Transverse Energy Density in High Energy Heavy Ion Collisions

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Abstract. A phenomenological model describing the transverse energy distribution (E_T) of nuclear collisions is first studied in detail by fitting it on E_T data for O-Pb collisions at $\sqrt{s_{NN}}$ = 200 GeV per nucleon obtained from the NA35 collaboration. Next, the model is used to fit the E_T data for Pb-Pb collisions at LHC energies of $\sqrt{s_{NN}}$ = 2.76 TeV per nucleon obtained from the ATLAS collaboration. From the fits, we determine an upper bound for the energy density for Pb-Pb collisions at LHC energies of $\sqrt{s_{NN}}$ = 2.76 TeV per nucleon.

1 Introduction

According to the Standard Model, hadronic particles are bound states of quarks. The quarks in the hadronic particles interact via the strong force and are bound by the strong force. Although currently, no isolated quark has been observed, results from deep inelastic scattering show that the quark model is highly viable [1].

Current theories say that as temperature increases, the strong force weakens. The opposite is also true. This is known as Asymptotic freedom in QCD. The implication of Asymptotic freedom is this: at high enough temperatures, we might have a possible new state of matter, where the quarks exist not as bound states, but as free particles. This new state of matter has been termed "Quark Gluon Plasma". Going from hadronic particles to Quark Gluon Plasma at high temperatures is known as Deconfinement, whereas going from Quark Gluon Plasma to hadronic particles at low temperatures is known as Hadronisation. Both Hadronisation and Deconfinement can be throught of as phase transitions [2]. Current theoretical calculations using Lattice QCD show that this phase transition occurs at a critical energy density of 0.7 ± 0.2 GeV fm^{-3} [3] and at a temperature of 140-190 MeV [4].

According to Big Bang Cosmology, the early universe was both extremely hot and extremely dense. This means that Quarks and Gluons probably existed as Quark Gluon Plasma in the early universe. Then, as the universe expanded and cooled, the Quark Gluon Plasma underwent Hadronisation to give us the hadrons that we see today. While various people have tried to model this process of Hadronisation, such as in [10], this process of Hadronisation is still under active investigation. Clearly, a better understanding of Quark Gluon Plasma would help us achieve a better understanding of Hadronisation. Thus, if we are able to make Quark Gluon Plasma in the lab, we will be able to better study the Hadronisation process.

From as early as the 1980s, the collision of heavy ions at as high energies as possible was seen as a viable way to experimentally replicate the hot and dense conditions of the

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early universe, which is needed for the formation of Quark Gluon Plasma. Such heavy ion collisions were first done in colliders like the RHIC (Relativistic Heavy Ion Collider), and are done today in experiments like ALICE (A Large Ion Collider Experiment) in CERN. There are certain theoretical signposts that tell us if we have indeed created Quark Gluon Plasma in our heavy ion collision. Firstly, we need to first check if the energy density in the heavy ion collision is greater than the theoretical critical energy density. Next, we need to check for other theoretical signs of Quark Gluon Plasma, such as Strangeness enhancement [5], amongst many others.

The remainder of this paper is to explore the first point above with respect to Pb-Pb collisions at LHC energies of $\sqrt{s_{NN}} = 2.76$ TeV per nucleon obtained from the ATLAS collaboration. A phenomenological model will be used for this purpose.

1.1 Purpose of this paper

We want to be determine the energy density in Pb-Pb collisions at LHC energies of $\sqrt{s_{NN}} =$ 2.76 TeV per nucleon obtained from the ATLAS collaboration. In other words, from the collider data, we want to extract the energy density. There are no “first principles" (i.e QCD) model to do this yet, thus the problem has to be treated phenomenologically.

The collider data that we want to study is shown below in figure 1. In this report, we show that the upper bound for the energy density for the collision data in figure 1 is 0.006 TeV. Future work will be done to determine the exact energy density.

