

## VIII. MICROWAVE ELECTRONICS\*

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### RESEARCH OBJECTIVES

#### 1. Magnetron Injection, Hollow-Beam Guns

Parametric experimental studies of the magnetron injection gun will be made. Particular attention will be paid to the noise-generation process. The effect of axially tapered magnetic fields on perveance and beam shape will be studied. Finally, the rf properties of the hollow beams, thus formed, will be studied in a demountable, two-cavity klystron.

#### 2. Focused Hollow Beams from Internally Coated Cathodes

The internally coated (hollow) cathode has received little attention during the past five years. Recent experiments in this laboratory indicate that well-focused hollow beams of small diameter and large current density can be produced with these cathodes. Experimental and theoretical gun design studies will be made.

#### 3. Large-Signal Klystron Theory

The theory of superbunching of an electron beam has been generalized to include the effect of all harmonics present in the electron bunch. An approximate solution has been obtained for the variation of circuit reactance with distance along the beam. The design of such a circuit and the possible experimental verification of the theory of superbunching will be studied.

#### 4. Theory of Klystron Gap Loading

The formulation of a theory of gap interaction that takes into account the depression of the time-average potential in the beam is being studied. The evaluation of Bers' theory of gap loading, which accounts for both electromagnetic power flow and kinetic power flow, will continue.

#### 5. Waves on High-Density Electron Beams

We have recently established the properties of backward waves and backward traveling waves in dense electron beams. Studies will be made of possible double-stream instabilities in systems that have such electron streams.

L. D. Smullin, H. A. Haus, A. Bers

### A. TWO-GAP KLYSTRON CAVITY CALCULATIONS

Calculations of the theoretical operation of the two-gap klystron output cavity previously described (1) are being programmed for execution by the IBM 709 computer at the Computation Center, M. I. T. The calculations are based on the equivalent circuit that

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was given in another report (1) and on the small-signal space-charge wave equations for the electron beam-gap interactions.

Signal-flow graph techniques were used to determine the expressions for the ratios of the gap voltages and power output to an initial drive current at the first gap. The circuit parameters were evaluated from the results of experimental tests on the cavity.

The programming for the ratio of the voltage in the first gap to the drive current versus frequency is now complete, and programs for the ratios of the voltage in the second gap and of the power output to the drive current are being written. No numerical results have yet been obtained.

D. L. Morse

### References

1. D. L. Morse, Two-gap klystron cavities, Quarterly Progress Report No. 57, Research Laboratory of Electronics, M. I. T., April 15, 1960, pp. 79-81.

## B. LARGE-SIGNAL BEHAVIOR OF ELECTRON BEAMS

A study is being made of the bunching and rebunching of electron beams by klystron gaps. A one-dimensional model of the beam is used, and the effect of harmonics in the bunching process is accounted for. Two processes are considered in this report: the rebunching by a single infinitesimal gap, and the rebunching by a distributed circuit. Our theory includes space charge, and most of the results obtainable in closed form are applicable to beams with weak space charge and small initial velocity modulations.

### 1. Single Intermediate Gap

According to Haus (1), the beam current resulting from an initial velocity modulation of the form  $v_1 \sin \omega t_0$  is given by

$$i(z, t) = i_0 \sum_{n=-\infty}^{+\infty} J_n(nX_0 \sin \theta) \exp[jn\omega(t-z/v_0)]$$

where  $J_n(X)$  is the Bessel function of  $n^{\text{th}}$  order,  $i_0$  is the dc current,  $X_0 = \omega v_1 / \omega_p v_0$  is the "bunching parameter," and  $\theta = (\omega_p / v_0)z$  is the plasma angle. This expression is valid only if there has been no overtaking of electrons. It can be shown that for no overtaking  $X_0 \sin \theta \leq 1$ . A plot of the harmonic amplitudes of the beam current is shown in Fig. VIII-1. It can be seen from this plot that the beam current can be quite rich in harmonics under large-signal conditions.

