

V. MICROWAVE ELECTRONICS*

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A. HIGH-PERVEANCE HOLLOW ELECTRON-BEAM STUDY

This experimental program has been started to determine the nature of the interaction between a dense, hollow beam and an external circuit. A magnetron injection gun will be used to form a beam of perveance $5-10 \times 10^{-6}$, with densities that are such that $\omega_p/\omega \sim 1$, or larger.

A gun has been designed and is now being built for use in a demountable vacuum system. The beam cross section will be observed by the heating of a tungsten wire screen.

A. Poeltinger, A. Bers

B. ELECTRONIC LOADING IN A SYMMETRIC LLEWELLYN GAP

Large electronic loading of klystron resonators by high-density electron beams has recently been observed experimentally. Beaver, Demmel, Meddaugh, and Taylor (1) attempted to account for this loading by considering the effect of the potential depression in the gap caused by space charge. Their approximate method was to consider a step in beam velocity at the input and output planes of the gap region, and thus find the effect of increased transit time on electronic loading. Kinematic theory, in which the effect of space charge is neglected in the interaction, was employed.

It is possible to solve the small-signal equations for the behavior of a beam in a one-dimensional gridded gap. The results for kinematic theory are well known. Bers (2) solved the problem for a beam neutralized by immobile positive ions, but he accounted for the time-variant net space charge caused by bunching of the beam. The problem of the one-dimensional gridded gap with no gross neutralization of the beam is a special case of the diode equations given by Llewellyn (3).

The relation between the gap voltage and the induced total gap current given by Llewellyn (in mks units) is

$$V = I \left\{ \frac{e}{m} \frac{I_0}{\epsilon_0} \frac{1}{\omega^4} [2 - 2 e^{-j\omega\tau} - j\omega\tau - j\omega\tau e^{-j\omega\tau}] + \frac{l}{j\omega\epsilon_0} \right\} \quad (1)$$

where V is the ac gap voltage at radian frequency ω , I is the total ac gap current, τ is

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the average transit time of an electron, ℓ is the gap width, and I_o is the time-average convection current of the beam.

Let ρ_o be the time-average electron density, and v_o the time-average velocity at the input plane. We now introduce the following parameters:

$$\begin{aligned} V_o &= (mv_o^2)/2e && \text{beam potential at input plane} \\ \omega_p &= (\rho_o e)/(\epsilon_o m) && \text{plasma frequency at input plane} \\ G_o &= I_o/V_o && \text{beam conductance at input plane} \\ \tau_o &= \ell/v_o && \text{transit time in the absence of space-charge fields} \end{aligned}$$

Introducing these parameters into Eq. 1, we obtain

$$\frac{I}{V} = \frac{1}{2} G_o \left(\frac{\omega}{\omega_p} \right)^2 \frac{1}{-j\omega\tau_o + (\omega_p/\omega)^2 (2 - 2e^{-j\omega\tau} - j\omega\tau e^{-j\omega\tau})} \quad (2)$$

Now, I represents the total induced ac gap current. To find the electronic loading, we must subtract from the total current

$$I_{\text{disp}} = \frac{j\omega\epsilon_o}{\ell} V = \frac{1}{2} G_o \left(\frac{\omega}{\omega_p} \right)^2 \left(-\frac{1}{j\omega\tau_o} \right)$$

which is the displacement current. The electronic loading is then given by

$$\frac{I - I_{\text{disp}}}{V} = Y_{el} = \frac{1}{2} G_o \left(\frac{\omega}{\omega_p} \right)^2 \left[\frac{1}{-j\omega\tau_o + (\omega_p/\omega)^2 (2 - 2e^{-j\omega\tau} - j\omega\tau e^{-j\omega\tau}) + \frac{1}{j\omega\tau_o}} \right] \quad (3)$$

We can now split Y_{el} into real and imaginary parts.

