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Modal expansion of optical far-field quantities using quasinormal modes

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Abstract. We discuss an approach for modal expansion of optical far-field quantities based on quasinormal modes (QNMs). The issue of the exponential divergence of QNMs is circumvented by contour integration of the far-field quantities involving resonance poles with negative and positive imaginary parts. A numerical realization of the approach is demonstrated by convergence studies for a nanophotonic system.

Introduction

For the study of physical phenomena in nano-optical systems, a modal description is the most instructive approach. Modal expansion techniques using QNMs [1–3] have been proposed to analyze light-matter interaction in nanoresonators [4–7]. As the QNMs are the solutions to open systems, they decay in time and are characterized by complex eigenfrequencies. State-of-the-art approaches use the electromagnetic fields of the QNMs in the near-field region of the resonant systems to expand near-field quantities of interest. However, far-field properties of optical systems are important for many applications because typical experiments perform measurements in the far-field region. QNMs diverge exponentially in the far-field region [2, 3], which is a key issue for modal expansion techniques. Approaches using model approximations with real-valued frequencies have been proposed to overcome the divergence problem [8–10].

Here, we discuss an approach for modal expansion of optical far-field quantities [11]. The approach is based on the complex eigenfrequencies of QNMs. The divergence issue in the far-field region is circumvented by introducing contour-integral-based expressions of the far-field quantities involving resonance poles with negative and positive imaginary parts. In this way, one can derive nondiverging expansions of the far-field quantities while the model with complex-valued frequencies of the resonant systems can be retained. We demonstrate the approach by convergence studies for a nanophotonic system.

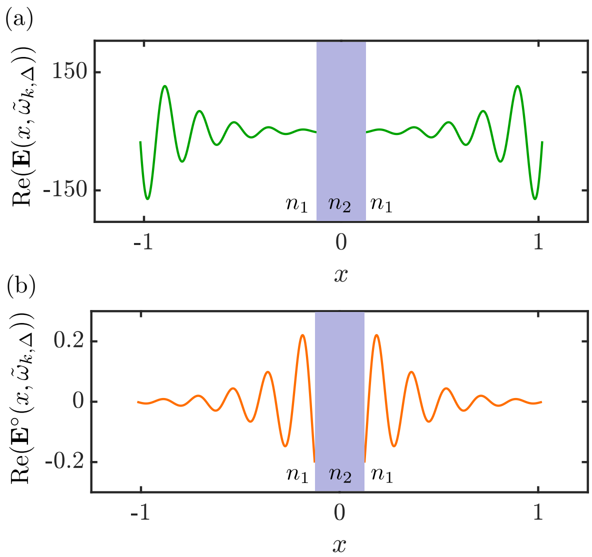

Figure 1. One-dimensional resonator defined by different refractive indices, where $n_2 > n_1$. Solving the Helmholtz equation with a source term corresponding to incoming plane waves yields solutions for the electric field, $\mathbf{E}(x, \omega)$ and $\mathbf{E}^\circ(x, \omega)$. For simplification, only the real parts of the scattered fields (a.u.) outside the resonator are shown. (a) Diverging field $\mathbf{E}(x, \tilde{\omega}_{k,\Delta}) = Ae^{i(n_1\tilde{\omega}_{k,\Delta}/c)|x|}$, where $\tilde{\omega}_{k,\Delta} = \tilde{\omega}_k + \Delta\tilde{\omega}_k$ is a frequency close to $\tilde{\omega}_k$. The frequency $\tilde{\omega}_k$ is a resonance pole of $\mathbf{E}(x, \omega)$. (b) Nondiverging field $\mathbf{E}^\circ(x, \tilde{\omega}_{k,\Delta}) = Be^{-i(n_1\tilde{\omega}_{k,\Delta}/c)|x|}$.