Preliminary investigations showed that the electronic efficiency could be 100 per cent if the intermediate cavity interacts with all of the beam-current harmonics. The

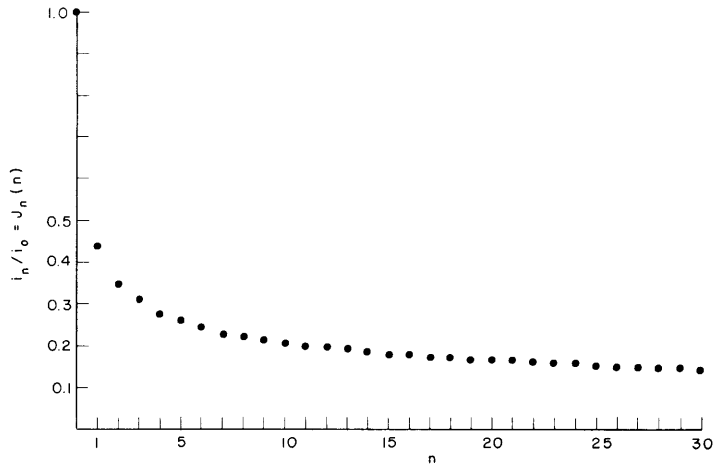


Fig. VIII-1. Current harmonics resulting from initial velocity modulation ( $\theta=90^\circ$ ,  $X_0=1$ ).

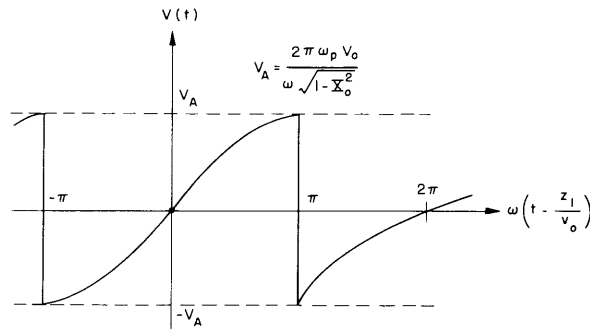


Fig. VIII-2. Voltage waveform for 100 per cent efficiency with one intermediate gap.

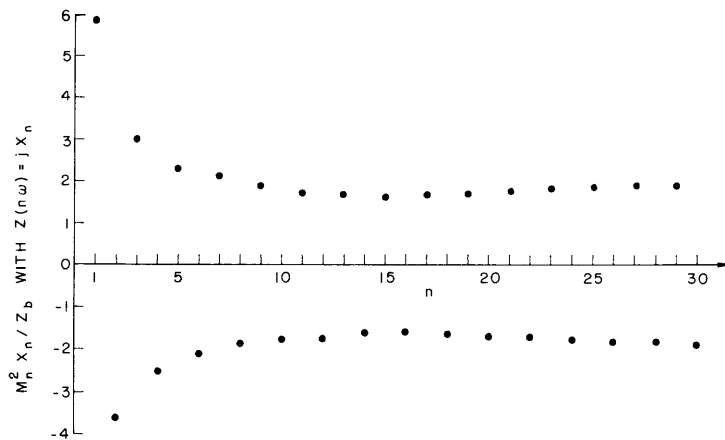


Fig. VIII-3. Impedance of intermediate gap.

