VI. MICROWAVE ELECTRONICS*

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A. SPACE-CHARGE MODE THEORY OF GAP INTERACTION

We have already presented (1) a linear space-charge theory of gap interaction for a thin electron beam. We shall now take account of the space variation of the space-charge fields and present the interaction for both infinite and Brillouin magnetic-focusing fields.

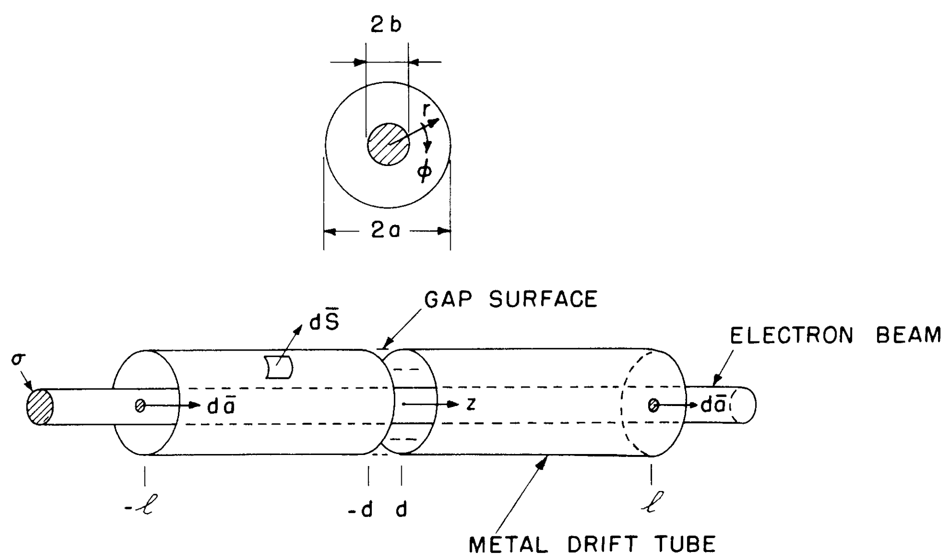

Fig. VI-1. Electron beam in a drift tube with circuit fields coupled through a gap.

A system consisting of an electron stream and a gap region is shown in Fig. VI-1. We have defined (1) the gap voltage, V_g, and gap current, I_g, at the surface. Excitations in the electron stream will be represented by kinetic voltage V, and current I. In matrix form, they are given by

$$\underline{\underline{B}} = \begin{bmatrix} V \\ I \end{bmatrix} \tag{1}$$

By superposition we can write

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$$\begin{bmatrix} \underline{\underline{B}}_2 \\ \\ I_g \end{bmatrix} = \begin{bmatrix} \underline{\underline{D}} & \underline{\underline{K}} \\ \\ \underline{\underline{\Gamma}} & Y_{e\ell} \end{bmatrix} \begin{bmatrix} \underline{\underline{B}}_1 \\ \\ V_g \end{bmatrix} \tag{2}$$

1. The Undriven System: $V_g = 0$

We shall assume that the gap surface is a perfect electric short. The electromagnetic fields of interest are the TM_{0n} modes; that is, $\partial/\partial\phi = 0$. We also assume that $\omega_p/\omega \ll 1$, and disregard the nonpropagating (cutoff) fields.

Case a. $B_{oz} = \infty$

We have found (2) an infinite set of modes

$$E_z(z, r) = \sum_n A_{\pm n} \exp(-j\beta_{\pm n}z)\, J_o(p_n r) \tag{3}$$

in which

$$\beta_{\pm n} = \beta_e \mp \beta_{qn} \tag{4}$$

$$p_n = \gamma \left[\left(\frac{\omega_p}{\omega_{qn}}\right)^2 - 1\right]^{1/2} \tag{5}$$

are determined from the boundary conditions, with

$$\beta_e = \frac{\omega}{v_o}, \quad \beta_q = \frac{\omega_q}{v_o}, \quad \gamma^2 = \beta_e^2 - k^2 \qquad \left(k^2 = \frac{\omega}{c}\right)$$

Case b. Brillouin focusing (nonrelativistic)

For this case we find an infinite number of degenerate modes at β_p, to which the external fields of the gap cannot couple, and a surface-wave mode (3):

$$E_z(z, r) = A_{\pm B}\, I_o(\beta_e r) \exp(-j\beta_{\pm B}z) \tag{6}$$

$$\beta_{\pm B} = \beta_e \mp \beta_{qB} \tag{7}$$

with β_{qB} determined from the boundary conditions.

