CERN – EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CLIC-Note-707

MEASUREMENT OF S PARAMETERS OF AN ACCELERATING STRUCTURE WITH DOUBLE-FEED COUPLERS

R. Fandos, A. Grudiev, W. Wuensch

Abstract

A method for measuring the transmission and reflection coefficients of an accelerating structure with double-feed input and output couplers using a 2 port network analyzer is presented. This method avoids the use of magic Ts and hybrids, whose symmetry is not obvious. The procedure is extended to devices with n symmetrical input and m symmetrical output ports. The method to make bead pull measurements for such devices is described.

Keywords: RF measurements, accelerating structure, double-feed couplers

Geneva, Switzerland
November 2006

1. INTRODUCTION

Early versions of CLIC accelerating structures for high gradient testing had single feed input and output couplers. Double fed couplers have been introduced in order to have lower and more symmetrical surface electric field pattern in the couplers, which improves high gradient performance.

This change created the need to measure the transmission and reflection coefficients of a 4 port accelerating structure. Since Network analyzers typically have only two ports, this measure was usually done by using magic Ts or hybrids, assuming they are symmetric. However this introduces an uncertainty in the measurements from the extra elements.

A method has been devised where the matching of the structure and its transmission coefficient can be determined from a series of direct 2 port measures. Instead of using magic Ts or hybrids as extra elements, this new technique requires adapted loads. This is a major advantage, since good matching of a load over a large range of frequencies can be much easier achieved than perfect symmetry of hybrids. The technique is derived and examples are given in this note.

Moreover, the changes needed to adapt the standard bead pull method to a multiple port device are described and an example is shown.

2. REFLECTION MEASUREMENT

2. 1. Derivation of the reflection coefficient of a 4 port device from direct 2 port S parameter measures.

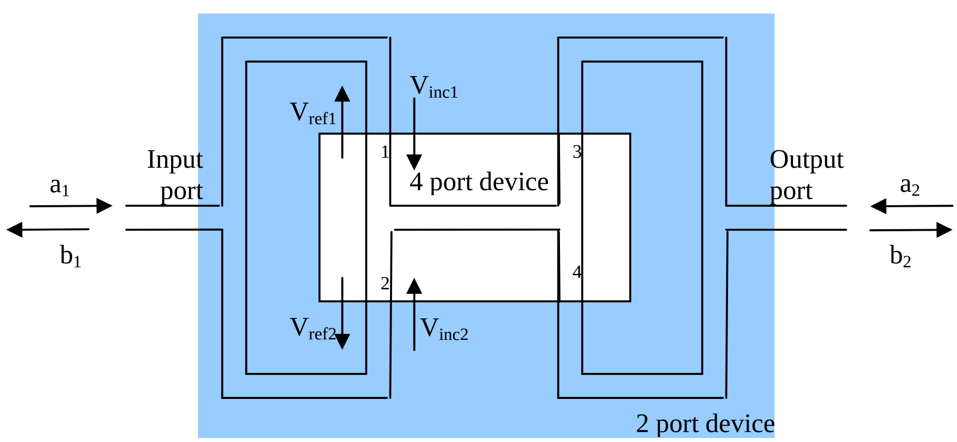

Figure 1: The graphical representation of the correspondence between ports of the 4 port and 2 port devices and the identification of the signals needed for the reflection coefficient calculation.

The input port reflection coefficient of a 2 port device is by definition:

$$R_{in} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad \text{eq. 1}$$

Where a_1 is the normalized incident complex voltage at the input port and b_1 is the normalized reflected complex voltage at the input port:

$$b_1 = \frac{V_{ref}}{\sqrt{Z_o}}, \qquad a_1 = \frac{V_{inc}}{\sqrt{Z_o}} \qquad \text{eq. 2}$$

Where Z_0 is the characteristic impedance. Thus, the reflection coefficient R_{in} can also be expressed as

$$R_{in} = \left.\frac{V_{ref}}{V_{inc}}\right|_{a_2=0} \qquad \text{eq. 3}$$

Equating a_2 to 0 implies terminating the output port with a matched load.

