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Incoherent High-frequency Schottky Signals in LHC

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Abstract

A detector for high-frequency Schottky signals in the LHC is currently being designed in the framework of the US-LARP collaboration. The detector will work at 4.8 GHz, where coherent beam signals should be rather low and the incoherent Schottky bands should not yet overlap. This note discusses the expected properties of the incoherent Schottky signals in this frequency range for various types of proton and ion beams in the LHC. The use of gating to increase the signal-to-noise ratio for the pilot beam is also discussed.

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1 Introduction

In order to optimize the integrated luminosity of the LHC, the beam emittance must remain as small as possible during the collision runs. Since the synchrotron radiation damping times required for any reduction in emittance are very long (of the order of 10 hours), the use of beam instrumentation that needs an excitation of the beam, hence leading to an increase in emittance, has to be limited [1]. This applies in particular to standard methods for the measurement of incoherent tune and chromaticity that require various sources of beam excitation. A Schottky monitor on the other hand allows the measurement of these quantities without any additional excitation of the beam. In the following, the properties of the Schottky sidebands expected in the LHC for the proposed monitor are discussed.

2 Calculations

Two cases were considered for the calculations, i.e. LHC beam at injection and at collision energy. In both cases calculations were performed for a pilot bunch and a nominal proton beam.

2.1 Machine, beam and Schottky detector parameters

The machine and beam parameters relevant for the calculation of the Schottky signals are given in Tab. 1 for nominal proton beam. The pilot beam used for commissioning consists of a single proton bunch with 5e9 particles. It may have a smaller emittance than nominal beam, but the beam parameters and the machine settings should be the same as for nominal beam. The parameters for lead ion beams and for the Schottky monitor follow in Tab. 2 and 3 [1]. For commissioning an early ion scheme is foreseen with only 62 bunches in the machine but the same parameters otherwise.

		Unit	Symbol	Injection	Collision
Beam parameters	Proton energy	GeV	E	450	7000
	Relativistic gamma		γ	479.6	7461
	Number of bunches			2808	
	Number of particles per bunch			1.15e11	
	Bunch length, 1 sigma	cm	σ_L	11.2 (injection) 12.8 (after filam.)	7.55
	Normalized vertical emittance	m*rad	ε_n	3.5e-6	3.75e-6
	Energy spread		dp/p	3.06e-4 (injection) 4.4e-4 (after filam.)	1.129e-4
Machine parameters	Horizontal tune		Q_H	64.28	64.31
	Vertical tune		Q_V	59.31	59.32
	Slip factor		η	3.182e-4	3.225e-4
	Chromaticity		$Q\xi$	<10 in the beginning ≈2 nominal	between 1 and 2 in both planes
	Synchrotron frequency	Hz	f_S	63.7	23.0
	Revolution frequency	Hz	f_0	11245.475	11245.500
	β function at Schottky monitor	m	β	about 400	

Tab. 1: Beam and machine parameters for nominal LHC proton operation

	Unit	Symbol	Injection	Collision
Lead energy per nucleon	GeV	E	177.4	2759
Relativistic gamma		γ	190.5	2963.5
Number of bunches			592	
Number of particles per bunch			7e7	
Bunch length, 1 sigma	cm	σ_L	9.97	7.94
Normalized vertical emittance	m*rad	ε_n	1.4e-6	1.5e-6
Energy spread		dp/p	3.9e-4	1.1e-4

Tab. 2: Nominal Pb ion beam parameters. Only the parameters different from nominal proton beam are listed; the LHC optics is unchanged for ions beams, except for the collision points.

	Unit	Symbol	Value
Schottky monitor operating frequency	GHz	f	4.8
Beam pipe diameter in monitor	cm	d	6
Length of waveguides	cm	L	120

Tab. 3: LHC Schottky monitor parameters. The calculated sensitivity is given in Fig. 2.

2.2 Bunched versus unbunched Schottky signals

A very comprehensive treatment of Schottky diagnostics is presented in [2,3]. The LHC Schottky detector will be operated at 4.8 GHz, corresponding to a harmonic number, *n*, of about 427000. For such a high *n* the significant bandwidth of the bunched beam Schottky signals will be the same as for an unbunched beam with the same momentum spread [3]. For the ease of display only the unbunched beam Schottky spectra are plotted in this note, neglecting a possible underlying synchrotron motion structure. In practice, such spectra are observed on a spectrum analyser when the interfrequency filter bandwidth is set larger than the synchrotron frequency. Then the phase modulation spectrum of each band is averaged to yield the smooth spectrum expected for unbunched beam.

