XV. PHYSICAL ACOUSTICS[*]

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A. TEMPERATURE DEPENDENCE OF SOUND ATTENUATION IN ALUMINUM

The object of this experiment was to measure the attenuation of sound waves in commercial aluminum as a function of temperature in the range between the temperature of boiling nitrogen (78°K) and room temperature (300°K).

Several attenuation mechanisms have been known for some time; for instance, thermal loss mechanisms, which occur when a solid is compressed. The compressed parts become warmer than the extended parts, and the heat flow that occurs between these parts abstracts energy from the wave. Another example is the scattering of sound in polycrystalline materials in which the random grain orientation causes an impedance mismatch between adjacent grains and produces a scattering of sound and a loss of energy. The tendency of atoms to interchange position under thermal agitation can also cause an ultrasonic attenuation.

In recent years, attenuation effects for which there is no theoretical explanation have been observed. The presence of a type of imperfection in metals, known as a dislocation, has provided a partial answer. Dislocation phenomena have been classified into two types. The first, an edge dislocation, is a region in which a plane of atoms has slipped into or out of an otherwise perfect crystal. The other type of dislocation, known as a screw dislocation, is the boundary between a slipped and an unslipped region of the lattice.

Measurements at low temperatures indicate the presence of attenuation maxima whose position on the temperature scale is a function of the frequency of measurement. It has been suggested that these peaks may be caused by a relaxation mechanism whereby dislocations are displaced by thermal agitation between their pinning points into adjacent minimum energy wells to form pairs of kinks. These kinks can be formed by stresses less than the Peierls stress. If the angular frequency of the applied stress is greater, or less, than the frequency of thermal formation of kinks, very little energy is lost during each stress cycle (attenuation is small). A maximum attenuation occurs when the angular frequency of the applied stress is close to the frequency of kink formation.

The apparatus used in measuring the attenuation of acoustical waves in metal bars
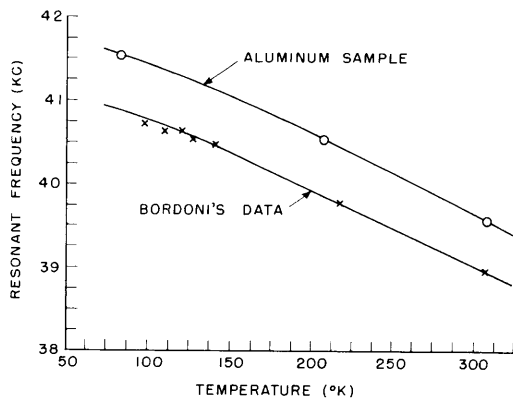
Fig. XV-1.   Resonant frequency of aluminum bar as a function of temperature.
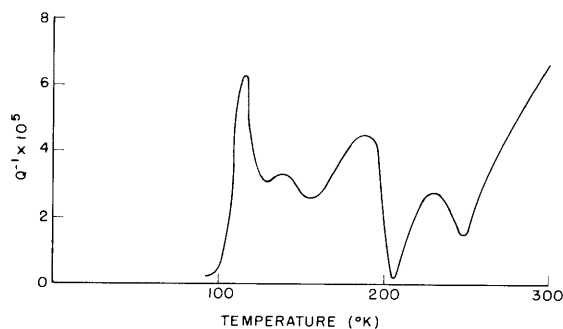

Fig. XV-2.   Inverse Q of aluminum bar as a function of temperature.

is patterned after that of Bordoni (1).   This apparatus is described in more detail by Schaefer (2).

An aluminum bar, 2.56 inches long and 0.5 inch in diameter, was mounted in a hollow brass container by means of small pin points placed at the center (nodal plane) of the aluminum bar.   The container was placed in a dewar flask containing liquid nitrogen and was allowed to warm up slowly until it reached room temperature.   The exact temperature of the bar was obtained by measuring the resonant frequency of the bar at certain fixed temperatures and then comparing these points with the curve that Bordoni obtained.   The sample was electrostatically driven by an oscillator, and as the temperature of the bar slowly changed, the decay of the attenuated signal was recorded.   This was done for many resonant frequencies as the temperature of the bar rose slowly from 78°K to 300°K.

Figure XV-1 shows a comparison of our temperature calibration curve with that of Bordoni.   Figure XV-2 shows a plot of inverse Q as a function of temperature. Inverse Q, which is the internal friction, is a direct measure of the attenuation of

acoustical waves in solids.

