Electromagnetic scattering beyond the weak regime: Solving the problem of divergent Born perturbation series by Padé approximants

Thomas van der Sijs[1], *Omar* El Gawhary[1,2,*], and *Paul* Urbach[1]

[1]Optics Research Group, Delft University of Technology, P.O. Box 5046, 2600 GA Delft, The Netherlands.
[2]Dutch Metrology Institute (VSL), Thijsseweg 11, 2600 AR Delft, The Netherlands.

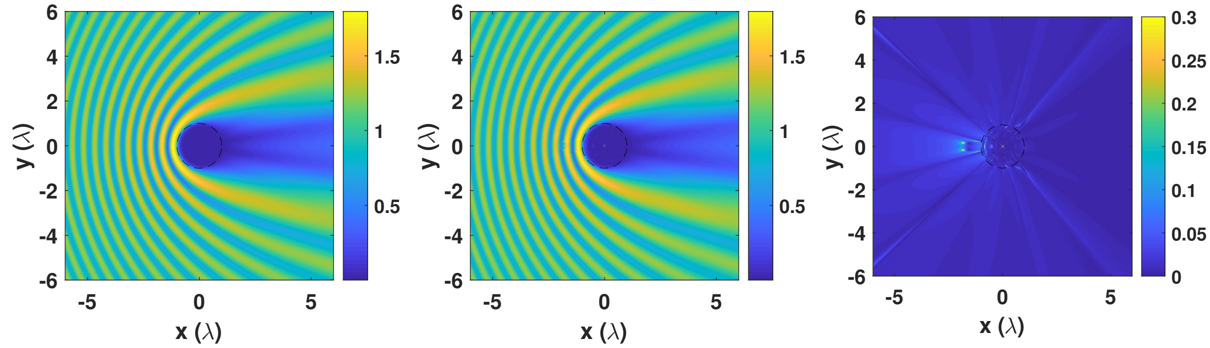
Abstract. Electromagnetic scattering is the main phenomenon behind all optical measurement methods where one aims to retrieve the shape or physical properties of an unknown object by measuring how it scatters an incident optical field. Such an inverse problem is often approached by solving, several times, the corresponding direct scattering problem and trying to find the best estimate of the object which is compatible with a set of measurements. In the direct scattering problem, two regimes can be distinguished depending on the size of the object and the permittivity contrast: the weak-scattering regime and the strong-scattering regime. Generally, the presence of the scatterer alters the form of the incident field inside the scatterer. If that effect is neglected in the physical model, then one speaks of the so-called single-scattering regime or, more often, the Born approximation. The regime in which this approximation is valid is the weak-scattering regime. The corresponding inverse problem, that aims to retrieve the object from scattering data, becomes linear in this case. Linearizing the problem simplifies the method to solve it, but also introduces limitations to the maximum spatial resolution achievable in the reconstruction of the object. In the strong-scattering regime, multiple-scattering effects are not neglected and the inverse problem is treated in its full non-linear nature, which makes finding its solution a far more challenging task.

Despite the existence of numerical methods, a powerful way to solve those direct problems would be to use a perturbation approach where the field is expressed as a series, known as the Born series. The advantage of a perturbation approach stems from the fact that each term of the series has a clear physical meaning and can unveil much more about the scattering process than a purely numerical approach can offer. Unfortunately, the series solution turns out to be strongly divergent in the strong-scattering regime, making it an unpractical approach for problems under these strong-scattering conditions. Thus, despite the fact that multiple scattering could, in principle, allow resolving sub-wavelength details of the unknown object, this possibility is in practice hampered by the divergent nature of the higher-order terms of the Born series.

In this work, we show how to solve this problem by employing Padé approximants and how to treat electromagnetic problems well beyond the weak-scattering regime and provide an accurate evaluation of the scattered field even under strong-scattering conditions. Padé approximants are rational functions that can offer improvements in two ways, namely series acceleration of converging series and analytic continuation of a series outside its region of convergence. In the case of a symmetric approximant of order N, the approximant is calculated from $2N + 1$ terms in the Born series, therefore incorporating multiple-scattering effects to which these higher-order corrections in the Born series correspond. We apply the method to two scalar scattering problems: that of a one-dimensional slab and that of an infinitely long cylinder, which reduces to a two-dimensional problem under normal incidence. In particular, we treat cases in the strong-scattering regime where the Born series diverges, but where Padé approximation retrieves a valuable result. In Fig. 1 the case of a cylinder is shown which is well beyond the weak-scattering regime, but where the most accurate Padé approximant gives a good result for the field.

The presented approach incorporates multiple-scattering effects and can therefore represent an important building block to the application of the Born series to direct and inverse problems, with potential applications in superresolution, optical metrology, and phase retrieval.

*e-mail: o.elgawhary@tudelft.nl



(a) Analytical solution.

(b) Padé approximant P_4^4.

(c) Error of P_4^4.

Figure 1: Padé approximation in the two-dimensional problem of a cylinder on which a unit plane wave impinges under normal incidence. The cylinder has a radius of 1 λ and has the permittivity of silver $\varepsilon_r = -4.42 + 0.201i$, which is a case that is well outside the weak-scattering regime. The approximant that is shown is P_4^4, the symmetric approximant of order $N = 4$, which was found to give the best approximation to the actual solution.