In our particular case, the input port of the 2 port device consists of the combination of ports 1 and 2 of the 4 port device (see fig. 1). Thus,

$$V_{inc} = V_{inc1} + V_{inc2}$$
$$V_{ref} = V_{ref1} + V_{ref2} \qquad \text{eq.4}$$

Symmetry of the two input ports is assumed. Therefore,

$$V_{inc1} = V_{inc2} = \frac{V_{inc}}{2} \qquad \text{eq. 5}$$

And the reflected voltages can be calculated as follows:

$$V_{ref1} = S_{11}V_{inc1} + S_{21}V_{inc2} = (S_{11} + S_{21})\frac{V_{inc}}{2}$$
$$V_{ref2} = S_{22}V_{inc2} + S_{12}V_{inc1} = (S_{22} + S_{12})\frac{V_{inc}}{2} \qquad \text{eq. 6}$$

Where S_{11}, S_{12}, S_{21} and S_{22} are the standard s-parameters of the 4 port device and therefore can be measured with a 2 port network analyzer (see section 2.2). For symmetry reasons $S_{11}=S_{22}$ and because the device is reciprocal $S_{21}=S_{12}$. Therefore $V_{ref1}=V_{ref2}$ and

$$V_{ref} = V_{ref1} + V_{ref2} = (S_{11} + S_{21})V_{inc} = (S_{22} + S_{21})V_{inc} \qquad \text{eq. 7}$$

And the input port reflection coefficient

$$R_{in} = \frac{V_{ref}}{V_{inc}} = S_{11} + S_{21} = S_{22} + S_{21} \qquad \text{eq. 8}$$

By analogy the output port reflection coefficient can be calculated as

$$R_{out} = S_{33} + S_{43} = S_{44} + S_{34} \qquad \text{eq. 9}$$

2. 2. Description of the network analyzer settings needed for the reflection coefficients calculation.

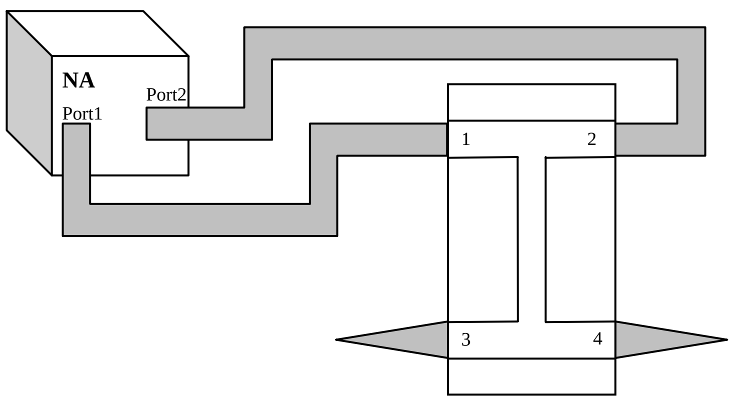

Figure 2: Setting for the measurement of the input reflection coefficient. Ports 3 and 4 are terminated. The NA ports are connected to ports 1 and 2 of the structure.

Terminating the output ports 3 and 4 of the device and connecting the two ports of the network analyzer to the input ports 1 and 2 (see fig. 2), the S parameters S_{11}, S_{21} and S_{22} are measured. It is straightforward to calculate the reflection coefficient following eq. 8.

As said above, symmetry of the ports 1 and 2 is assumed. In order to have an idea of how symmetric the device is in practice, the reflection coefficient calculated as $S_{11}+S_{21}$ can be compared with the reflection coefficient calculated as $S_{22}+S_{21}$.

To calculate the reflection from the output, we terminate now ports 1 and 2 and connect the ports of the network analyzer to ports 3 and 4 of the device, obtaining S_{33}, S_{34} and S_{44}. The output reflection coefficient can be calculated regarding eq. 9.

3. TRANSMISSION MEASUREMENT

3. 1. Derivation of the transmission coefficient of a 4 port device from 2 port s parameter measures.