2.3 Comparison of formulas

Longitudinal Schottky signals

Using the CERN definition the signal to noise ratio (*SNR*) for protons over a Schottky sideband is derived from the longitudinal sensitivity S_l as [2]

$$S_l = \frac{V_{out}}{i_{rms}},$$

where V_{out} is measured in the system impedance R_0 and i_{rms} is the rms beam current per band.

$$i_{rms} = 2ef_0\sqrt{N/2}$$

where e is the charge of the electron, N the number of particles in the beam, f_0 the revolution frequency.

The beam power in one Schottky sideband is then given by

$$P_{beam} = \frac{V_{out}^2}{R_0} = \frac{(i_{rms} S_l)^2}{R_0} = \frac{S_l^2}{R_0}\left(2ef_0\sqrt{N/2}\right)^2 = \frac{S_l^2}{R_0} 2e^2 f_0^2 N$$

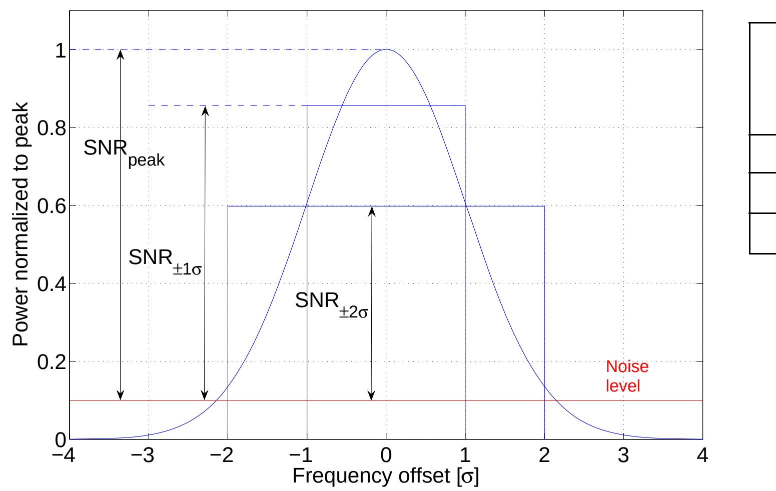
It has to be compared with the noise power over the sideband width Δf

$$P_{noise} = kT\Delta f$$

with the Boltzmann constant k and the temperature T. The signal to noise ratio is then

$$SNR_L = \frac{P_{beam}}{P_{noise}} = \frac{2e^2 N f_0^2}{kT\Delta f} \frac{S_l^2}{R_0} \qquad (1)$$

The noise power will be calculated over the full ±1 sigma width of the Schottky band Δf and the beam power is averaged over this bandwidth ($SNR_{\pm 1\sigma}$ in Fig. 1). Since this band only contains 68% of the total beam signal power, the calculated SNR will somewhat overestimate $SNR_{\pm 1\sigma}$, which is neglected in subsequent calculations. There are other common definitions, e.g. with the ±2 sigma width or the peak of the Gaussian distribution and the noise level. The conversion factor is given in the table in Fig. 1.



Relation between the SNR definitions on the left and the calculation in Equation 1
$SNR_{peak} = SNR_{calc} - 1$ dB
$SNR_{\pm 1\sigma} = SNR_{calc} - 1.7$ dB
$SNR_{\pm 2\sigma} = SNR_{calc} - 3.2$ dB

Fig. 1: The different definitions of the signal to noise ratio for one Schottky band

Fermilab defines the SNR as the ratio of the power *per revolution band* when there is beam in the machine to when there is no beam. In this case $\Delta f = f_0$ and the above equation becomes:

$$SNR_{L,f_0} = \frac{2e^2 N f_0^2}{kTf_0} \frac{S_1^2}{R_0} = \frac{2e^2 N f_0}{kT} \frac{S_1^2}{R_0}$$