$$Y_{el} = G_{el} + jB_{el} \quad (4)$$

where

$$G_{el} = \frac{1}{2} G_o \frac{4 \sin \frac{\omega\tau}{2} \left(\sin \frac{\omega\tau}{2} - \frac{\omega\tau}{2} \cos \frac{\omega\tau}{2} \right)}{\left[(\omega\tau_o)^2 - 8\omega\tau_o (\omega_p/\omega)^2 \left(\sin \frac{\omega\tau}{2} - \frac{\omega\tau}{2} \cos \frac{\omega\tau}{2} \right) \cos \frac{\omega\tau}{2} + 16(\omega_p/\omega)^4 \left(\sin \frac{\omega\tau}{2} - \frac{\omega\tau}{2} \cos \frac{\omega\tau}{2} \right)^2 \right]} \quad (5)$$

and

$$B_{e\ell} = \frac{1}{2} G_0 \frac{4 \left(\sin \frac{\omega\tau}{2} - \frac{\omega\tau}{2} \cos \frac{\omega\tau}{2} \right) \left[\omega\tau_0 \cos \frac{\omega\tau}{2} - 4(\omega_p/\omega)^2 \left(\sin \frac{\omega\tau}{2} - \frac{\omega\tau}{2} \cos \frac{\omega\tau}{2} \right) \right]}{\omega\tau_0 \left[(\omega\tau_0)^2 - 8\omega\tau_0 (\omega_p/\omega)^2 \left(\sin \frac{\omega\tau}{2} - \frac{\omega\tau}{2} \cos \frac{\omega\tau}{2} \right) \cos \frac{\omega\tau}{2} + 16(\omega_p/\omega)^4 \left(\sin \frac{\omega\tau}{2} - \frac{\omega\tau}{2} \cos \frac{\omega\tau}{2} \right)^2 \right]} \quad (6)$$

Finally, we need to relate τ to τ_0 . We can find τ by solving the cubic equation

$$\tau^3 - \frac{12}{\omega_p} \tau + \frac{12}{\omega_p} \tau_0 = 0$$

For $0 \leq \omega_p \leq 4/3\tau_0$, the real root in the range $\tau_0 \leq \tau \leq 3\tau_0/2$ should be chosen. For $\omega_p > 4/3\tau_0$, electrons are reflected in the gap, and the Llewellyn theory breaks down.

Mihran (4) has obtained an equivalent expression for $G_{e\ell}$ as a function of the Llewellyn space-charge parameter and the angle $\omega\tau$.

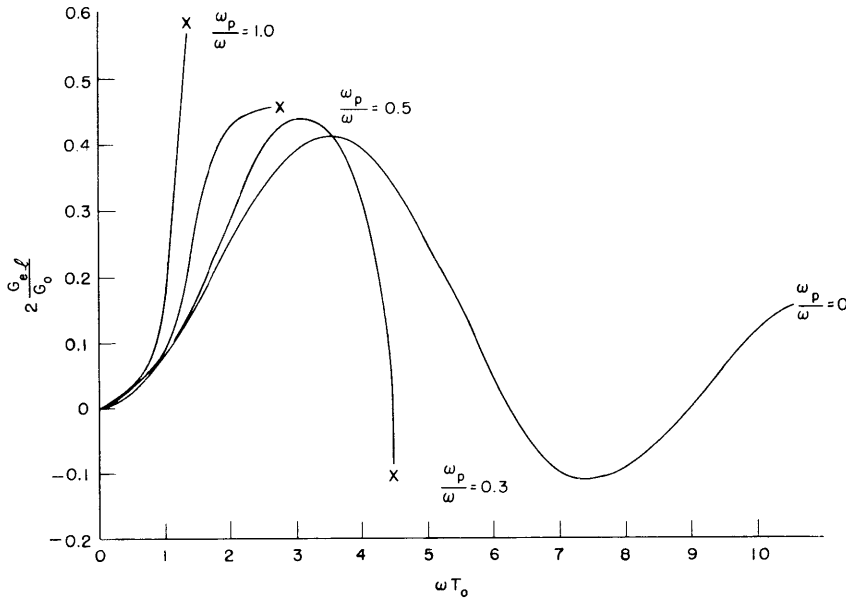


Fig. V-1. Electronic-loading conductance. The point where electrons are turned around by space-charge fields is indicated by x.

Figure V-1 is a plot of $2G_{e\ell}/G_0$ versus $\omega\tau_0$ with (ω_p/ω) as a parameter. It is interesting to note that in the theory of loading for a neutralized beam, an increase in space charge produces a decrease in normalized electronic loading conductance at small transit angles (2) (less than 3 radians), while the unneutralized beam theory predicts an increase in normalized conductance with increasing electron density.