The first attenuation maximum, shown in Fig. XV-2 at 117°K, is in agreement with the attenuation maximum measured by Bordoni for his sample at approximately 100°K. However, there are two significant differences between Bordoni's plot and this one. First, Bordoni's attenuation peak at 100°K is much broader than that of the peak represented here at 117°K. Second, this plot shows several relatively smaller attenuation maxima that are absent in Bordoni's plot.

The first attenuation maximum is thought to be caused by dislocations in the solid. Since very little is known of the imperfections in the internal composition of solids, no plausible explanation can be submitted for the appearance of these peaks other than that used to explain Bordoni's attenuation peak at 100°K.

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B. DIFFRACTION OF LIGHT BY HYPERSONIC COMPRESSION WAVES

The possibility of using diffraction techniques to detect traveling compression waves with wavelengths of the order of 10^{-4} cm was studied. A general expression for the intensities of the diffraction orders was developed, and the solutions to the resulting difference-differential equation were investigated in the limits of very long and very short wavelengths.

It was found that, at low frequencies, the diffraction pattern resembles that produced by a diffraction grating with a grating width equal to one wavelength of sound. As the frequency of the traveling wave increases, the diffraction pattern becomes increasingly asymmetrical and has a marked dependence on the angle between the wave front of the traveling wave and the incident light beam.

The detection of traveling compression waves by the diffraction method becomes increasingly difficult to observe as the wavelength of the hypersound decreases. The characteristic angles become extremely critical, and only one diffraction order can be observed. Furthermore, the traveling wave behaves like a very sharp monochromator. Only one wavelength is diffracted out of an incident spectrum, and it is determined by the angle between the traveling wave and the light beam.

1. Description of the Apparatus

The hypersonic waves to be detected are assumed to be traveling in a region that is roughly perpendicular to the incident light. They can be produced by the action of a microwave electric field on a quartz crystal, or by an ultrahigh-frequency crystal in a tank of water. In either case, compression waves in the medium cause a corresponding change in the index of refraction. It is this periodically varying index of refraction that causes the diffraction pattern.

2. General Theory

Maxwell's equations governing the propogation of light in a medium of varying index of refraction are:

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \tag{1}$$

$$\nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \tag{2}$$

$$\nabla \cdot \vec{D} = 0 \tag{3}$$

$$\nabla \cdot \vec{H} = 0 \tag{4}$$

where

$$\vec{D} = k(x, z, t) \vec{E} \tag{5}$$

and

$$k(x, z, t) = \mu^2(x, z, t) \tag{6}$$

If $\vec{H}$ is eliminated from Eqs. 1 and 2,

$$-\frac{1}{c^2}\frac{\partial^2 \vec{D}}{\partial t^2} = \nabla \times \nabla \times \vec{E} \tag{7}$$

By a vector identity

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \tag{8}$$

and

$$\nabla \cdot \vec{D} = \nabla(k\vec{E}) = k(\nabla \cdot \vec{E}) + \vec{E} \cdot \nabla k \tag{9}$$

Therefore, from Eqs. 3 and 9, we obtain

$$\nabla \cdot \vec{E} = -\vec{E} \cdot \frac{\nabla k}{k} \tag{10}$$

and the equation governing the propagation of an electromagnetic disturbance becomes:

$$\frac{1}{c^2} \frac{\partial^2 (k\vec{E})}{\partial t^2} = \nabla^2 \vec{E} + \nabla\left(\frac{\vec{E} \cdot \nabla k}{k}\right) \tag{11}$$

We shall consider the effects of a compression wave traveling in the medium in the xz-plane at an angle ϕ with the x-axis. The light beam travels along the z-axis. An asterisk is used here to denote quantities that are related to the traveling compression wave.

In order to simplify the wave equation, there are several approximations that can be made. The traveling wave certainly has a frequency much smaller than that of light; that is, $\nu^* \ll \nu$. Therefore,

$$\frac{\partial k}{\partial t}, \frac{\partial^2 k}{\partial t^2} \ll \frac{\partial^2 E}{\partial t^2} \tag{12}$$

and these terms can be neglected. If this approximation is made, Eq. 11 becomes

$$\frac{k(x, z, t)}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \nabla^2 \vec{E} + \nabla\left(\frac{\vec{E} \cdot \nabla k}{k}\right) \tag{13}$$

If the change in the index of the refraction is small in a wavelength of light, that is, if $\lambda \ll \lambda^*$, then

$$\frac{\nabla k}{k} \ll \lambda \tag{14}$$

and can be neglected. Physically, this means that reflection terms can be neglected.