Modal expansion of far-field quantities

In nano-optics, QNMs, $\tilde{\mathbf{E}}(\omega_0) \in \mathbb{C}^3$, are solutions to the time-harmonic Maxwell's equations in second-order form,

$$\nabla \times \mu_0^{-1} \nabla \times \tilde{\mathbf{E}}(\omega_0) - \omega_0^2 \epsilon(\omega_0) \tilde{\mathbf{E}}(\omega_0) = 0, \qquad (1)$$

where $\omega_0 \in \mathbb{R}$ is the angular frequency, μ_0 is the vacuum permeability, and $\epsilon(\omega_0)$ is the permittivity tensor. For simplification of the notation, we omit the spatial dependence of the quantities. The eigenfrequencies $\tilde{\omega}_k \in \mathbb{C}$ corresponding to the QNMs have negative imaginary parts as the QNMs have to satisfy outgoing radiation conditions.

The approach proposed in [11] is demonstrated by decomposing the energy flux density,

$$s(\mathbf{E}(\omega_0), \mathbf{E}^*(\omega_0)) = \frac{1}{2}\mathrm{Re}\left(\mathbf{E}^*(\omega_0) \times \frac{1}{i\omega_0\mu_0} \nabla \times \mathbf{E}(\omega_0)\right) \cdot \mathbf{n},$$

where the field $\mathbf{E}^*(\omega_0)$ is the complex conjugate of the electric field $\mathbf{E}(\omega_0)$ and $\mathbf{n}$ is the normal vector on the associated far-field sphere. The Riesz projection expansion (RPE) is used to expand $s(\mathbf{E}(\omega_0), \mathbf{E}^*(\omega_0))$ into modal contributions [7, 12]. The RPE is based on complex contour integration, which means that $s(\mathbf{E}(\omega_0), \mathbf{E}^*(\omega_0))$ has to be evaluated for complex frequencies. This is not straightforward as $s(\mathbf{E}(\omega_0), \mathbf{E}^*(\omega_0))$ is nonholomorphic. This challenge has been addressed by exploiting the relation $\mathbf{E}^*(\omega_0) = \mathbf{E}(-\omega_0)$ for $\omega_0 \in \mathbb{R}$, which is also a solution to Eq. (1). The field $\mathbf{E}(-\omega_0)$ has an analytical continuation into the complex plane $\omega \in \mathbb{C}$, denoted by $\mathbf{E}^\circ(\omega)$. This field yields the required analytical continuation given by $s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))$. With this, Cauchy's integral formula,

$$s(\mathbf{E}(\omega_0), \mathbf{E}^\circ(\omega_0)) = \frac{1}{2\pi i} \oint_{C_0} \frac{s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))}{\omega - \omega_0} \mathrm{d}\omega,$$

is exploited for the closed integration path C_0 around ω_0, where $s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))$ is holomorphic inside of C_0. Cauchy's residue theorem leads to

$$\begin{aligned} s(\mathbf{E}(\omega_0), \mathbf{E}^\circ(\omega_0)) = &- \sum_{k=1}^{K} \frac{1}{2\pi i} \oint_{\tilde{C}_k} \frac{s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))}{\omega - \omega_0} \mathrm{d}\omega \\ &- \sum_{k=1}^{K} \frac{1}{2\pi i} \oint_{\tilde{C}_k^*} \frac{s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))}{\omega - \omega_0} \mathrm{d}\omega \\ &+ \frac{1}{2\pi i} \oint_{C_\mathrm{r}} \frac{s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))}{\omega - \omega_0} \mathrm{d}\omega, \end{aligned} \quad (2)$$

where $\tilde{C}_1, \ldots, \tilde{C}_K$ are contours around the resonance poles of $\mathbf{E}(\omega)$, given by $\tilde{\omega}_1, \ldots, \tilde{\omega}_K$, and $\tilde{C}_1^*, \ldots, \tilde{C}_K^*$ are contours around the resonance poles of $\mathbf{E}^\circ(\omega)$, given by $\tilde{\omega}_1^*, \ldots, \tilde{\omega}_K^*$. The contour C_r comprises ω_0, the resonance poles $\tilde{\omega}_1, \ldots, \tilde{\omega}_K$ and $\tilde{\omega}_1^*, \ldots, \tilde{\omega}_K^*$, and no further poles. The Riesz projections

$$\begin{aligned} \tilde{s}_k(\mathbf{E}(\omega_0), \mathbf{E}^\circ(\omega_0)) = &- \frac{1}{2\pi i} \oint_{\tilde{C}_k} \frac{s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))}{\omega - \omega_0} \mathrm{d}\omega \\ &- \frac{1}{2\pi i} \oint_{\tilde{C}_k^*} \frac{s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))}{\omega - \omega_0} \mathrm{d}\omega \end{aligned}$$

are modal contributions for the energy flux density. The contribution

$$s_\mathrm{r}(\mathbf{E}(\omega_0), \mathbf{E}^\circ(\omega_0)) = \frac{1}{2\pi i} \oint_{C_\mathrm{r}} \frac{s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))}{\omega - \omega_0} \mathrm{d}\omega$$

is the remainder containing nonresonant components as well as contributions corresponding to eigenfrequencies outside of the contour C_r.