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C. PROPERTIES OF WAVES IN ELECTRON-BEAM WAVEGUIDES

In Quarterly Progress Report No. 59, pages 58-63, we established the conditions under which slow waves with negative phase velocity may exist in electron-beam waveguides with infinite magnetic fields. In the present report we shall establish the group velocity of these waves, that is, the direction in which power is carried by these waves.

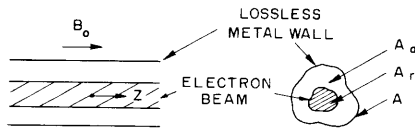


Fig. V-2. Electron-beam waveguide that is uniform in the z-direction.

In the cylindrical system of a uniform lossless electron-beam waveguide (Fig. V-2), if we use the linearized equations of motion and Maxwell's equations, a propagating wave with time and z-dependence of $\exp[j(\omega t - \beta z)]$ satisfies the small-signal Poynting theorem and the small-signal energy theorem (1). From these theorems we find

$$\frac{\partial}{\partial z} \frac{1}{2} \int_A (\hat{\underline{E}} \times \hat{\underline{H}}^* + \hat{\underline{V}} \hat{\underline{J}}^*) \cdot d\bar{a} = 0 \quad (1)$$

$$\frac{1}{4} \int_A [\epsilon_o |\hat{\underline{E}}|^2 - \mu_o |\hat{\underline{H}}|^2 - \epsilon_o |\beta_p \hat{\underline{V}}|^2] da = 0 \quad (2)$$

$$\frac{\partial \omega}{\partial \beta} = \frac{\frac{1}{2} \text{Re} \int_A (\hat{\underline{E}} \times \hat{\underline{H}}^* + \hat{\underline{V}} \hat{\underline{J}}^*) \cdot d\bar{a}}{\frac{1}{4} \int_A [\mu_o |\hat{\underline{H}}|^2 + \epsilon_o |\hat{\underline{E}}|^2 - \epsilon_o |\beta_p \hat{\underline{V}}|^2 + \frac{2}{v_o} \text{Re}(\hat{\underline{V}} \hat{\underline{J}}^*)] da} \quad (3)$$

where $\hat{\underline{V}} = (m/e) \bar{v}_o \cdot \hat{\underline{v}}$, with v_o the time-average electron velocity and v the small-signal complex amplitude of the velocity; the circumflex on the field quantities indicates that

they are functions of the transverse coordinates only; A is the cross section of the waveguide; $\beta_p = \omega_p/v_o$, with ω_p the electron plasma frequency. Equation 1 gives the balance between the small-signal electromagnetic power flow

$$P_e = \frac{1}{2} \operatorname{Re} \int_A \hat{\underline{E}} \times \hat{\underline{H}}^* \cdot d\bar{a} \quad (4)$$

and the corresponding kinetic power flow

$$P_k = \frac{1}{2} \operatorname{Re} \int_A \hat{\underline{V}} \hat{\underline{J}}^* \cdot d\bar{a} \quad (5)$$

Equation 3 is an expression for the group velocity $\partial\omega/\partial\beta$; the numerator is the total power flow, the denominator is the total energy storage per unit length of waveguide. Equation 2 gives the balance among the partial energies of the system which is required for the existence of a purely propagating wave.

Combining Eqs. 2, 3, 4, and 5, we obtain for the group velocity

$$\frac{\partial\omega}{\partial\beta} = v_o \left[\frac{1 + (P_e/P_k)}{1 + [(2w_m v_o)/P_k]} \right] \quad (6)$$

where $w_m = 1/4 \mu_o \int_A |\hat{\underline{H}}|^2 da$. In the one-dimensional electron beam, $P_e = 0$ and $\hat{\underline{H}} = 0$, hence the group velocity is v_o , the time-average electron velocity, independent of both density and frequency. For the confined-flow electron-beam waveguide, we find

$$\frac{\partial\omega}{\partial\beta} = v_o \left[\frac{1 + (P_e/P_k)}{1 + \left(\frac{v_o/c}{\beta/k} \right) \frac{P_e}{P_k}} \right] \quad (7)$$

