL", FERMILAB-TM-2275-AD

[11] R. Assmann, "Collimation and Cleaning: Could this limit the LHC Performance?", CERN, Chamonix XII, 2003

[12] S. Fartoukh, O. Brüning, "Field Quality Specification for the LHC Main Dipole Magnets", LHC Project Report 501

[13] G. Golub and C. Reinsch, "Handbook for automatic computation II, Linear Algebra", Springer, NY, 1971

[14] K.G.J. Jacobi, "Über ein leichtes Verfahren, die in der Theorie der Säcularstörungen [..]", Crelle's Journal, 1846, 30 p. 51-94, 97-144

[15] D. C. Youla et al., "Modern Wiener-Hopf Design of Optimal Controllers", IEEE Trans. on Automatic Control,1976, Vol. 21-1,pp. 3-13 & 319-338

[16] O. Smith, "Feedback Control Systems", McGraw-Hill, 1958

[17] Goodwin, Graebe, Salgado, "Control System Design", Prentice Hall, 2000

[18] B. Anderson, "From Youla-Kucera to Identification, Adaptive and Non-linear Control", Automatica, 1998, Vol. 34, No. 12, pp. 1485-1506

[19] M.Gasior, R.Jones, "The principle and first results of betatron tune measurement by direct diode detection", CERN-LHC-Project-Report-853

[20] R. Jones et al., "Towards a Robust Phase Locked Loop Tune Feedback System", DIPAC'05, Lyon, France, 2005

[21] R.J. Steinhagen et al., "Influence of Varying Chromaticity on Robustness of the LHC Tune PLL and its Application for Continuous Chromaticity Measurement.", tbp. at PAC'07, 2007