After these approximations have been made, the wave equation for a plane wave advancing in the z direction becomes

$$\frac{k(x, z, t)}{c^2} \frac{\partial^2 E}{\partial t^2} = \nabla^2 E \tag{15}$$

or

$$\frac{\mu^2(x, z, t)}{c^2} \frac{\partial^2 E}{\partial t^2} = \nabla^2 E \tag{16}$$

Since any disturbance caused by the traveling compression wave will appear as a modulation on the incident beam, let

$$E = e^{2\pi i \nu t} \chi(x, z, t) \tag{17}$$

where χ varies in the same way as the traveling wave. Because the sound frequency is much less than the frequency of light,

$$\frac{\partial \chi}{\partial t}, \quad \frac{\partial^2 \chi}{\partial t^2} \ll \nu \tag{18}$$

Therefore the wave equation to be satisfied by χ becomes:

$$-\frac{4\pi^2}{c^2} \mu^2(x, z, t) \chi(x, z, t) = \frac{\partial^2 \chi}{\partial x^2} + \frac{\partial^2 \chi}{\partial z^2} \tag{19}$$

A plane compression wave traveling at an angle ϕ with the x-axis (Fig. XV-3) can be represented in the form

$$\mu(x, z, t) = \mu_0 - \mu \sin \frac{2\pi x}{\lambda^*} \tag{20}$$

where μ is the maximum change in the index of refraction caused by the traveling wave. Because the sound wave is periodic, the disturbance it causes also must be
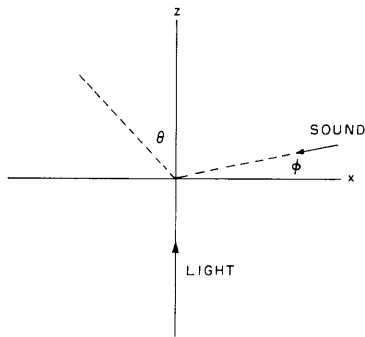

Fig. XV-3. Diffraction of light by sound.

periodic. By expanding χ into a double Fourier series in x and t, an expression for χ is found in the form

$$E = \exp(2\pi i \nu t) \chi = \exp\left[-2\pi i \left(\frac{\mu_0}{\lambda} z - \nu t\right)\right] \sum_{n=-\infty}^{\infty} \Phi_n(\xi) \exp\left[2\pi i n\left(\frac{x \cos \phi + z \sin \phi}{\lambda^*}\right) - \nu^* t\right] \tag{21}$$

The differential equation satisfied by these Fourier coefficients is found by inserting the expansion of χ into the wave equation. After dropping terms of the order of μ^2, then Eq. 19 becomes:

$$2 \frac{d\Phi_n(\xi)}{d\xi} - i(n^2 \rho - 2na \sin \phi) \Phi_n = \Phi_{n-1} - \Phi_{n+1} \tag{22}$$

where

206

$$\xi = \frac{2\pi\mu z}{\lambda}; \quad \rho = \frac{\lambda^2 \cos^2 \phi}{\mu_0 \mu \lambda^{*2}}; \quad a = \frac{\lambda}{\mu \lambda^*}$$

Equation 22 was first derived by Nath [1] and is the basis for most calculations at low frequencies. It has been well validated experimentally for long wavelengths.

The optical disturbance (Eq. 21) also can be written

$$E = \sum_{-\infty}^{\infty} \Phi_n(\xi) \exp - 2\pi i \left[\left(\frac{\mu_0}{\lambda} - \frac{n \sin \phi}{\lambda^*} \right) z - \frac{n \cos \phi}{\lambda^*} x - (\nu - n\nu^*) t \right] \qquad (23)$$

This expansion suggests a series of plane waves of amplitude $\Phi_n(\xi)$ propagating at an angle θ from the z-axis (Fig. XV-3), which is given by

$$\tan \theta = \frac{n \cos \phi}{\dfrac{\mu \lambda^*}{\lambda} - n \sin \phi} \qquad (24)$$

For small values of ϕ, which indicate near-normal incidence,

$$\tan \theta = \frac{n\lambda}{\lambda^*}$$

These are the well-known diffraction angles. It is apparent that each term in Eq. 23 represents a diffraction order with amplitude $\Phi_n(\xi)$ and frequency $\nu - n\nu^*$.