Equations 4 and 5 can be evaluated from the field equations given previously (2). Green's first theorem applied to the $\hat{\underline{E}}_z$ field over the nonuniform cross section of the waveguide gives

$$\begin{aligned} p^2 \int_{A_p} |\hat{\underline{E}}_z^p|^2 da_p + q^2 \int_{A_a} |\hat{\underline{E}}_z^a|^2 da_a \\ = \int_{A_p} |\nabla_T \hat{\underline{E}}_z^p|^2 da_p + \int_{A_a} |\nabla_T \hat{\underline{E}}_z^a|^2 da_a \end{aligned} \quad (8)$$

where p and q are the transverse wave numbers for the fields in the beam and in the air spaces, respectively. We find

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$$\frac{P_e}{P_k} = -\beta(\beta - \beta_e) \frac{|\beta - \beta_e|^2 - \beta_p^2}{|q^2| \beta_p^2} \frac{p^2}{|p^2|} \left[1 + \frac{q^2 \int_{A_a} |\hat{E}_z^a|^2 da_a}{p^2 \int_{A_p} |\hat{E}_z^p|^2 da_p} \right] \quad (9)$$

where $\beta_e = \omega/v_o$. The bracketed terms of Eq. 9 depend upon the geometry of the system.

As an example, consider an electron beam filling the waveguide. In this case, the expression in brackets in Eq. 9 equals 1, and p^2 is a positive real number. Equation 9 becomes

$$\frac{P_e}{P_k} = \frac{\beta_k}{v_c} \frac{(1 - v_c \beta_k)}{1 - \beta_k^2} \left| 1 - \frac{(1 - v_c \beta_k)^2}{\omega_{pn}} \right| \quad (10)$$

where $\beta_k = \beta/k$, $v_c = v_o/c$, and $\omega_{pn} = \omega_p/\omega$. From Eqs. 7 and 10 we note that four types of slow ($\beta > k$) waves are possible: two forward traveling waves,

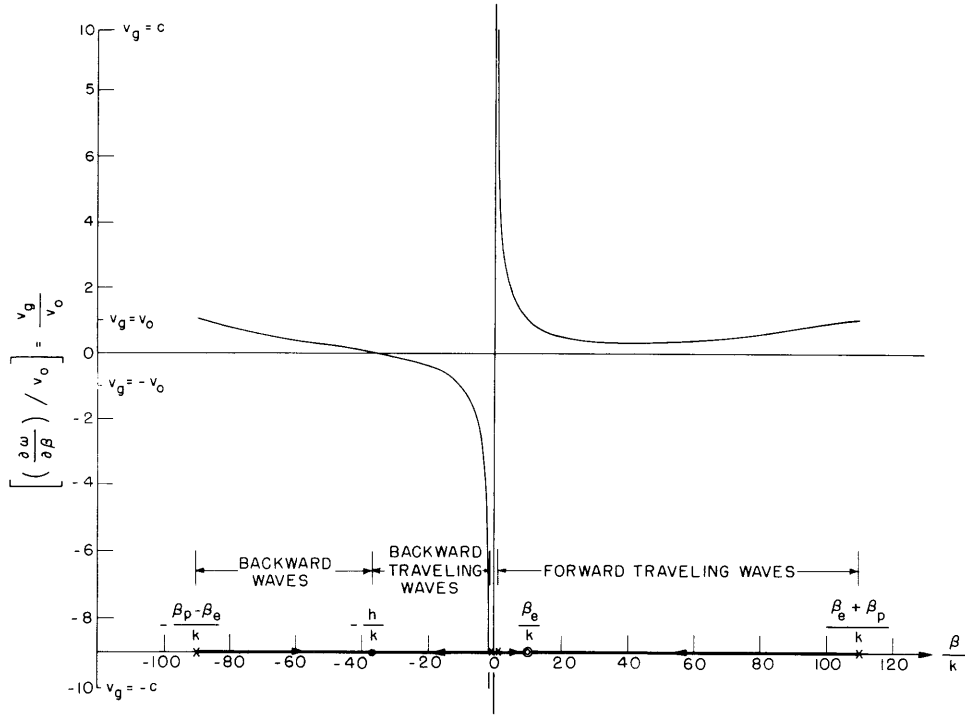


Fig. V-3. Group velocity versus propagation constant for an electron beam filling a cylindrical waveguide ($\omega_p/\omega = 10$; $v_o/c = 0.1$). The arrows indicate the location of the β -roots as a function of increasing p .

