

# Ultrashort Laser Pulse Phenomena

Fundamentals, Techniques, and Applications on a  
Femtosecond Time Scale

Second Edition

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AMSTERDAM • BOSTON • HEIDELBERG • LONDON  
NEW YORK • OXFORD • PARIS • SAN DIEGO  
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Academic Press is an imprint of Elsevier



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