For case a and case b the kinetic voltage and beam current for each mode satisfy a set of transmission-line equations:

$$\frac{D}{Dz} V_{n,B} = jZ_{0n,B}\, \beta_{qn,B}\, I_{n,B} \tag{8}$$

$$\frac{D}{Dz} I_{n,B} = jY_{0n,B}\, \beta_{qn,B}\, V_{n,B} \tag{9}$$

For case a,

$$V_n(z,r) = \frac{m}{e} v_o v_n(z,r) \tag{10}$$

$$I_n(z,r) = \sigma J_n(z,r) \tag{11}$$

$$Y_{0n} = \frac{1}{2} \frac{I_o}{V_o} \frac{\omega}{\omega_{qn}} \tag{12}$$

For case b,

$$V_B(z) = \frac{m}{e} v_o v_z(z,b) \tag{13}$$

$$I_B(z) = 2\pi b K_z(z) \tag{14}$$

$$Y_{0B} = \frac{1}{2} \frac{I_o}{V_o} \frac{\omega}{\omega_{qB}} \frac{2I_1(\beta_e b)}{(\beta_e b)\, I_o(\beta_e b)} \tag{15}$$

In Eqs. 8-15, v_o is the z-component of the time-average electron velocity; $v_n(z,r)$ and $v_z(z,b)$ are the first-order electron velocities for the n^{th} space-charge mode and surface-wave mode, respectively; $J_n(z,r)$ is the current density of the n^{th} mode; $K_z(z)$ is the z-component of the surface current; σ is the cross-section area of the beam; I_o and V_o are the dc current and voltage of the beam; and $I_1(\beta_e b)$ and $I_o(\beta_e b)$ are modified Bessel functions.

The solutions of Eqs. 8 and 9 give the elements of the $\underline{\underline{D}}$-matrix:

$$\underline{\underline{D}}_{n,B} = \begin{pmatrix} A_{n,B} & B_{n,B} \\ C_{n,B} & A_{n,B} \end{pmatrix} \tag{16}$$

$$A_{n,B} = \frac{1}{2}\left[\exp(-j\beta_{+n,B}\, 2\ell) + \exp(-j\beta_{-n,B}\, 2\ell)\right] \tag{17}$$

$$B_{n,B} = \frac{1}{2} Z_{0n,B} \left[\exp(-j\beta_{+n,B}\, 2\ell) - \exp(-j\beta_{-n,B}\, 2\ell)\right] \tag{18}$$

$$C_{n,B} = \frac{1}{2} Y_{0n,B} \left[\exp(-j\beta_{+n,B}\, 2\ell) - \exp(-j\beta_{-n,B}\, 2\ell)\right] \tag{19}$$

2. Circuit-to-Beam Coupling

We assume that the circuit field, $E_c(z,r)$, can be identified in the presence of the electron stream. This is consistent with our assumption $(\omega_p/\omega) \ll 1$, and allows us to use a weak-coupling formalism. We have

$$\sum_n \frac{D}{Dz} V_{n,B} = \left[\sum_n jZ_{0n,B}\, \beta_{qn,B}\, I_{n,B}\right] + E_c(z,r) \tag{20}$$

$$\sum_n \frac{D}{Dz} I_{n,B} = \sum_n jY_{0n,B}\, \beta_{qn,B}\, V_{n,B} \tag{21}$$