The longitudinal sensitivity, S_l, of the monitor can be replaced by the longitudinal impedance of the monitor, a parameter which is calculated by Fermilab for the LHC Schottky pick-up structure and is defined as follows (“old definition” [5]):

$$Z_\Sigma = \frac{S_l^2}{2R_0}$$

$$\Rightarrow SNR_L = \frac{e^2 N f_0^2}{kT} \frac{Z_\Sigma}{\Delta f}$$

Taking into account the overall processing system noise figure, N_f , one obtains

$$SNR_L = \frac{e^2 N f_0^2}{kT\Delta f} \frac{Z_\Sigma}{N_f}$$

For ions with a charge of *Ze* the *SNR* is Z^2 times higher since the *Z* charges of each ion add up coherently.
The RMS width of a single longitudinal Schottky line is given as

$$df = n f_0 \eta \frac{dp}{p},$$

with the slip factor η and the rms momentum spread *dp/p*. The full width of the band (±1 sigma) is then simply $\Delta f = 2df$.

Transverse Schottky signals

For transverse Schottky signals, the *SNR* for protons can be shown [2] to be given with the transverse sensitivity S_d as

$$S_d = \frac{V_{out}(measured\ in\ R_0)}{d_{rms}}$$

where d_{rms} is the rms dipole moment and is given by

$$d_{rms} = e f_0 a_{rms} \sqrt{N/2}$$

with a_{rms} being the rms amplitude of the oscillation. This is related to the beam emittance as follows:
$a_{rms} = \sqrt{(\varepsilon\beta/2)}$, with β the twiss beta function at the location of the monitor. Hence:

$$P_{beam} = \frac{V_{out}^2}{R_0} = \frac{(d_{rms} S_d)^2}{R_0} = \frac{S_d^2}{R_0}\left(e f_0 a_{rms}\sqrt{N/2}\right)^2 = \frac{S_d^2}{R_0} e^2 f_0^2 a_{rms}^2 N/2$$

and with the noise power over one beam sideband

$$P_{noise} = kT\Delta f$$

we get for the transverse signal to noise ratio

$$SNR_T = \frac{P_{beam}}{P_{noise}} = \frac{e^2 N f_0^2 a_{rms}^2}{2kT\Delta f} \frac{S_d^2}{R_0}$$

The sensitivity can be defined in terms of the Fermilab delta mode transfer impedance Z_Δ ("old definition [5]), where

$$S_d^2 = \frac{4 Z_\Delta R_0}{d^2},$$

with *d* being the beam pipe diameter at the location of the detector.
Using this formalism and taking into account the overall processing system noise figure, N_f, one obtains:

$$SNR_T = \frac{e^2 N f_0^2 a_{rms}^2}{kT\Delta f} \frac{2 Z_\Delta}{d^2 N_f}$$

As in the longitudinal case the signal for ions with charge number Z is higher by a factor Z^2.

The rms width of the two transverse Schottky bands at the n[th] harmonic is given as

$$df = f_0 \frac{dp}{p}\left((n \pm q)\eta \pm Q\xi\right),$$

where q designates the fractional tune, Q the full integer tune, ξ the chromaticity, dp/p the rms momentum spread and η the momentum compaction factor. The full ±1 sigma width is again given by $\Delta f = 2df$.

2.4 Detector sensitivity

The simulated transfer impedance of the LHC Schottky detector [7] is plotted in Fig. 2. The design is centered at 4.8 GHz, which corresponds to the maximum of the transverse sensitivity. However, there is still substantial longitudinal sensitivity at this frequency. Working off the maximum transverse sensitivity at lower frequencies (about 4.68 GHz), where the longitudinal sensitivity is very poor, might therefore be advantageous to the operation of the instrument as it would enhance the suppression of the sum mode signal. From past experience it is estimated that the frequency offset between the simulated and the real pick-up response may be of the order of 50 MHz [7].