3. Solutions to Nath's Equation

Nath's equation for the amplitude of the nth diffraction order (Eq. 22) has the boundary conditions

$$\Phi_0(0) = 1, \quad \Phi_n(0) = 0$$

and the condition imposed by the nature of diffraction patterns

$$\sum_{-\infty}^{\infty} \Phi_n^2(\xi) = 1$$

In order to investigate solutions to Eq. 22, the magnitude of the constants involved should first be estimated:

$$n^2 \rho - 2na \sin \phi = 2na \left(\frac{n\rho}{2a} - \sin \phi \right)$$

$$= \frac{2n\lambda}{\mu \lambda^*} \left(\frac{n\lambda \cos^2 \phi}{2\mu_0 \lambda^*} - \sin \phi \right) \qquad (25)$$

207

Since $\lambda/\lambda^* < 1$, the solutions, for the most part, will depend on the parameter

$$a = \frac{\lambda}{\mu\lambda^*}$$

Because μ is the change in index of refraction caused by the traveling wave, its value could be expected to be very small, that is, approximately 10^{-6}. For short hypersonic wavelengths, $a = \lambda/\mu\lambda^* \gg 1$. For very long wavelengths, $a \ll 1$, and in this case $\underline{a}$ can be neglected altogether. Then Nath's equation becomes

$$2\frac{d\Phi_n}{d\xi} = \Phi_{n-1} - \Phi_{n+1} \tag{26}$$

which is an identity of the Bessel functions. That is, for $a = 0$

$$\Phi_n(\xi) = J_n(\xi) \tag{27}$$

and the intensity of the nth diffraction order is given by

$$I_n = J_n^2(\xi)$$

and, by a known theorem,

$$\sum_{-\infty}^{\infty} J_n^2(\xi) = 1$$

This result is precisely that derived by Raman and Nath in their first series of papers on the subject (2, 3, 4). This expression for the diffraction intensities at long wavelengths of sound has been justified experimentally (5).

However, for the hypersonic region, in which the wavelength of sound approaches the order of magnitude of the wavelength of light, $a \gg 1$, and the low-frequency approximation is invalid.

If the Laplace transform of $\Phi_n(\xi)$ is defined as

$$\overline{\Phi}_n(p) = \int_0^\infty e^{-p\xi} \Phi_n(\xi)\, d\xi \tag{28}$$

Nath's difference-differential equation becomes

$$2p\overline{\Phi}_n(p) - 2\delta_{no} - i(n^2\rho - 2an \sin \phi)\,\overline{\Phi}_n = \overline{\Phi}_{n-1} - \overline{\Phi}_{n+1} \tag{29}$$

where

$$\delta_{no} = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

The equations for the $\overline{\Phi}_n$ can be expressed in terms of an infinite matrix equation:

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 2p - ic_{-2} & +1 & 0 & 0 & \cdot \\ \cdot & -1 & 2p - ic_{-1} & +1 & 0 & \cdot \\ \cdot & 0 & -1 & 2p & +1 & \cdot \\ \cdot & 0 & 0 & -1 & 2p - ic_{1} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \bar{\Phi}_{-2} \\ \bar{\Phi}_{-1} \\ \bar{\Phi}_{o} \\ \bar{\Phi}_{1} \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot \\ 0 \\ 0 \\ 2 \\ 0 \\ \cdot \end{bmatrix} \tag{30}$$

where

$$c_n = n^2 \rho - 2an \sin \phi$$

When diffraction effects are slight, it should be possible to neglect all but the first diffraction orders. When $a \gg 1$, it is also possible to neglect the negative order, as will be justified later. In that case, Eq. 30 becomes:

$$2p\bar{\Phi}_o + \bar{\Phi}_1 = 2 \tag{31}$$

$$-\bar{\Phi}_o + (2p-ic_1) \, \bar{\Phi}_1 = 0 \tag{32}$$

Equations 31 and 32 can be solved to give

$$\bar{\Phi}_1 = \frac{2}{4p^2 - 2ic_1 p + 1} = \frac{1}{2} \frac{1}{(p-p_1)(p-p_2)} \tag{33}$$

where

$$p_1 = \frac{ic_1}{4} + i\left[\left(\frac{c_1}{4}\right)^2 + 1\right]^{1/2}$$

$$p_2 = \frac{ic_1}{4} - i\left[\left(\frac{c_1}{4}\right)^2 + 1\right]^{1/2}$$

If we take the inverse Laplace transform of Eq. 33, we obtain

$$\Phi_1(\xi) = \frac{1}{2} \frac{1}{p_1 - p_2} \left(e^{p_1 \xi} - e^{p_2 \xi}\right) \tag{34}$$