We use the orthogonality properties of the space-charge modes (4), and neglect the electromagnetic power flow because it is small compared with the kinetic power flow. Then, the excitation of each mode is given by

$$\frac{D}{Dz} \hat{V}_{n,B} = jZ_{0n,B}\, \beta_{qn,B}\, \hat{I}_{n,B} + C_{n,B} E_c(z) \tag{22}$$

$$\frac{D}{Dz} \hat{I}_{n,B} = jY_{0n,B}\, \beta_{qn,B}\, \hat{V}_{n,B} \tag{23}$$

where the circumflex denotes that the r-dependence is omitted, and

$$C_n = \frac{\int_\sigma F_c(r)\, J_o(p_n r)\, da}{\int_\sigma J_o^2(p_n r)\, da} \tag{24}$$

$$C_B = F_c(b) \tag{25}$$

In Eqs. 24 and 25 we have written

$$E_c(z,r) = F_c(r)\, E_c(z)$$

Equations 22 and 23 can be solved by finding the impulse response $(E_c(z) = u_o(z))$ and then using the superposition integral. The elements of the K-matrix

$$\underline{\underline{K}}_{n,B} = \begin{bmatrix} a_{n,B} \\ \\ b_{n,B} \end{bmatrix} \tag{26}$$

are found to be

$$a_{n,B} = \frac{1}{2} \left[M_{+n,B} \exp(-j\beta_{+n,B}\, \ell) + M_{-n,B} \exp(-j\beta_{+n,B}\, \ell)\right] C_{n,B} \tag{27}$$

$$b_{n,B} = \frac{1}{2} Y_{0n,B} \left[M_{+n,B} \exp(-j\beta_{+n,B}\, \ell) - M_{-n,B} \exp(-j\beta_{-n,B}\, \ell)\right] C_{n,B} \tag{28}$$

where

$$M_{\pm n,B} = \int_{-\infty}^{\infty} \mathscr{E}_{\pm n,B}(\theta_{\pm n,B}) \exp(j\theta_{\pm n,B})\, d\theta_{\pm n,B} \tag{29}$$

as defined (1) previously.

3. Beam-to-Circuit Coupling

The kinetic power theorem

$$\mathrm{Re}\left(V_g I_g^*\right) = \mathrm{Re}\int_\sigma \left[(VI^*)_\ell - (VI^*)_{-\ell}\right] da \tag{30}$$

imposes certain relationships (5) upon the matrices of Eq. 2. From Eqs. 30 and 2 we find that

$$\underline{\underline{\Gamma}}_{n,B} = \left[c_{n,B} \quad d_{n,B}\right] \tag{31}$$

$$c_{n,B} = \frac{1}{2} Y_{0n,B}\left[M_{+n,B}^* \exp(-j\beta_{+n,B}\,\ell) - M_{-n,B}^* \exp(-j\beta_{-n,B}\,\ell)\right] K_{n,B} \tag{32}$$

$$d_{n,B} = \frac{1}{2}\left[M_{+n,B}^* \exp(-j\beta_{+n,B}\,\ell) + M_{-n,B}^* \exp(-j\beta_{-n,B}\,\ell)\right] K_{n,B} \tag{33}$$

where

$$K_n = \frac{1}{\sigma}\int F_c(r)\, J_o(p_n r)\, da \tag{34}$$

$$K_B = C_B \tag{35}$$

Equation 3 also gives the real part of the electronic loading admittance for each mode:

$$G_{e\ell\, n,B} = \frac{1}{4} Y_o \left[\left|M_{+n,B}\right|^2 - \left|M_{-n,B}\right|^2\right] C_{n,B} K_{n,B} \tag{36}$$

The imaginary part of the electronic loading admittance is readily obtained from Eq. 2, in conjunction with the following kinetic energy theorem (5)

$$\begin{aligned}\mathrm{Im}\left(V_g I_g^*\right) = \mathrm{Im}\int_\sigma \left[(VI^*)_\ell - (VI^*)_{-\ell}\right] da \\ + \omega\int_\tau \epsilon_o \left|\beta_p V\right|^2 d\tau \\ + \omega\int_\tau \left[\mu_o |H|^2 - \epsilon_o |E|^2\right] d\tau\end{aligned} \tag{37}$$

Finally, Eqs. 2 and 37 yield

$$\begin{aligned}B_{e\ell\, n,B} = \frac{1}{2} Y_{0n,B}\, \mathrm{Im}\left[M_{+n,B} M_{-n,B}^* \exp(j2\beta_{qn,B}\,\ell)\right] C_{n,B} K_{n,B} \\ + \omega\epsilon_o \beta_p \sigma \frac{K_{n,B}}{C_{n,B}} \int a_{n,B}(z)\, dz\end{aligned} \tag{38}$$

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