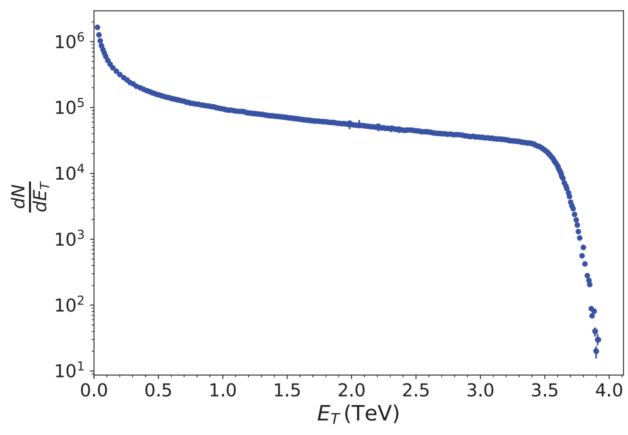

Figure 1. The transverse energy distribition for Pb-Pb collisions at centre of mass energy 2.76 TeV per nucleon. The data here is in the pseudorapidity range of $3.2 < |\eta| < 4.9$, and is taken from the ATLAS collaboration [6].

1.2 Approach

As mentioned, a phenomenological approach to finding the energy density will be taken. Thus, we will first construct a phenomenological model to model heavy ion collisions. The model will take in as input the transverse energy distribition data as shown in figure 1, and produce as the output a number that can be reasonably interpreted as the energy density of the heavy ion collision. Next, we will test the model we constructed on an old and well-investigated set of data, shown in figure 2, to try and reproduce past results. Lastly, if the previous step is successful, we will test the model on the real data shown in figure 1.

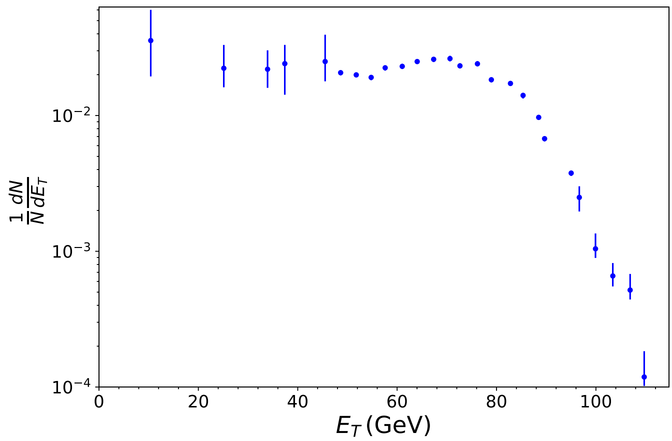

Figure 2. The data for ^{16}O on ^{208}Pb at energies of $\sqrt{s_{NN}} = 200$ GeV per nucleon in pseudorapidity range $2.2 < \eta < 3.8$. Data is from the NA35 collaboration [9], and has already been studied in detail in [7].

2 The Phenomenological model considered

As was mentioned previously, we will first need to find a suitable phenomenological model. We will adapt a geometrical model used before in the past to analyse heavy ion collisions [7],[8]. A rough schematic of the geometrical model is shown in figure 3. There are three key

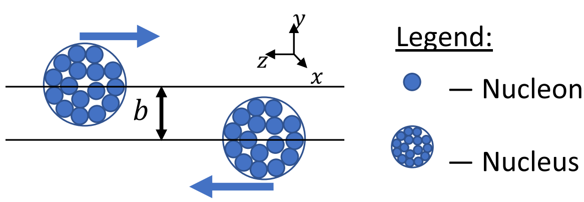

Figure 3. Geometrical model considered. The two colliding nuclei are considered as spheres colliding at a given impact parameter b.

ideas behind this geometrical model. They are:

1. Treat nuclei-nuclei collisions as many individual nucleon-nucleon collisions
2. Let each nucleon-nucleon collision collision have a transverse energy distribution $\rho(E_T)$, with mean transverse energy per collision ϵ
3. Let the number of nucleon-nucleon collisions n take a Poisson distribution about the mean $\bar{n}(b)$