$\beta > 0$ with $\partial\omega/\partial\beta > 0$; a backward wave, $\beta < 0$ with $\partial\omega/\partial\beta > 0$; and a backward traveling wave, $\beta < 0$ with $\partial\omega/\partial\beta < 0$. An example is shown in Fig. V-3.

A. Bers

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D. LARGE-SIGNAL KLYSTRON THEORY

Some results of a theoretical investigation of several schemes for improving the electronic bunching efficiency of klystron amplifiers were presented in Quarterly Progress Report No. 60, pages 98-104. In particular, the rebunching of an electron beam by a succession of closely spaced (nonpropagating) gap circuits was analyzed. It was shown that 100 per cent electronic bunching efficiency could be realized if the gap circuits could interact with all of the beam-current harmonics. In the present report we wish to present some results for gap circuits that interact with only the fundamental component of the beam current. The equation of motion of the electrons in the distributed circuit is

$$\frac{\partial^2 v(t, t_1)}{\partial t^2} + \omega_p^2 v(t, t_1) = \omega_p^2 v_0 + \frac{e}{m} \frac{\partial E_c(t, t_1)}{\partial t}$$

where v is the electron velocity, t_1 is the time of entrance of a particular electron into the circuit, ω_p is the electron plasma frequency, and E_c is the electric field that arises from the induced charges on the circuit grids. In the present case, we would like to constrain the circuit field, E_c , to be a sinusoidal function of t for any fixed z . To carry out the analysis in closed form, however, it is necessary to assume (as in the multi-harmonic case) that the circuit field does not vary with time for a particular electron. That is, we shall assume that

$$E_c(t, t_1) = E_c(t_1) = E_0 \sin \omega \left(t_1 - \frac{z_1}{v_0} \right)$$

where z_1 is the entrance position of the distributed circuit relative to the input gap. The difficulty with this last assumption in the present case is that it is inconsistent with the first assumption of a sinusoidal field at some point $z > z_1$. However, one finds that it is

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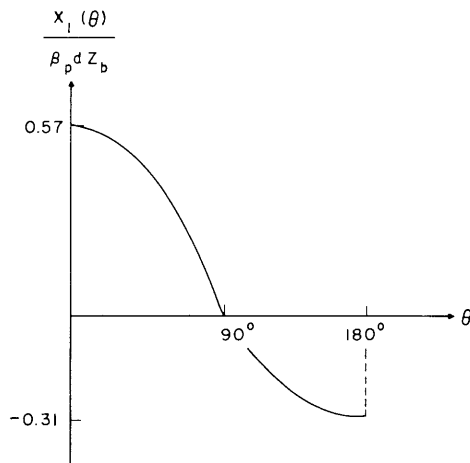


Fig. V-4. Gap reactance at the fundamental frequency versus plasma transit angle. ($Z = jX_1$ is the impedance of the gap; $Z_b = (2V_o \omega_p)/I_o \omega$; d is the length of one gap; $\beta_p = \omega_p/v_o$; θ is the plasma angle.)

approximately sinusoidal, so the results were taken as an approximation to a true single harmonic theory. Our theory does take account of all of the beam-current harmonics that enter the circuit.

For such a distributed circuit of gaps, we found that the electronic bunching efficiency could be 70 per cent with approximately zero velocity spread at the output gap. The length of the circuit should (as in the multiharmonic case) be 180° of plasma angle. Previous theories (1), which neglected all components of the beam current other than the fundamental, predicted a theoretical efficiency of around 83 per cent. The form of the gap reactance (which includes the free-space capacitance of the gaps) is plotted as a function of plasma angle in Fig. V-4. It is qualitatively the same as the fundamental gap reactance in the multiharmonic case (1); however, the magnitude is approximately one-half as much.

The analysis given in this report and the analysis in a previous report (2) formed the basis of the author's Master of Science thesis (3).

R. J. Briggs

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