XVII. PHYSICAL ACOUSTICS*

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RESEARCH OBJECTIVES

In general terms, our research objectives concern the physical problems involved in the generation, propagation, and absorption of sound and vibrations in matter. Specifically, our program of study includes:

1. Acoustics of moving media.
2. Acoustical studies of molecular structure of liquids.
3. Generation of very high frequency sound waves ("hypersonics").
4. Nonlinear acoustics.

U. Ingard

A. SCATTERING OF SOUND BY SOUND

The ordinary equations describing the propagation of sound in a fluid are obtained by linearization of the hydrodynamic equations of the fluid. That is, the sound fields are assumed to be everywhere small, and terms of quadratic or higher order in the sound variables are neglected. To take account of the neglected terms, the sound equations can be thought of as merely the first step in the solution by successive approximations of the exact hydrodynamic equations. The second step is to assume a solution that is the sum of the small sound fields and a still smaller correction, to linearize the equations again, and to solve for the correction. If the primary sound field is sinusoidal, characterized by a single frequency, the correction, or secondary field, contains a time-independent term and a sinusoidal term of twice the frequency. Physically, these terms are simply additional sound waves which may be thought of as being generated by an interaction of the primary sound wave with itself. If the primary field consists of two sinusoidal waves of different frequencies, the secondary field contains terms that oscillate with the sum and difference of these two frequencies. Physically, these "sum and difference" terms are also sound waves, although, as will be shown later, they do not always travel with the usual speed of sound. These "sum and difference" terms may be thought of as arising from an interaction of one sound wave with the other (or the scattering of one sound wave by the other). In the following discussion, these "sum and difference" terms will be extracted from the complete second-order solution, and the terms representing the interaction of the sound waves with themselves will be discarded.

The equations for the sound pressure, p, and the first correction, p_s, are

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$$\left(\nabla^2 - \frac{1}{c_o^2}\frac{\partial^2}{\partial t^2}\right) p = 0 \tag{1}$$

$$\left(\nabla^2 - \frac{1}{c_o^2}\frac{\partial^2}{\partial t^2}\right) p_s = -\rho_o \operatorname{div}\left[\underline{v} \operatorname{div} \underline{v} + \frac{1}{2} \operatorname{grad} v^2\right] + \frac{\Lambda}{\rho_o c_o^4}\frac{\partial^2 p^2}{\partial t^2} \tag{2}$$

where p is pressure, ρ is density, $\underline{v}$ is velocity, and

$$\Lambda = \frac{\rho_o c_o^4}{2} \left.\frac{d^2\rho}{dp^2}\right]_{p=p_o}$$

Equation 2 can be simplified by letting

$$p_s = p_s' + p_s'' \tag{3}$$

where

$$p_s'' = -\frac{\rho_o v^2}{2} - \frac{p^2}{2\rho_o c_o^2} - \frac{1}{\rho_o c_o^2}\frac{\partial p}{\partial t}\int p\,dt \tag{4}$$

and then p_s' satisfies the equation

$$\left(\nabla^2 - \frac{1}{c_o^2}\frac{\partial^2}{\partial t^2}\right) p_s' = -\frac{1}{\rho_o c_o^4}(1-\Lambda)\frac{\partial^2 p^2}{\partial t^2} \tag{5}$$

The part of the solution to Eq. 2 that is represented by p_s'' can be immediately written, once the primary sound fields are known. Then Eq. 5 must be solved for the remainder (usually the most interesting part) of the solution.

To give some idea of how p_s' behaves for different geometries, the solutions of Eq. 5 for three of the more simple cases are given below. In each case,

$$p_s' = p_+' + p_-'; \quad \omega_\pm = \omega_a \pm \omega_b; \quad k_\pm = \omega_\pm/c_o \tag{6}$$

where p_+' and p_-' represent the "sum and difference" components of the complete solution which arise from scattering of one sound wave by the other.