The presented approach is based on computing the quantity $s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))$ by solving Eq. (1) for ω and for $-\omega$. Due to the compensation of the factors $e^{i(n\omega/c)r}$ and $e^{-i(n\omega/c)r}$ of the fields in the far-field region, this yields a nondiverging quadratic form $s(\mathbf{E}(\omega), \mathbf{E}^\circ(\omega))$, where a product of $\mathbf{E}(\omega)$ and $\mathbf{E}^\circ(\omega)$ is involved. In this way, modal expansions of far-field quantities can be computed.

To illustrate this, a one-dimensional resonator with the fields $\mathbf{E}(x, \omega)$ and $\mathbf{E}^\circ(x, \omega)$ fulfilling the corresponding Helmholtz equation is considered. Figure 1(a) sketches the diverging field $\mathbf{E}(x, \tilde{\omega}_{k,\Delta})$, which relates to a QNM of the problem as $\tilde{\omega}_{k,\Delta} = \tilde{\omega}_k + \Delta\tilde{\omega}_k$ is a frequency close to the eigenfrequency $\tilde{\omega}_k$. Figure 1(b) shows the nondiverging field $\mathbf{E}^\circ(x, \tilde{\omega}_{k,\Delta})$ outside of the resonator. Note that the frequency $\tilde{\omega}_{k,\Delta}$ represents a point on an integration contour $\tilde{C}_k$ from Eq. (2). The product $\mathbf{E}(x, \tilde{\omega}_{k,\Delta}) \cdot \mathbf{E}^\circ(x, \tilde{\omega}_{k,\Delta})$ shows a nondiverging behavior and relates to the energy flux density. The approach also applies to arbitrary three-dimensional problems, where, in the far-field region, $\mathbf{E}(\mathbf{r}, \omega) \sim e^{i(n\omega/c)r}(1/r)$ and $\mathbf{E}^\circ(\mathbf{r}, \omega) \sim e^{-i(n\omega/c)r}(1/r)$.

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References

[1] M. Zworski, Notices Amer. Math. Soc. **46**, 319 (1999)
[2] P. Lalanne, W. Yan, K. Vynck, C. Sauvan, J.P. Hugonin, Laser Photonics Rev. **12**, 1700113 (2018)
[3] P.T. Kristensen, K. Hermann, F. Intravaia, K. Busch, arXiv:1910.05412 (2019)
[4] E.A. Muljarov, W. Langbein, R. Zimmermann, EPL **92**, 50010 (2010)
[5] F. Zolla, A. Nicolet, G. Demésy, Opt. Lett. **43**, 5813 (2018)
[6] W. Yan, R. Faggiani, P. Lalanne, Phys. Rev. B **97**, 205422 (2018)
[7] L. Zschiedrich, F. Binkowski, N. Nikolay, O. Benson, G. Kewes, S. Burger, Phys. Rev. A **98**, 043806 (2018)
[8] P.Y. Chen, D.J. Bergman, Y. Sivan, Phys. Rev. Appl. **11**, 044018 (2019)
[9] R.C. Ge, P.T. Kristensen, J.F. Young, S. Hughes, New J. Phys. **16**, 113048 (2014)
[10] J. Ren, S. Franke, A. Knorr, M. Richter, S. Hughes, Phys. Rev. B **101**, 205402 (2020)
[11] F. Binkowski, F. Betz, R. Colom, M. Hammerschmidt, L. Zschiedrich, S. Burger, arXiv:2003.11305 (2020)
[12] F. Binkowski, L. Zschiedrich, M. Hammerschmidt, S. Burger, Phys. Rev. B **100**, 155406 (2019)