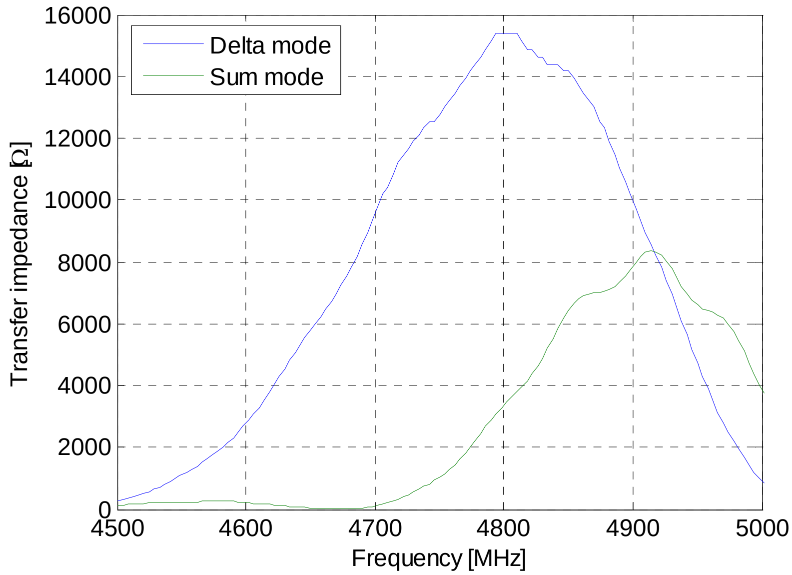


Fig. 2: Simulated transfer impedance for LHC Schottky monitor

2.5 Gating

When the beam fills only a small fraction of the machine circumference, the pick-up will detect noise for most of the time. In particular for the pilot beam the noise level may become comparable to, or even bigger than, the Schottky sidebands. In this case the effective noise level can be decreased by gating, where the input signal is switched “on” only for a relatively short time during the passage of the beam. In practice, however, the input signal cannot be totally switched “off”, and there will always be thermal noise coming from the line termination or the switch itself. A cryogenic load could be used as a low-noise termination, which at 1 K would reduce the noise power by a factor 300 or 25 dB.

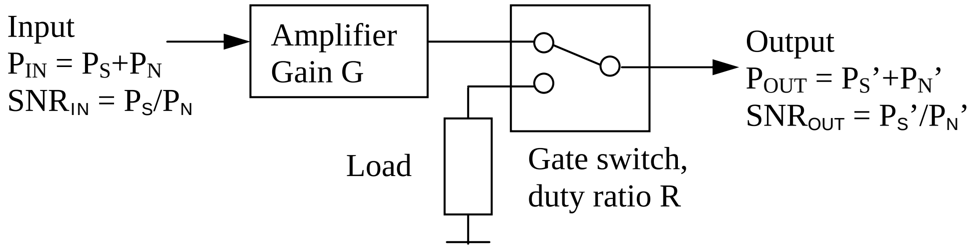


Fig. 3: Gating after preamplification can be used to reduce the effective noise level

Another option is to use preamplification before the gate (Fig. 3). This way first *both* the input noise and the beam signal are boosted. After this amplification, the gate switch can be used to pick out the amplified beam signal plus noise during the passage of the beam. The amplified noise during the rest of the time is removed by the gate and is replaced by the thermal noise of the gate in the “off” position, so corresponding to an effective reduction of the overall noise level.

The pick-up signal at the input of the gating section in Fig. 3 is composed of the signal power P_S and the noise power P_N, $P_{IN} = P_S + P_N$. The input signal-to-noise ratio is given by $SNR_{IN} = P_S/P_N$. After the amplification the power is $GP_S + GP_N$. The gate selects short sections of the input signal containing most of the beam signal with a duty ratio R. During the remaining time, thermal noise comes into the system from the room temperature termination at the second switch entry. The resulting output power can be expressed as:

$$P_{OUT} = GP_S + RGP_N + (1-R)P_N = P_S' + P_N'.$$

Therefore, we have at the output an effective signal power given by

$$P_S' = GP_S$$

and an effective noise power given by

$$P_N' = (RG+1-R)P_N$$

Hence the SNR ratio at the output can be expressed as

$$SNR_{OUT} = \frac{GP_S}{(RG+1-R)P_N}$$

This can be compared to the SNR of the input signal, allowing the increase in signal-to-noise ratio due to gating, SNR' , to be written as

$$\boxed{SNR' = \frac{SNR_{OUT}}{SNR_{IN}} = \frac{G}{RG+1-R}}$$

From this formula it can be seen that

- For small G the gain in signal to noise is proportional to G. In this case the noise power coming from the load is the dominant noise contribution.
- For large G the gain in signal to noise converges to $1/R$. The output SNR is then determined by the noise power that comes through the gate when it is open. As expected

the output SNR can be maximized by minimizing the gate duty ratio, provided that the gate aperture is still wide enough to let through most of the beam signal.