For two plane waves propagating in the same direction, we have

$$p = \rho_o c_o\left[v_a \cos(k_a x - \omega_a t) + v_b \cos(k_b x - \omega_b t)\right] \tag{7}$$

$$p_\pm' = \rho_o v_a v_b (1-\Lambda)(k_\pm/2)\, x \sin(k_\pm x - \omega_\pm t) \tag{8}$$

For the simplest corresponding cylindrical case, we have

$$p = \rho_o c_o \left[v_a \left(\frac{a}{r}\right)^{1/2} \cos(k_a r - k_a a - \omega_a t) + v_b \left(\frac{a}{r}\right)^{1/2} \cos(k_b r - k_b a - \omega_b t) \right] \tag{9}$$

$$p'_{\pm} = -\rho_o v_a v_b (1-\Lambda)(k_{\pm} a) \sin(k_{\pm} r - k_{\pm} a - \omega_{\pm} t) \tag{10}$$

For the simplest corresponding spherical case, we have

$$p = \rho_o c_o \left[v_a \frac{a}{r} \cos(k_a r - k_a a - \omega_a t) + v_b \frac{a}{r} \cos(k_b r - k_b a - \omega_b t) \right] \tag{11}$$

$$p'_{\pm} = \rho_o v_a v_b (1-\Lambda)\, k_{\pm} a^2 (2r)^{-1} \ln (r/a) \sin(k_{\pm} r - k_{\pm} a - \omega_{\pm} t) \tag{12}$$

In the cylindrical and spherical cases, the asymptotic forms of the solutions are given. Solutions for the more general cases are more difficult to obtain. An exception is the case of two plane waves traveling in different directions, which will be discussed further.

The solution for two plane waves traveling in the same direction has been discussed by several authors (1, 2, 3), and there is good experimental evidence to support it (3). This author has carried out experiments that seem to verify Eq. 10, the case for two concentric cylindrical waves (4).

Consider, now, the case of two plane waves traveling at right angles. If the x and y axes are chosen to lie in the directions of propagation, the sound pressure and velocity can be written

$$\begin{aligned} v_x &= v_a \cos(k_a x - \omega_a t) \\ v_y &= v_b \cos(k_b y - \omega_b t) \end{aligned} \tag{13}$$

$$p = \rho_o c_o [v_a \cos(k_a x - \omega_a t) + v_b \cos(k_b y - \omega_b t)] \tag{14}$$

It can be shown that the waves arising from the interaction of the two primary waves are themselves sound waves in all respects, except for the fact that they do not propagate with the normal velocity of sound, c_o (4). The velocities with which they do propagate are given by

$$c_{\pm} = c_o \left[1 \pm \frac{2\omega_a \omega_b}{\omega_a^2 + \omega_b^2} \right]^{1/2} \tag{15}$$

Finally, consider two beams of plane waves having equal square cross sections and intersecting in a cubical volume. Within this volume the waves will interact, and secondary waves characterized by the sum and difference frequencies of the primary beams will be generated, as we have described. If we assume that upon reaching the boundary of the interaction region these "sum and difference" waves are equivalent to normal sound waves in their ability to be transmitted and reflected, scattered waves will be seen outside the interaction region. The sum and difference frequency waves that are generated inside the interaction region propagate with a speed different from that of sound by virtue of the forcing term on the right-hand side of Eq. 2. Any wave arising from transmission or reflection at the boundary of the interaction region is no longer subject to this forcing term and, therefore, travels with the normal speed of sound. Under these assumptions, we can immediately calculate the angles at which scattered waves leave the interaction region by matching surfaces of constant phase at the boundary.

For the scattered wave whose frequency is the sum of the frequencies of the primary beams there are four such angles, where θ is measured from the direction of propagation of the beam of frequency toward the beam of lower frequency. Thus

$$\theta_+ = \sin^{-1}\left(\frac{\omega_b}{\omega_+}\right); \quad \pi - \sin^{-1}\left(\frac{\omega_b}{\omega_+}\right); \quad \pm \cos^{-1}\left(\frac{\omega_a}{\omega_+}\right) \tag{16}$$

For the scattered wave whose frequency is the difference of the frequencies of the primary beams there are no such angles, unless one primary beam frequency is at least twice the other. If this is the case, there are two such angles.

$$\theta_- = -\sin^{-1}\left(\frac{\omega_b}{\omega_-}\right); \quad \pi + \sin^{-1}\left(\frac{\omega_b}{\omega_-}\right) \qquad \text{if } \omega_a \geq 2\omega_b \tag{17}$$

The absence of some of the scattered wave components in this case results from the fact that the difference frequency wave generated in the interaction region travels with less than the normal speed of sound, and there is the possibility of "total reflection." However, since the reflected and transmitted waves travel with the same speed, the normal speed of sound, the angle of reflection equals the angle of transmission. Therefore, at certain angles of incidence, normally those for total reflection, both the transmitted and reflected waves decay exponentially with distance from the boundary.

The number of scattered waves of various frequencies and the angles at which they should be found, which have resulted from this research, are in agreement with those predicted from a different theoretical approach by Ingard and Pridmore-Brown (5). The existence of such scattered waves outside the interaction region has still not been firmly established (4, 5, 6).

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