

**Can there be an elegant spin-orbital decomposition of the nucleon magnetic moment?**Xiang-Song Chen,<sup>1,\*</sup> Wei-Min Sun,<sup>2</sup> Di Qing,<sup>3</sup> and Hong-Shi Zong<sup>2,4</sup><sup>1</sup>*Department of Physics, Sichuan University, Chengdu 610064, China*<sup>2</sup>*Department of Physics, Nanjing University, Nanjing 210093, China*<sup>3</sup>*CERN, CH-1211 Geneva 23, Switzerland*<sup>4</sup>*CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China*

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Recently, Mekhfi [Phys. Rev. D **72**, 114014 (2005)] remarked that when studying spin-orbital separation of the nucleon magnetic moment with the Gordon decomposition, one should keep a time-derivative term because the quark fields depend on time. We clarify that this term vanishes identically in a rigorous formulation of the nucleon magnetic moment, which then can be elegantly separated into a spin part related to quark tensor charge, and an orbital part related to quark convection angular momentum. In a quark model description of the nucleon, however, such a time-derivative term might contribute because it is hard to construct a true Hamiltonian eigenstate of relativistic interacting quarks.

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In Ref [1], we used the Gordon decomposition to derive an elegant relation between magnetic moment and angular momentum for a relativistic system. This relation unambiguously separates the nucleon intrinsic magnetic moment into a spin part related to the quark tensor charge, and an orbital part related to the quark “convection” angular momentum. Recently, Mekhfi [2] remarked that our derivation erroneously omitted a time-derivative term. We supplement here why this term vanishes identically when one rigorously studies the intrinsic magnetic moment of a particle, either fundamental or composite. We also call attention that in a phenomenological model description of a composite particle this term might nevertheless be non-zero, because the model wave function may not be a true Hamiltonian eigenstate. In such a case one must be cautious at which magnetic moment formula to use.

Gordon decomposition separates the Dirac vector current  $j^\mu = \bar{\psi}\gamma^\mu\psi$  into a convection part and a spin part:

$$\bar{\psi}\gamma^\mu\psi = \frac{i}{2m}\bar{\psi}\overleftrightarrow{D}^\mu\psi + \frac{1}{2m}\partial_\nu(\bar{\psi}\sigma^{\mu\nu}\psi) \equiv j_C^\mu + j_S^\mu, \quad (1)$$

where  $m$  is the mass of the Dirac field,  $\overleftrightarrow{D}^\mu = D^\mu - \overleftarrow{D}^\mu$  is the covariant derivative. The Gordon decomposition follows directly from the equation of motion. In the case of free field,  $D^\mu$  is replaced with  $\partial^\mu$ .

In [1], we remarked that the time-derivative term in Eq. (1) does not contribute to the nucleon magnetic moment. However, Mekhfi argues in [2] that this time-derivative term cannot be thrown away because the quark fields depend on time in a nucleon. To clarify this issue, one must keep in mind that the *intrinsic* magnetic moment of a particle is defined in its Hamiltonian eigenstate with momentum close to zero [3]. In such states, a time-

derivative term can be discarded identically: For any Heisenberg operator  $\mathcal{O}$ , we have the Heisenberg equation of motion  $\partial_t\mathcal{O} = i[H, \mathcal{O}]$ , where  $H$  is the *total* Hamiltonian of the system. When taking expectation value in an eigenstate of  $H$ ,  $\partial_t\mathcal{O}$  vanishes for any operator  $\mathcal{O}$ .

The above nonperturbative conclusion can be verified perturbatively if one knows how to construct a Hamiltonian eigenstate of the particle. For example, one can easily check by straightforward calculation that at 1-loop order the time-derivative term in Eq. (1) does not contribute to the anomalous magnetic moment of the electron (despite that the electron field is nontrivially time-dependent when interacting with the photon field).

After dropping the time-derivative term in Eq. (1), we can elegantly decompose the magnetic moment operator  $\vec{\mu} = \frac{1}{2}\int d^3x\vec{r}\times\vec{j}$  into an orbital part and a spin part [1]:

$$\begin{aligned} \vec{\mu} &= \frac{1}{2m}\int d^3x\vec{r}\times\bar{\psi}\frac{1}{2i}\overleftrightarrow{D}\psi + \frac{1}{2m}\int d^3x\bar{\psi}\vec{\Sigma}\psi \\ &\equiv \vec{\mu}_L + \vec{\mu}_S. \end{aligned} \quad (2)$$

The advantage of this expression is that the spin part is related to the quark tensor charge, which can be accessed experimentally and calculated reliably with lattice QCD [1]. However, one should not promptly employ this expression in a phenomenological quark model calculation. The dilemma is that it is hard to construct a true eigenstate of the total Hamiltonian of a relativistic interacting system, hence a time-derivative term  $\partial_t\mathcal{O} = i[H, \mathcal{O}]$  might be non-zero in this model and Eq. (2) might be invalid.

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\*Electronic address: [cxs@scu.edu.cn](mailto:cxs@scu.edu.cn)

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