For a single LHC bunch with a duty factor of R = 25ns/89μs ≈ 3e-4, the maximum attainable gain in signal to noise through gating is ≈35 dB. With a gain before gating of 35 dB, it should therefore be possible to improve the SNR through gating by about 30 dB for single bunch measurements.

2.6 Results

The width and the expected SNR of the incoherent Schottky bands in LHC are given in Tab. 4 and Tab. 5 for a chromaticity $Q\xi = 2$. Results for fully stripped lead ions (Pb^{82+}) for LHC ion operation are also included. The values for the SNR are at the output of the detector after combination in an ideal hybrid, i.e. the noise figure of the signal processing chain is not included. For perfectly centered beam the longitudinal signal can be observed only at the Sum output of the hybrid; the Delta output gives the transverse signal. For the pilot beam the transverse emittance may be smaller than nominal by up to a factor 3 [1]. This leads to a decrease in transverse SNR by up to about 5 dB compared to nominal. Both values are quoted in Tab. 5. As shown in Section 2.5, gating should allow the SNR for a single pilot bunch to be improved by ≈30 dB.

	RMS width of lower/upper Schottky bands Δf [Hz]	
	protons	Pb ions
injection	948/921	1120/1085
collision	355/344	345/335

Tab. 4: Two sigma width of incoherent Schottky bands in LHC.

		Longitudinal Schottky SNR [dB]	Transverse Schottky SNR [dB]	Transverse Schottky SNR [dB] with gating
Nominal p^+ beam	injection	60	36	
	collision	64	28	
Pilot p^+ beam	injection	12	-17 to -13	+13 to +17
	collision	16	-25 to -20	+5 to +10
Nom. Pb ion beam	injection	58	34	
	collision	64	28	
Early Pb ion beam	injection	49	24	
	collision	54	18	

Tab. 5: Calculated raw SNR of the incoherent Schottky band for nominal beam and pilot in the LHC. The noise figure of the signal processing chain and the possible improvements by gating are not included. The range given for the pilot beam corresponds to minimum and maximum transverse emittance. An SNR increase of 30 dB due to gating was assumed.