In terms of c_1, we have

$$\Phi_1(\xi) = \frac{e^{ic_1/4}}{2\left[\left(\frac{c_1}{4}\right)^2 + 1\right]^{1/2}} \sin\left[\left(\frac{c_1}{4}\right)^2 + 1\right]^{1/2} \xi \tag{35}$$

209

The intensity of the first diffraction order is given by

$$I_1 = |\Phi_1|^2 = \frac{\sin^2\left[\left(\frac{c_1}{4}\right)^2 + 1\right]^{1/2} \xi}{4\left[\left(\frac{c_1}{4}\right)^2 + 1\right]} \tag{36}$$

If $c_1 \gg 1$ (that is, $a \gg 1$), the intensity practically vanishes for

$$\xi = \pm \frac{4\pi}{c_1}$$

and

$$\sin \phi - \frac{\lambda \cos^2 \phi}{2\mu_0 \lambda^*} = \pm \frac{\lambda^*}{z}$$

Thus the diffraction line vanishes almost completely unless the oblique angle ϕ is such that

$$\sin \phi = \frac{\lambda}{2\mu_0 \lambda^*} \cos^2 \phi$$

or, for small ϕ

$$\sin \phi = \frac{\lambda}{2\mu_0 \lambda^*} \tag{37}$$

These oblique angles are recognized as the characteristic Bragg angles of reflection, although reflection was eliminated from consideration at the start. If the oblique angle varies by as much as λ^*/z from this characteristic angle the diffraction line vanishes.

Because the Bragg angle for the negative diffraction order is merely the negative of that for the first order, it cannot have a maximum at the same time as the first order. Therefore the initial assumption is justified.

If the light is maintained at the first Bragg angle, the intensity of the first diffraction order is given by

$$I_1 = \frac{\sin^2 \xi}{4}$$

Thus the intensity of the diffraction line reaches a maximum periodically with ξ.

Using the results obtained for Φ_1 and the original matrix equation (Eq. 30), we can make immediate predictions as to the intensities of the higher diffraction orders.

The general equation for $\bar{\Phi}_n$ is

$$(2p-ic_n) \, \bar{\Phi}_n = \bar{\Phi}_{n-1} - \bar{\Phi}_{n+1}$$

The value of $\bar{\Phi}_n$ depends on multiplying the difference of its neighbors by a fraction $\leq 1/c_n$. If the sound wave front is not tilted at the correct Bragg angle, that is, the angle that makes $c_n = 0$, then its intensity will be negligible. Thus all the neighboring diffraction orders vanish except the zeroth, so that the first diffraction orders are the only ones that appear.

The first-order line will vanish for oblique angles that vary from the Bragg reflection angle by as little as λ^*/z.

This prediction has been clearly verified by Bhagavantum and Rao (6). They observed wavelengths of the order of 8.5×10^{-4} cm in water and found that it was possible to see only the first diffraction order on the side favored by the Bragg reflection angle.

Because the Bragg angle condition is satisfied for only one wavelength out of an incident spectrum, the traveling wave acts as a sharp monochromator.

4. Conclusion

Our object was the investigation of an optical method of detecting hypersonic compression waves. It was found that traveling waves do produce a diffraction effect, and that the diffraction orders are found at the characteristic angles

$$\tan \theta = \frac{n\lambda}{\lambda^*}$$

The parameter that distinguishes the character of the solutions is $a = \lambda/\mu\lambda^*$, where λ and λ^* are the wavelengths of light and the compression wave, respectively, and μ is the maximum change in the index of refraction caused by the sound wave.

When $a \ll 1$, the diffraction orders are symmetrical, and their intensities are proportional to $J_n^2(\xi)$, where $\xi = (2\pi\mu z)/\lambda$. When $a \gg 1$, the diffraction effects become asymmetrical, and their intensities depend on the oblique angle between the sound and light beams. In the limit, only one diffraction order is visible, and that only when the oblique angle is equal to the first Bragg angle. If the oblique angle varies by as little as λ^*/z, the effect vanishes altogether.

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