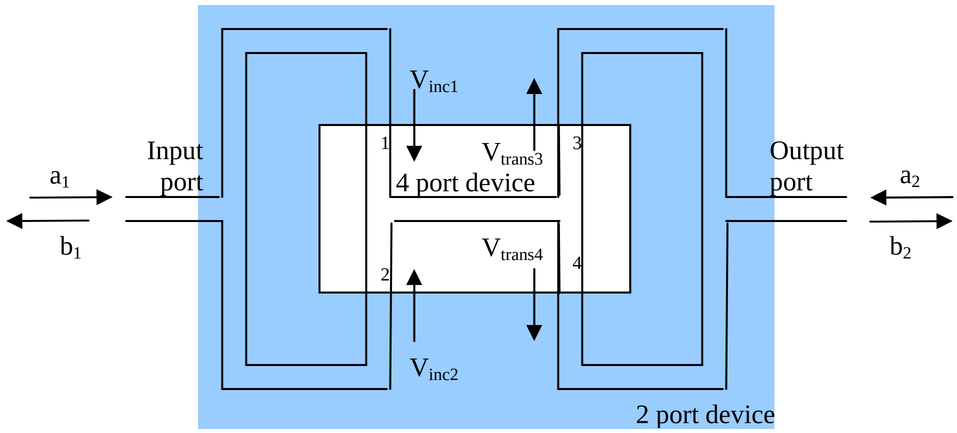

Figure 4: Graphical representation of the correspondence between ports of the 4 port and 2 port devices. Identification of the signals needed for the transmission coefficient calculation.

The transmission coefficient of a 2 port device is by definition:

$$T = \left. \frac{b_2}{a_1} \right|_{a_2=0} \qquad \text{eq. 10}$$

Where a_2 is the normalized incident complex voltage at the output port and b_2 is the normalized reflected complex voltage at the output port:

$$b_2 = \frac{V_{trans}}{\sqrt{Z_o}}, \qquad a_1 = \frac{V_{inc}}{\sqrt{Z_o}} \qquad \text{eq. 11}$$

Therefore, the transmission coefficient T can also be expressed as

$$T = \left. \frac{V_{trans}}{V_{inc}} \right|_{a_2=0} \qquad \text{eq. 12}$$

In our particular case, the input port of the 2 port device consists of the combination of ports 1 and 2 of the 4 port device, and the output port of the 2 port device consists of the combination of ports 3 and 4 of the 4 port device (see fig. 4). Hence,

$$V_{inc} = V_{inc1} + V_{inc2}$$
$$V_{trans} = V_{trans3} + V_{trans4} \quad \text{eq. 13}$$

Because of the symmetry of the input ports

$$V_{inc1} = V_{inc2} = \frac{V_{inc}}{2} \quad \text{eq. 14}$$

And because of the symmetry of the output ports

$$V_{trans3} = V_{trans4} = \frac{V_{trans}}{2} \quad \text{eq. 15}$$

The transmitted voltages can be calculated as follows:

$$V_{trans3} = S_{31}V_{inc1} + S_{32}V_{inc2} = (S_{31} + S_{32})\frac{V_{inc}}{2}$$
$$V_{trans4} = S_{41}V_{inc1} + S_{42}V_{inc2} = (S_{41} + S_{42})\frac{V_{inc}}{2} \quad \text{eq. 16}$$

Where $s_{31,}$ s_{32}, s_{41} and s_{42} are the standard s-parameters of the 4 port device and therefore can be measured with a 2 port network analyzer (see section 3.2). For symmetry reasons $S_{31}=S_{42}$ and $S_{32}=S_{41}$. Therefore $V_{trans3}=V_{trans4}$ and

$$V_{trans} = V_{trans3} + V_{trans4} = (S_{31} + S_{32})V_{inc} = (S_{41} + S_{42})V_{inc} \quad \text{eq. 17}$$

And the transmission coefficient

$$T = \frac{V_{trans}}{V_{inc}} = S_{31} + S_{32} = S_{41} + S_{42} \quad \text{eq. 18}$$

Since the device is reciprocal, the transmission coefficient from the output to the input is identical.

3. 2. Description of the network analyzer settings needed for the transmission coefficients calculation.

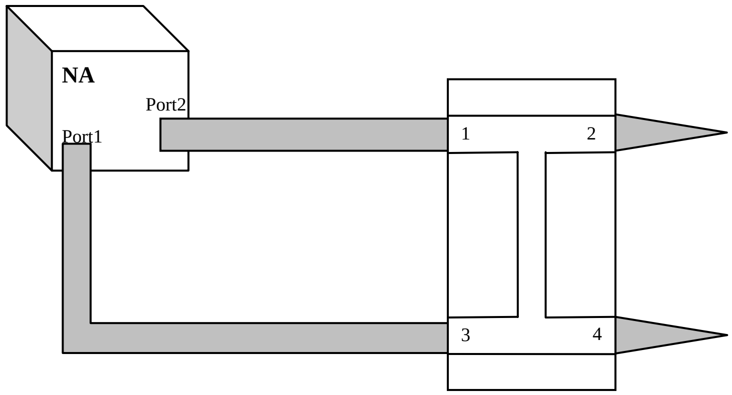

Figure 5: Setting for the measurement of the transmission coefficient. Ports 2 and 4 are terminated. The NA ports are connected to ports 1 and 3 of the structure.

