X. ARTIFICIAL INTELLIGENCE*

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A. MACHINE MANIPULATION OF ALGEBRAIC EXPRESSIONS

A list processing computer programming language (1, 3), called LISP, is used to program the IBM 704 computer to analyze a linear bilateral electric network whose components have literal or numerical values. LISP was chosen because it is particularly suited to the expression of mathematical and logical algorithms such as are associated with manipulations and simplification of algebraic expressions of the type found in electrical circuit analysis. Basically, our problem can be stated: Given a network made up of ideal resistors, capacitors, and inductors, calculate literal expressions for circuit characteristics such as the driving point and transfer impedance between given nodes.

A network is described to the machine as a list of circuit elements in which each element is described as a resistor, capacitor, or inductor connected between two nodes. Each element is also given a name that can be any algebraic expression. Programs (which take the form of functions in LISP) have been written to accept such a network; these programs will allow the calculation of a literal expression for the driving-point and transfer characteristics.

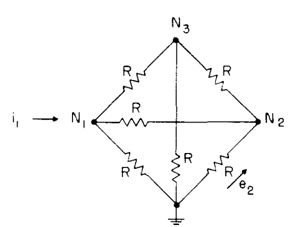

Fig. X-1. Simple resistor network used in a transfer impedance calculation.

In the following example (see Fig. X-1), the transfer impedance between N_1 and N_2 with respect to ground was calculated. This simple network was chosen because the computer result was easy to check and because it was a good test for our programs.

First, the machine constructed a 3 × 3 admittance matrix for the network and reduced this to a 2 × 2 matrix by the method of matrix postmultiplication. Next, the determinant of the 2 × 2 admittance matrix was found and used to find the transfer impedance:

$$\frac{e_2}{i_1} = - \frac{-1.33/R}{(2.67/R)(2.67/R) - (-1.33/R)^2} \tag{1}$$

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An important by-product of our electric-circuit-analysis problem was the simplification functions that were written in such a way that they could operate on an algebraic expression such as Eq. 1 and reduce it to its simplest form. The simplification functions were used to simplify the expression for the transfer impedance (Eq. 1) to 0.25 R, which is the correct transfer impedance. The computer took less than 18 seconds for this entire transfer impedance calculation. In a similar manner the driving-point impedance between N_1 and ground was found to be 0.5 R.

For the network shown in Fig. X-2, the machine required 2.2 minutes to compute the driving-point admittance,

$$\frac{e_1}{i_1} = \frac{2.0\,CRL^2s^3 + LRs + CL^3s^4 + L^2s^2}{2.0\,CLRs^2 + 2.0\,CL^2s^3 + 2.0\,RC^2L^2s^4 + Ls + C^2L^3s^5}$$

and approximately 3.1 minutes to calculate the transfer impedance,

$$\frac{e_2}{i_1} = \frac{CRL^3s^4 + RL^2s^2}{2.0\,RC^2L^2s^3 + CRLs + CLs^2 + CRL^2s^3 + CL^2s^3 + LRs + Ls + C^2L^2s^4 - R}$$

In this instance, a complicated program was required for clearing fractions. Because this program was run in the interpretive mode, it was slow. When the LISP compiler is available and the clear fractions program is compiled, the time will be greatly reduced.

The machine also calculated the driving-point impedance between nodes N_1 and N_0 of the circuit of Fig. X-3 in 2.9 minutes as

$$\frac{e_1}{i_1} = 0.872485\ R$$

The success of this method of electric circuit analysis depends to a great extent upon the ability of the simplification functions to find the simplest form of any algebraic expression. Simplification is accomplished by the systematic application of an algorithm that embodies the usual simplification rules. Some examples of algebraic simplification

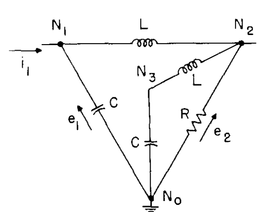

Fig. X-2. An example of an RLC network that is more complicated than that of Fig. X-1.

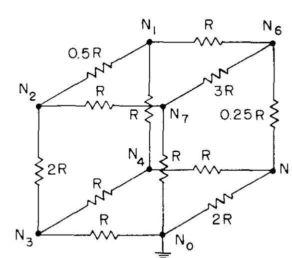

Fig. X-3. A complex nonplanar resistor network.

performed by the machine are:

$$(a+b)(c+d) - b(c+d) - ca = da \tag{2}$$

$$a^n + 1/a^2 + b + ca + d = d + b + a(c+1/a^3+a^{n-1}) \tag{3}$$

Each of these examples required less than 6 seconds of machine time.

During the evaluation of characteristics for complicated electric networks, the machine can produce algebraic expressions that are far too complicated for any human being to understand. Because simplifying assumptions such as eliminating negligible quantities are needed in most engineering problems, a LISP function, makaprox, that would make approximations by eliminating negligible addends in a given algebraic expression was written. Makaprox is given an expression, a list of approximate values, and a limiting ratio; addends should be ignored after the limiting number is reached. The expression

$$ab(b^c+b+e)\ [(a+b+e)^{2(b+e)d(a+c)}]\ (a+b+c) \tag{4}$$

was simplified with the approximate values

$$a \approx 3.2 \quad b \approx 5.7 \quad c \approx 0.04 \quad d \approx 2.0 \quad e \approx 0.02 \tag{5}$$

and a limiting ratio of 0.01. The simplified expression is

$$ba(b+b^c)(b+a)^{1+2db(c+a)} \tag{6}$$

This investigation has demonstrated that a digital computer is a practical means of performing algebraic processes, as well as numerical calculations. A more detailed description of the project will be found in the theses of Goldberg, Edwards, and Rubenstein (3, 4, 5).

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