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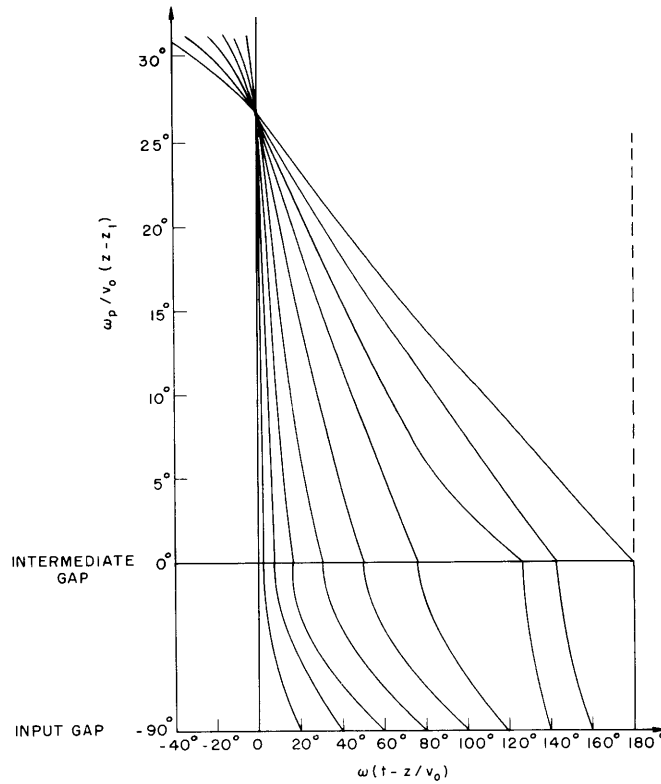


Fig. VIII-4. Electron paths for multiharmonic bunching ( $X_0 = 0.9$ ).

general form of the voltage waveform that should appear across the gap is shown in Fig. VIII-2; it is somewhat similar to the "saw-tooth waveform" that a kinematic analysis would predict for 100 per cent efficiency. The location of the intermediate gap should be  $90^\circ$  of plasma angle from the input gap. An expansion of the voltage waveform in a Fourier series gives the necessary gap impedance as

$$M_n^2 Z_n(n\omega) = j \frac{Z_b}{n J_n(nX_0) (1 - X_0^2)^{1/2}} [(1 - X_0) J_n(nX_0) - \cos n\pi]$$

where  $M_n = \sin(n\beta_e d)/(n\beta_e d)$ ,  $d/2$  is the gap length, and  $Z_b = 2\omega_p V_0/\omega I_0$  is the beam impedance that is met also in small-signal analysis. See Fig. VIII-3 for a plot of this impedance versus  $n$  for  $X_0 = 0.9$ .

The distance from the second gap to the position of maximum bunching is approximately  $26^\circ$  of plasma angle for  $X_0 = 0.9$ . The electron paths are shown in the form of an Applegate diagram in Fig. VIII-4. The velocity spread in the beam is quite considerable at the overtaking position; it is well known that the presence of such a velocity spread can lower the over-all energy-conversion efficiency.

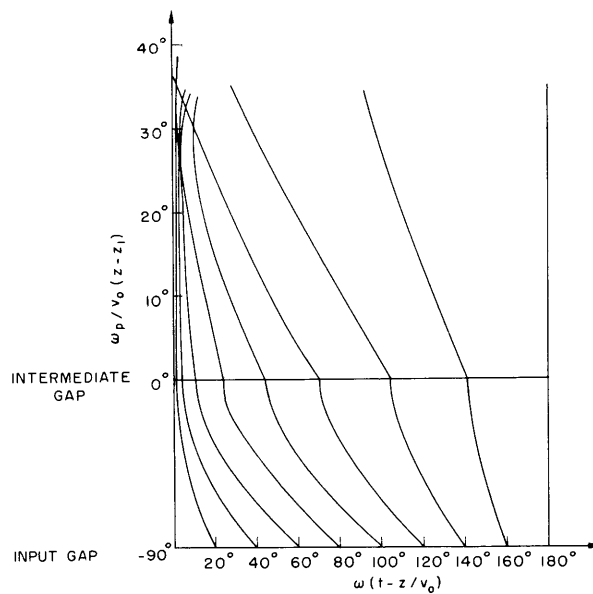


Fig. VIII-5. Electron paths for single-harmonic bunching ( $X_0=1$ ).