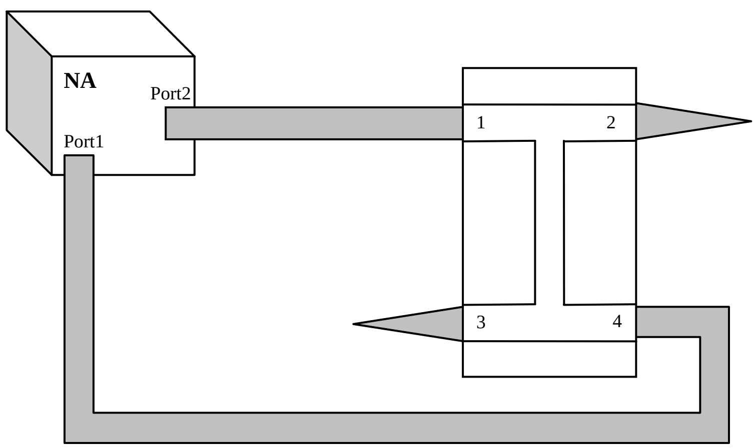

Figure 6: Setting for the measurement of the transmission coefficient. Ports 2 and 3 are terminated. The NA ports are connected to ports 1 and 4 of the structure.

In order to measure the transmission coefficient, two measurements are made with the network analyzer as shown in figs. 5 and 6, the former for S_{31} and the latter for S_{14}. Equivalently, we could measure S_{42} and S_{23}. The transmission coefficient can then be calculated as in eq. 18.

Symmetry of ports 1 and 2 and ports 3 and 4 is assumed for the transmission coefficient calculation. An estimation of how symmetric the device actually is can be done by comparison of the transmission coefficient calculated as S_{31}+S_{32} can be compared with the reflection coefficient calculated as S_{41}+S_{42}.

4. AN EXAMPLE: TRANSMISSION AND REFLECTION PARAMETERS OF THE HDS60 ACCELERATING STRUCTURE

This method has been used to measure the S parameters of the HDS60 accelerating structure, shown in fig. 7.

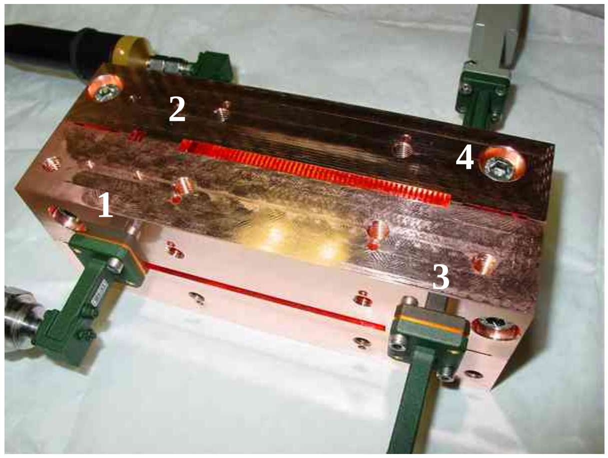

Figure 7: HDS60. Ports 3 and 4 are terminated. NA cables are connected to ports 1 and 2.

The measurements taken with the NA for the S_{11} and S_{44} parameters are shown in fig. 8. S_{11} and $S_{22,}$ are identical, due to the symmetry of the device. The same happens with S_{33} and $S_{44.}$ In figure 9, S_{21} and S_{34} are shown. By combining them as in eq. 8 and eq. 9, we can obtain the reflection coefficients both from the input and the output (figs. 10 and 11).