From these three key ideas, we can derive the following equation for transverse energy density:

$$\frac{1}{N}\frac{dN}{dE_T} = \left\langle \sum_{k=0}^{\infty} \rho^k(E_T) \frac{\bar{n}(b)^k}{k!} e^{-\bar{n}(b)} \right\rangle \quad (1)$$

where in equation 1, $\rho^k(E_T) \equiv \int \prod_{i=1}^{k} \rho(E_i)\delta(E_T - \sum_{a=1}^{k} E_a)$ denotes a k-fold convolution of ρ, and the angular brackets $\langle\ \rangle$ denote an average over the impact parameter b. Equation 1 has 1 free parameter, which is ϵ. Thus, we can vary the value of ϵ to best fit the data in figures 1 and 2. The value of ϵ that best fits the data is the value that we want. Since ϵ is the mean transverse energy per nucleon-nucleon collision, ϵ can be interpreted as the energy density during the Pb-Pb collision. Thus, the energy density during the nuclei-nuclei collision is determined.

Now, we see that to evaluate the RHS of equation 1, we need to calculate the mean number of collisions at a given impact parameter, $\bar{n}(b)$. This is the hardest part of evaluating equation 1, and this is why this model is known as a geometrical model.

2.1 Calculating $\bar{n}(b)$

Calculating $\bar{n}(b)$ is the key to evaluating equation 1. To do so, we firstly need to define the nuclear profile of the two nuclei. The Wood-Saxon nuclear density profile is used to get the results below, but it has been shown in [7] that the results obtained are pretty independent on the nuclear density profile used. This will be verified in a future work.

Using the Wood-Saxon nuclear density profile, we have

$$R = 1.19A^{1/3} - 1.61A^{-1/3} \tag{2}$$

$$\rho(r) \propto (1 + \exp(r - R)/s)^{-1} \tag{3}$$

where $\rho(r)$ is normalised such that $\iiint \rho(r)\, d\mathbf{r} = A$, where A is the atomic number of the nuclei. Here, s which is the surface thickness of the nucleus is taken to be 0.54 fm^{-3}.

Now, consider the case of two colliding nuclei A and B with nuclear density profiles $\rho_A(r)$ and $\rho_B(r)$ respectively. Also, let the nucleon-nucleon inelastic scattering cross section be σ_{nn}. By geometrical considerations, we would have:

$$\bar{n}(b) = \iint dxdy\, \sigma_{nn} \int dz_A\, \rho_A\Big(\sqrt{x^2 + y^2 + z_A^2}\Big) \int dz_B\, \rho_A\Big(\sqrt{x^2 + (y-b)^2 + z_B^2}\Big) \tag{4}$$

where b is the impact parameter of the two nuclei. The x-y integration is done over the overlap region of the two geometrical cross sections, as shown in figure 4.

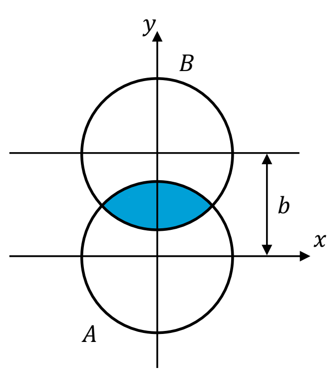

Figure 4. Overlap of the geometrical cross sections of the two nuclei. The z axis points into the page. $\bar{n}(b)$ can be determined from this overlap.

Using the Pb-Pb collision as an example, equation 4 would look like:

$$\bar{n}(b) = \int_0^{x_I} 2\,dx \int_{b-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dy\,\sigma_{nn} \int_0^{z_A} dz_A\,\rho_A\left(\sqrt{x^2+y^2+z_A^2}\right) \int_0^{z_B} dz_B\,\rho_A\left(\sqrt{x^2+(y-b)^2+z_B^2}\right)$$

where R is the radius of the Pb nuclei in the Wood-Saxon nuclear density profile, and

$$x_I = \sqrt{R^2 - \frac{b^2}{4}}$$
$$z_A = \sqrt{R^2 - x^2 - y^2}$$
$$z_B = \sqrt{R^2 - x^2 - (y-b)^2}$$

When the two nuclei have different radii, more care has to be done when using equation 4. In this case, these three cases have to be considered separately:

- $0 \le b < R_A - R_B$
- $R_A - R_B \le b < \sqrt{R_A^2 - R_B^2}$
- $\sqrt{R_A^2 - R_B^2} \le b < R_A + R_B$

where R_A is the radius of nuclei A and R_B is the radius of nuclei B, and we assume $R_A > R_B$.