For obvious practical reasons, an intermediate circuit that interacts only with the fundamental component of the beam current was considered. In this case, it was found that the maximum electronic efficiency is around 72 per cent. The distance from the second gap to the position of maximum bunching was, again, approximately  $26^\circ$  of plasma angle, and the velocity spread in the beam is still a problem. (See Fig. VIII-5 for the electron paths.) The gap impedance (which is nonzero for the first harmonic only) should be a pure reactance and inductive in nature. The necessary magnitude of the impedance can be reasonably approximated by cavities with  $Q$  of 50, or greater; this will result in a reactance that is 10 times greater than the real part of the gap impedance. The effect of this loss will, of course, lower the bunching efficiency.

## 2. A Distributed Circuit (Squeezer)

The idea of using a distributed circuit was first proposed by Professor L. D. Smullin as a means of attaining a highly bunched beam while retaining a very low velocity spread. This scheme was analyzed by Bers (2) who used a one-dimensional model and assumed that only the fundamental beam current entered the squeezer. The aim of this investigation is to remove the single-harmonic assumption. (See Fig. VIII-6.)

In this analysis, the electric field is considered as a superposition of the field arising from the space charge ( $E_s$ ) and the field arising from the charges on the gridded gaps ( $E_c$ ). The equation of motion for a particular electron entering the squeezer at time  $t_1$  is

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$$\frac{\partial^2 v(t, t_1)}{\partial t^2} + \omega_p^2 v(t, t_1) = \omega_p^2 v_0 + \frac{e}{m} \frac{\partial E_c(t, t_1)}{\partial t}$$

The optimum location of the front of the squeezer can be shown to be  $90^\circ$  of plasma angle from the input gap. The equation of motion was solved by requiring that the circuit field ( $E_c$ ) on a particular electron not change with time and by allowing the squeezer circuit to interact with all of the beam-current harmonics. We found that 100 per cent electronic efficiency could be realized with approximately zero velocity

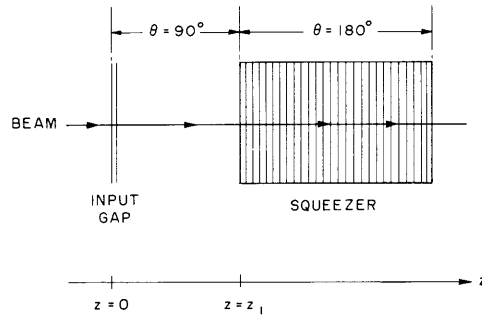


Fig. VIII-6. Squeezer circuit.

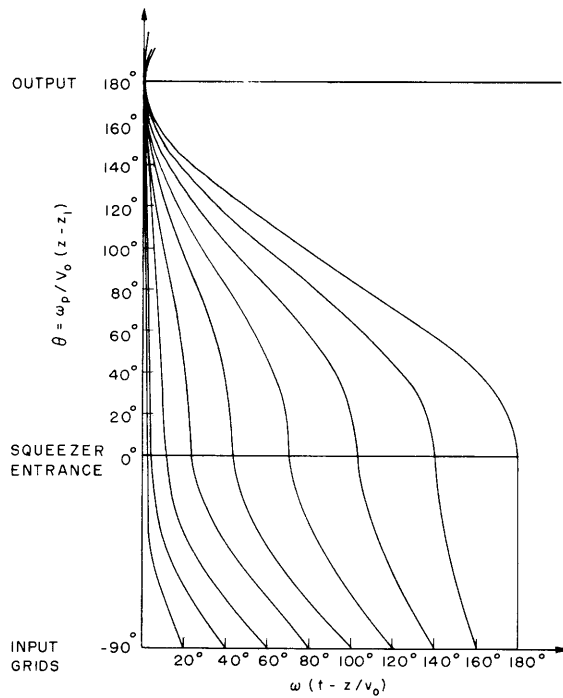


Fig. VIII-7. Electron paths in the squeezer.