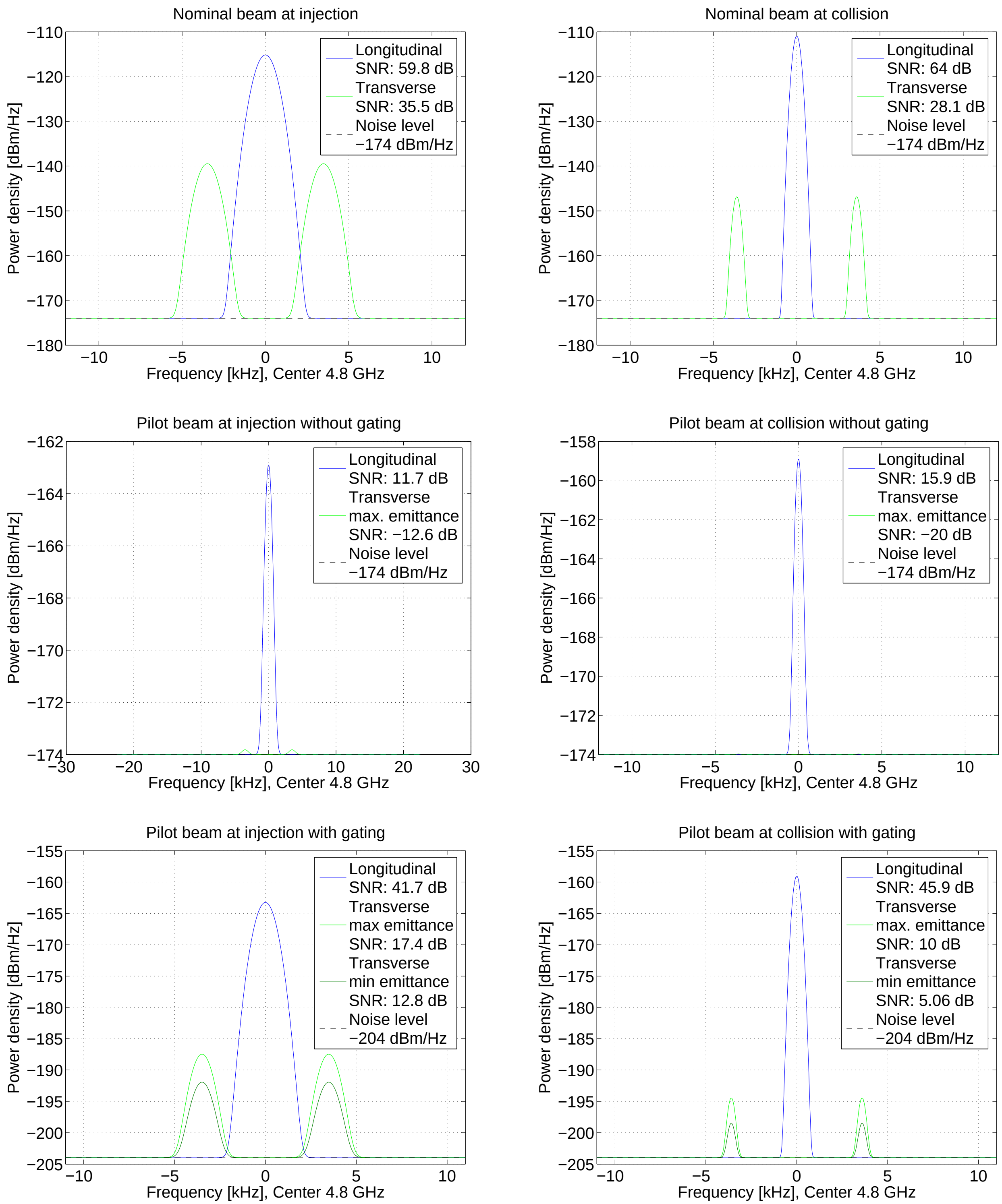


Fig. 4: The calculated vertical raw spectra for nominal and pilot proton beam. The signals are after combination with an ideal hybrid at the sum/difference output; the noise figure of the signal processing chain is not included. The chromaticity was neglected on account of its small effect on the bandwidth and SNR. Gating reduces the effective noise level for the Delta mode signal; it is not applied to the Sum mode signal.

