Prof. J. R. Zacharias	R. S. Badessa	E. L. Kamen
Prof. J. G. King	V. J. Bates	R. L. Kent
Prof. C. L. Searle	H. H. Brown, Jr.	J. C. Nowell
Prof. J. W. Graham	J. R. Cogdell	R. C. Rennick
Dr. R. F. C. Vessot	R. Golub	R. Weiss

RESEARCH OBJECTIVES

The distributions of electric charge and magnetism in an atomic nucleus are usually described in terms of multipole moments limited in number by the magnitude of the nuclear angular momentum. In this laboratory, atomic-beam techniques are used to determine such electric and magnetic moments. In addition, information about the radial distribution of nuclear magnetism can be obtained in cases in which more than one isotope is available. These techniques lend themselves to such precision that they were used in this laboratory for the development of the most accurate atomic clock. In turn, these clocks are being used to make studies on the nature of time itself. Precision apparatus is under construction to observe not only the dependence of atomic time on gravitational potential but also the epochal dependence of nuclear, gravitational, and atomic time. Similar studies are being made on the velocity of light.

J. G. King, J. R. Zacharias

A. DOPPLER-CANCELLATION TECHNIQUE FOR DETERMINING THE ALTITUDE DEPENDENCE OF GRAVITATIONAL RED SHIFT IN AN EARTH SATELLITE

According to the General Theory of Relativity, the time kept by a "proper" clock (or the frequency of a standard oscillator) is affected by the motion of the clock and by its local gravitational potential. The latter effect, known as the "gravitational red shift," is expected to result in variations of only 7 parts in 10^{10} in the rates of clocks in the vicinity of the earth. Attempts to obtain experimental verification have been stimulated by the development of highly stable frequency standards and the availability of earth satellites in orbits that provide a significant variation in gravitational potential.

The principle employed in the short-time measurement is described as follows: A ground transmitter transmits a frequency f. The satellite receives a frequency f + d, where d represents the Doppler shift. This signal is mixed with a frequency $2(f+\Delta)$ derived from a crystal oscillator on the satellite, the term 2Δ representing the change in oscillator frequency because of relativistic effects. The resulting lower-sideband signal, of frequency $f - d + 2\Delta$, is transmitted to the ground, where the received frequency is approximately $(f-d+2\Delta) + d = f + 2\Delta$. This frequency is compared with the original ground oscillator to determine the magnitude of Δ . A more detailed treatment has been given elsewhere (1).

^{*}This research was supported in part by Purchase Order DDL-B222 with Lincoln Laboratory, a center for research operated by M.I.T., which is supported by the U.S. Air Force under Air Force Contract AF19(604)-5200.

In order to obtain the altitude dependence of gravitational red shift, the measurement interval must be a small fraction of the orbital period. However, path-length variations make the performance of a short-time measurement in a one-way propagation experiment virtually impossible. But two-way propagation experiments offer definite possibilities. In transmission of signals to and from a radar target, the first-order Doppler shift is doubled. If it were possible to reverse the sense (i.e., direction) of one of these frequency shifts by a mixing operation, then essentially complete cancellation of firstorder Doppler shift could be effected. In the short-time measurement system that will be described, such a technique permits data for a single altitude to be obtained in an interval of 60 seconds, or less. These significant advantages are achieved:

1. By employing an elliptical orbit, frequency differences at various altitudes can be observed, and hence the variation of frequency with gravitational potential can be obtained.

2. If the measurement is repeated when the satellite is at a particular altitude (e.g., at perigee), a slow frequency drift between satellite and earth clocks can be determined and removed. Since the quantity of interest is the variation in frequency difference as a function of altitude, a fixed offset frequency between clocks is inconsequential. There-fore, quartz-crystal oscillators can probably be employed.

3. Continuous integration or counting of pulses over many orbital periods is not required. Skipped cycles or temporary failures would not invalidate the experimental results.

The frequency standards used in this experiment must meet two separate stability requirements. First, all random and periodic frequency excursions must be at least an order of magnitude smaller than the total effect that is to be measured. Second, any long-term frequency drift in one orbital period should not exceed 20 to 30 per cent of the total desired effect. The reasons for these stability requirements can best be illustrated by plotting a set of hypothetical data that might be obtained in this experiment. Figure IX-1 shows a plot of the hypothetical frequency difference against time. For simplicity, it is assumed that data are acquired alternately at apogee and at perigee for several successive sightings. The clusters of points at times A, B, C, and so on, represent several readings of frequency difference between the two dashed lines for apogee and perigee readings is expected to be approximately 4 parts in 10¹⁰. Therefore the undesired random and periodic frequency variations of the oscillators that are indicated by the dispersion of the clusters must be no greater than a few parts in 10¹¹.

The slope of either of the dashed lines in Fig. IX-1 represents the difference between the long-term drifts of the oscillators, and it is clear that a monotonic drift per orbital period of from 20 to 30 per cent of the total effect can easily be tolerated. For the proposed experiment, a drift of 1 part in 10^{10} in a few hours, or approximately 1 part

109

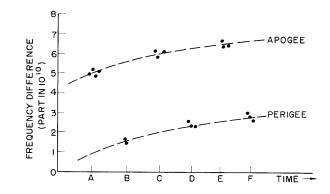


Fig. IX-1. Hypothetical data to illustrate oscillator stability requirements.

in 10^9 per day, is permissible. A small fixed offset frequency between the oscillators would merely shift the base line in Fig. IX-1, and therefore would have no effect upon the desired measurement.

Actual measurements taken on commercially available quartz-crystal oscillators indicate that these oscillators essentially meet the stability requirements. Table IX-1 shows some data obtained in a comparison test. The 1-mc output signals of two crystal oscillators were multiplied up and mixed down in such a way as to magnify their

10-sec counting interval; total elapsed time, approximately 45 minutes	100-sec counting interval; total elapsed time, approx- imately 15 minutes
93101	31005
93100	31006
93100	31006
93100	31005
93100	31006
	31006
	31006
93100	
93100	
93101	
93100	
93101	

Table IX-1. Frequency comparison data for two Hermes 1-mc crystal frequency standards.

frequency difference by a factor of 1000. The resulting beat frequency was then determined by means of a counter operating over periods of 10 and 100 seconds. The last digit of the counter output represents parts in 10^{10} for the 10-sec count, and parts in 10^{11} for the 100-sec count. In either case, the counter has a starting error of 1 count. Thus the differences indicated between the two oscillators may be entirely accounted for by the counter error. The oscillator error is shown to be no greater than 1 part in 10^{11} when it is averaged over successive 100-sec intervals.

The oscillators that were tested employed vacuum-tube circuits; data are not available for a transistor version that is now being developed. However, the stability is believed to be determined primarily by the crystal and its oven, and not by the oscillator circuitry. Therefore no degradation in stability is expected to result from a change to transistors.

R. S. Badessa, V. J. Bates, R. L. Kent, C. L. Searle

References

1. R. S. Badessa, R. L. Kent, and J. C. Nowell, Short-time measurement of time dilation in earth satellite, Phys. Rev. Letters <u>3</u>, 2 (1959).