3 Validating the geometrical model

Now that we have explained the geometrical model above, we fit equation 1 to the data in figure 2 to determine the value of ϵ, which will be interpreted as the energy density. Our results are shown figure 5.

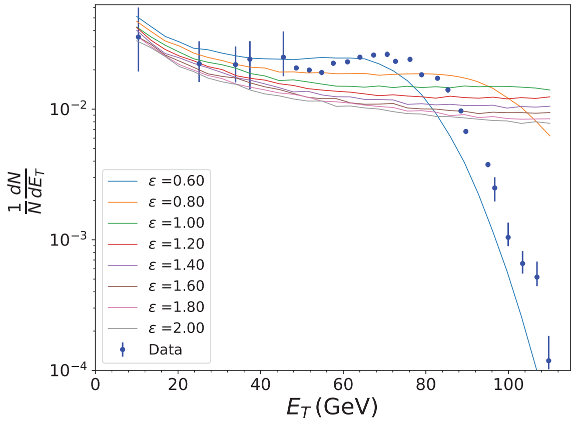

Figure 5. Equation 1 fitted with various values of ϵ to the data in figure 2.

Let us first examine the graphs of $\epsilon = 0.6$ GeV and $\epsilon = 0.8$ GeV. The graphs of $\epsilon = 0.6$ GeV and $\epsilon = 0.8$ GeV are unable to fit the data as the locations of the “shoulder" are too far to the left and too far to the right respectively. Thus, we infer that the value of ϵ that best fits the data is between 0.6 GeV and 0.8 GeV.

This is consistent with the results obtained from [7], who calculated the energy density of this set of data to be 0.79 GeV.

Another thing to note would be that as the value of ϵ increases, the location of the "shoulder" shifts rightwards. Thus, for the other values of ϵ plotted, the location of the "shoulder" is too far to the right to be seen on this diagram.

4 Preliminary results for Pb-Pb collision

Since our geometrical model could successively reproduce past results, we apply it to the our data in figure 1. Our results are shown in figure 6.

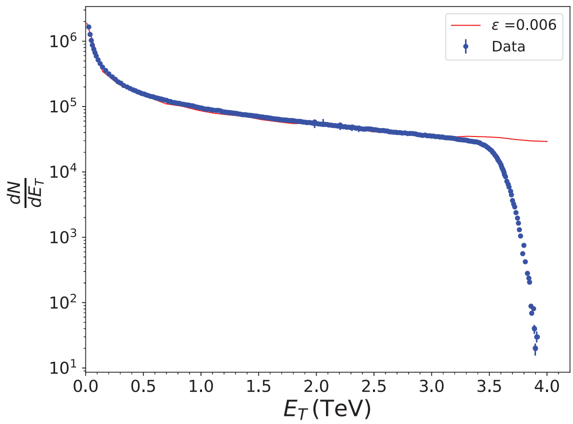

Figure 6. The fit of equation 1 with $\epsilon = 0.006$ TeV to the data in figure 1.

From figure 6, we see that the location of the "shoulder" as predicted by equation 1 with $\epsilon = 0.006$ TeV is too far right to be shown on the graph. Thus, we conclude that for the data in figure 1, the energy density must be lower that $\epsilon = 0.006$ TeV.

5 Conclusion

According to the predictions of both the Standard Model and Big Bang cosmology, Quark Gluon Plasma is possibly in the early universe. Then, as the universe cooled, the Quark Gluon Plasma hadronised to form hadrons. This provides ample motivation for studying high energy heavy ion collisions, which tries to mimic the conditions of the early universe to produce Quark Gluon Plasma. To determine the existence of Quark Gluon Plasma during such high energy heavy ion collisions, we need to check if the energy density reached during the collision exceeds the theoretical critical minimum value required for Hadronic matter to undergo a phase transition to Quark Gluon Plasma.

The problem we want to solve in this paper is this: from the heavy ion collision data in figure 1, we want to determine the energy density reached during the collision. To that end, a geometrical model is used for phenomenological calcuations. Preliminary calculations show that the energy density is lower than 0.006 TeV. Future work will have to be done to determine exactly what the energy density is.

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