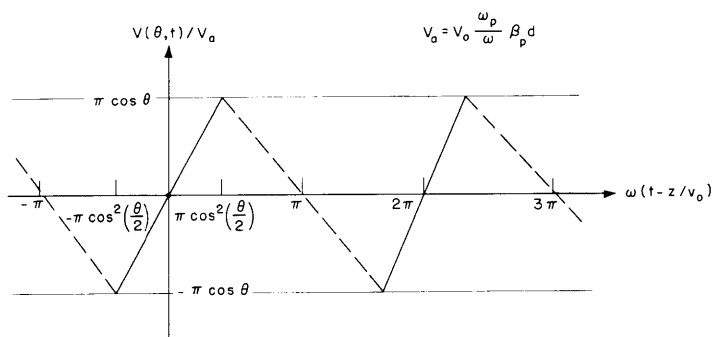


Fig. VIII-8. Voltage waveform in the squeezer.

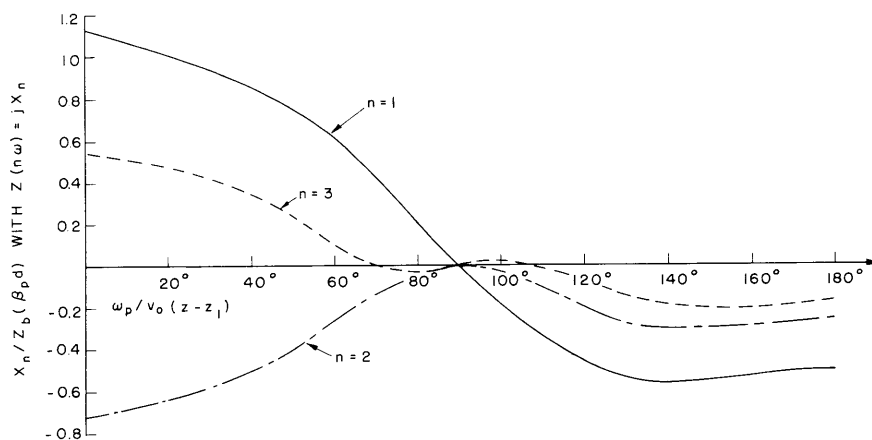


Fig. VIII-9. Squeezer impedance versus distance for the first three harmonics.

spread at the position of maximum bunching. The maximum bunch occurs 180° of plasma angle from the squeezer input. The electron paths are shown in Fig. VIII-7 for  $X_0 = 1$ .

The voltage waveform at any position in the squeezer is shown in Fig. VIII-8. The dotted portion is actually indeterminate because during this portion of the rf cycle, no electrons pass the point. The voltage waveform was expanded in a Fourier series by assuming the dotted connection as shown in the figure, and the beam current was also expanded in a Fourier series. This procedure gives the impedance as a function of distance  $[\theta = \omega_p(z-z_1)/v_0]$ :

$$Z_n(\theta) = j \frac{Z_b(\beta_p d) \cos \theta \sin [n\pi\alpha_n]}{2\pi n^2 \sin^2 \left(\frac{\theta}{2}\right) \cos^2 \left(\frac{\theta}{2}\right) J_{2n}(\alpha_n X_0)}$$

where  $\alpha_n = n \cos^2(\theta/2)$ , and  $\beta_p d = \omega_p d/v_0$  is the normalized length of any one of the

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gaps. In Fig. VIII-9, this impedance is plotted against distance for the first three harmonics (for  $X_0 = 1$ ).

Some investigations are in progress to determine the maximum efficiency obtainable from a squeezer structure that interacts only with the fundamental component of the beam current. All components of the beam current that enter the squeezer will be retained, however.

R. J. Briggs

### References

1. H. A. Haus, Propagation of Noise and Signals along Electron Beams at Microwave Frequencies, Sc.D. Thesis, Department of Electrical Engineering, M.I.T., May 21, 1954.
2. A. Bers, Interaction of Electrons with Electromagnetic Fields of Gaps with Application to Multicavity Klystrons, Sc.D. Thesis, Department of Electrical Engineering, M.I.T., June 1959.