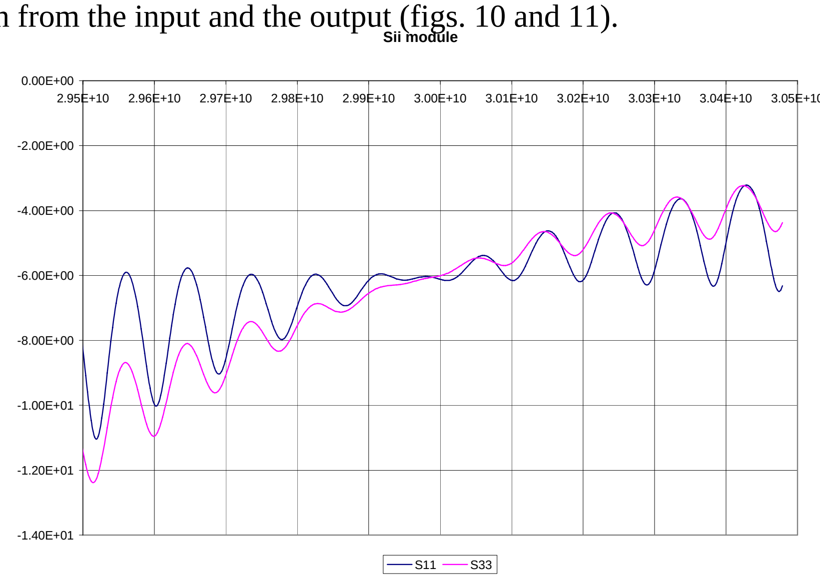

Figure 8: Network Analyzer measurements of the S_{11}, and S_{44} for the HDS60.

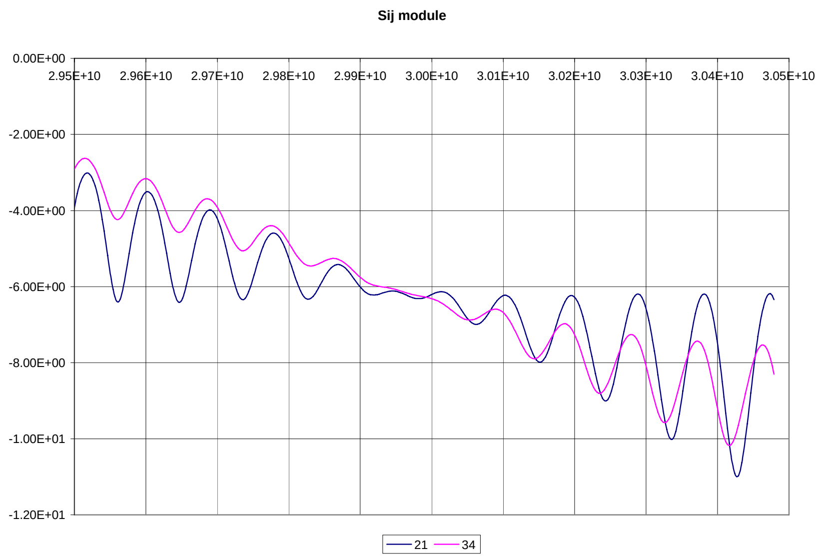

Figure 9: Network Analyzer measurements of the S_{21} and S_{34} for the HDS60.

Figure 10: Reflection from the input for the HDS60.

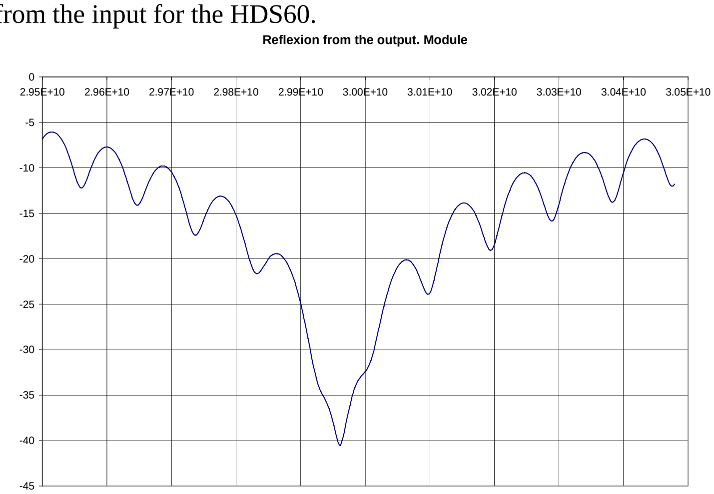

Figure 11: Reflection from the output for the HDS60.