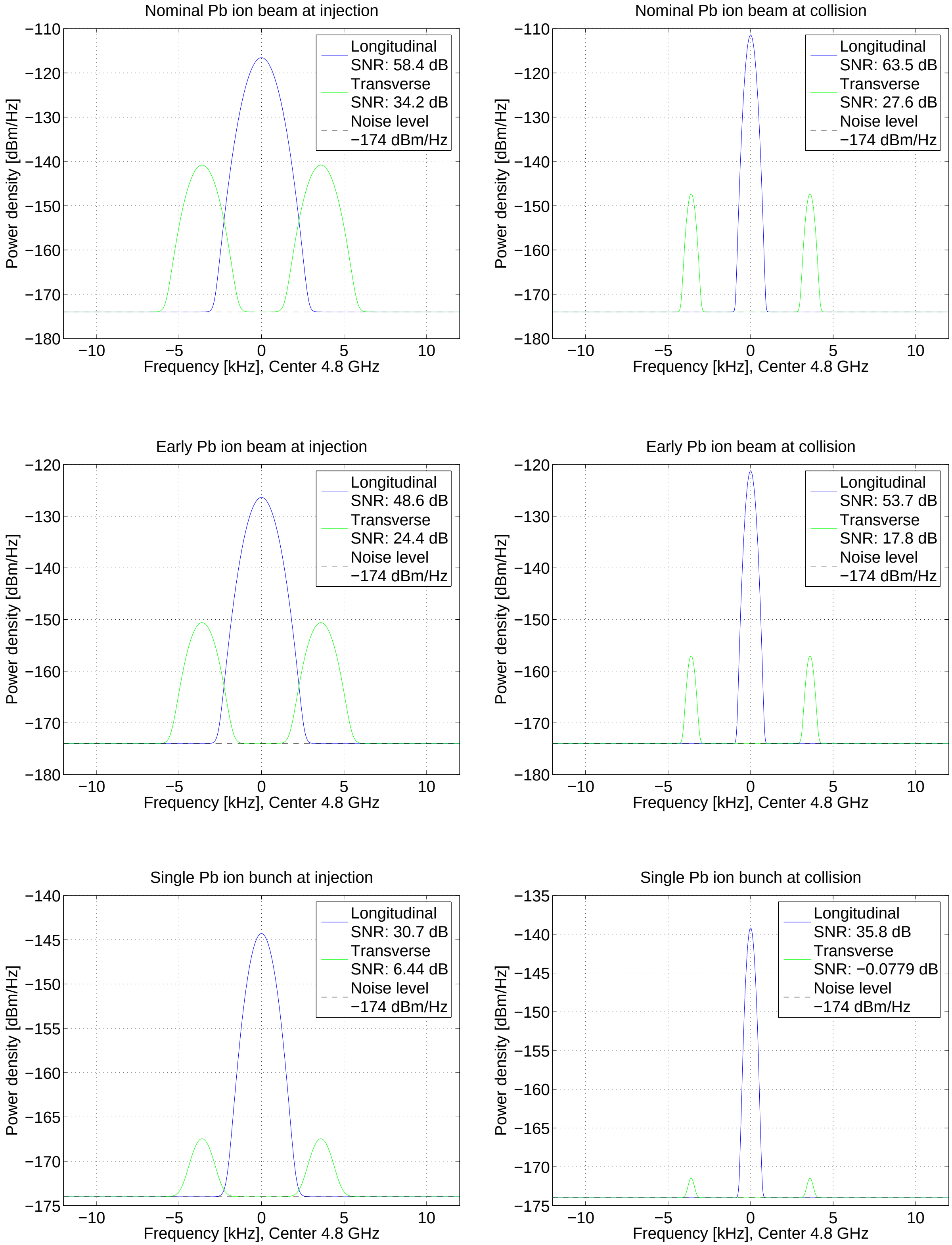


Fig. 5: The calculated vertical raw spectra for lead ion (Pb^{82+}) beam. The signals are after combination with an ideal hybrid at the sum/difference output; the noise figure of the signal processing chain is not included. Zero chromaticity was assumed here. Gating could be used to increase the SNR for a single bunch (7e7 ions).

The expected proton beam spectra for the vertical plane with zero chromaticity are plotted in Fig. 4. These are the “raw” signals at the output of the pick-up after signal recombination in an ideal hybrid. The inevitable decrease in SNR due to the noise figure of the detector electronics chain is not included.
For the nominal beam comfortably high transverse SNR levels of around 30 dB are expected. However, for the other extreme, the pilot bunch, the beam signal is well below noise level. With sufficient averaging a small bump on the noise floor should still be visible even when the SNR is below 0 dB, but interpretation would be very difficult for such small signals. Gating can be used to amend this situation. The two lowermost plots in Fig. 4 show the spectra expected when gating is used, giving an improvement in SNR of 30 dB. The effect of gating is illustrated by a reduction of the noise level. In this ideal case the SNR at the output of the detector is positive even for a pilot beam of small emittance (1.2e-6 m*rad), but the noise figure of the signal processing chain has yet to be included.
Realistically attainable noise figures for the complete electronics chain are of the order of 6 to 10 dB, which results in pilot bunch signals very close to the noise level. There are other uncertainties involved, such the exact value of the transfer impedance and the beam signal losses due to narrow gate apertures which might impact the expected performance by several dB.
Fig. 5 shows the spectra for various lead ion beams. Due to the coherent effect of the ion charges rather high signals are expected even though the DC beam current is generally much lower than for proton beams. Without gating the signal from single ion bunches containing 7e7 particles should be in the range of the noise level. Gating should render them visible.

3 Summary & Conclusion

A high frequency Schottky detector for LHC is currently being designed in the framework of the US-LARP collaboration. In this note the expected incoherent Schottky spectra are calculated using LHC design parameters and estimations of the key parameters of the Schottky monitor. For nominal LHC beam incoherent transverse sidebands about 30 dB above noise floor are expected. For the pilot beam on the other hand even for ideal signal processing without introduction of extra noise the transverse sidebands are of the order of 20 dB below the noise floor. Gating could be used to reduce the effective noise level and recover these weak signals. If a high SNR improvement by gating can be attained and the estimations of the pick-up characteristics hold, the transverse sidebands should rise above noise floor even for the pilot beam. To this end, the noise figure of the signal processing chain should be kept as low as possible, the gain before gating should be maximized and the gate aperture minimized while still capturing most of the beam signal. It is also expected that the transverse Schottky signals of lead ion beams can be observed.

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