In order to estimate the transmission coefficient of the structure, the S_{31} and S_{32} (or S_{41} and S_{42}, identical for symmetry reasons) parameters are needed (see eq. 20). The network analyzer measurements of these parameters are shown in fig. 12. The resulting transmission coefficient is shown in fig. 13. At our design frequency (29.979GHz), the measured value -1.98dB is very close to the analytical estimation of it, -1.965dB.

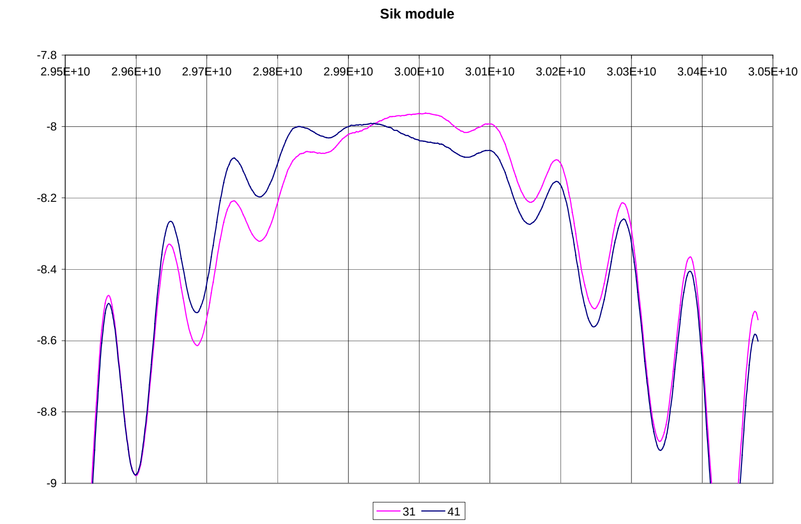

Figure 12: Network Analyzer measurements of the S_{31}, S_{32}, S_{41} and S_{42} for the HDS60.

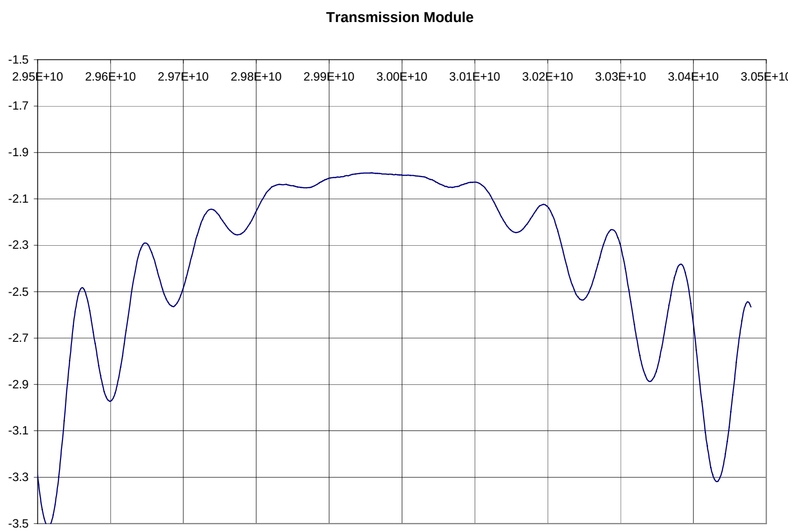

Figure 13: Transmission coefficient of the HDS60.

5. EXTENSION OF THE METHOD FOR AN N-PORT INPUT M-PORT OUTPUT SYMMETRIC DEVICE.

Let us consider now a device with n symmetric input ports (i=1..n) and m symmetric output ports (i=n+1..n+m), as in fig. 14. In order to obtain more general expressions, symmetry between input and output will not be assumed.

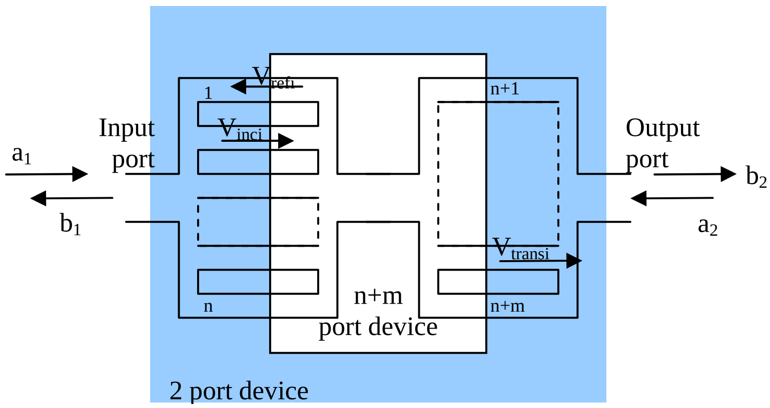

Figure 14: Graphical representation of a device with n input ports and m output ports. Identification of the signals need for the calculation of reflection and transmission coefficients.

We can easily extend the formulae from the previous sections to obtain general expressions for the reflection and transmission coefficients. Extending eq. 4 and 17,

$$\begin{aligned} V_{incIN} &= \sum_{i=1}^{n} V_{inci}, V_{incOUT} = \sum_{i=n+1}^{n+m} V_{inci} \\ V_{refIN} &= \sum_{i=1}^{n} V_{refi}, V_{refOUT} = \sum_{i=n+1}^{n+m} V_{refi} \\ V_{trans} &= \sum_{i=n+1}^{n+m} V_{transi} \end{aligned} \qquad \text{eq. 19}$$

5.1. The reflection coefficients

Symmetry between input and output is not assumed, so R_{in} and R_{out} can be different. We will first calculate the expression for R_{in} and then extend it for R_{out}.

Since the input ports are symmetric, V_{inci} and V_{refi}, i=1..n, are independent of the value of i and equal:

$$V_{inci} = \frac{V_{inc}}{n}$$

$$V_{refi} = \sum_{j=1}^{n} S_{ji} V_{incj} = \sum_{j=1}^{n} S_{ji} \frac{V_{inc}}{n} \qquad \text{eq. 20}$$

And the reflection coefficient

$$R_{in} = \left.\frac{V_{refIN}}{V_{incIN}}\right|_{a_{out}=0} = \frac{\sum_{i=1}^{n} V_{refi}}{V_{incIN}} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} S_{ji} \frac{V_{incIN}}{n}}{V_{incIN}} = \sum_{j=1}^{n} S_{ji} = S_{ii} + (n-1) S_{ji} \qquad \text{eq. 21}$$

where i can be any input port (between 1 and n) and j must be different from i but also an input port.

Analogically, the output reflection coefficient equals

$$R_{out} = S_{ii} + (m-1) S_{ji} \qquad \text{eq. 22}$$

where i can be any output port (between n+1 and n+m) and j must be different from i but also between an output port.

5.2. The transmission coefficient

Since the device is symmetric, V_{transi} (i=n+1..n+m), is independent of the value of i and equals:

$$V_{transi} = \sum_{j=1}^{n} S_{ji} V_{incINj} = \sum_{j=1}^{n} S_{ji} \frac{V_{incIN}}{n} \qquad \text{eq. 23}$$

And the transmission coefficient

$$T = \left.\frac{V_{trans}}{V_{inc}}\right|_{a_{out}=0} = \frac{\sum_{i=n+1}^{n+m} V_{transi}}{V_{incIN}} = \frac{\sum_{i=n+1}^{n+m}\sum_{j=1}^{n} S_{ji} \frac{V_{incIN}}{n}}{V_{incIN}} = \frac{m}{n}\sum_{j=1}^{n} S_{ji} = m S_{ji} \qquad \text{eq. 24}$$

where j is any input port and i any output port.

6. BEAD PULL OF A 4 PORT FEED STRUCTURE.

6.1. Description of the method.

The bead pull method is normally used in order to measure the cell to cell phase advance in an accelerating structure.

When using a 2 port feed structure the reflection coefficient is identical to the s_{21} parameter. We will therefore calculate the phase shift of this one directly from the network analyzer measurements.

When using a 4 port feed structure, we will first need to calculate either the input or output reflection coefficient R_{in} or R_{out} from the s parameters of the 4 port device, and then calculate its phase shift. For instance, we can connect the network analyzer as shown in fig. 2, obtaining s_{11}, s_{12} and s_{22}. The reflection coefficient for each position of the bead into the structure will be calculated as in eq. 10. Finally, the phase shift between the different reflection coefficients will be estimated.

6.2. An example: Bead pull of the HDS60 accelerating structure.

Fig. 15 shows the phase shift of the reflection coefficient from cell to cell of the HDS60 at the nominal frequency, calculated as described above. The nominal frequency shift from cell to cell is of 60 degrees (2pi/6) in the transmission coefficient, 120 degrees in the reflection coefficient.

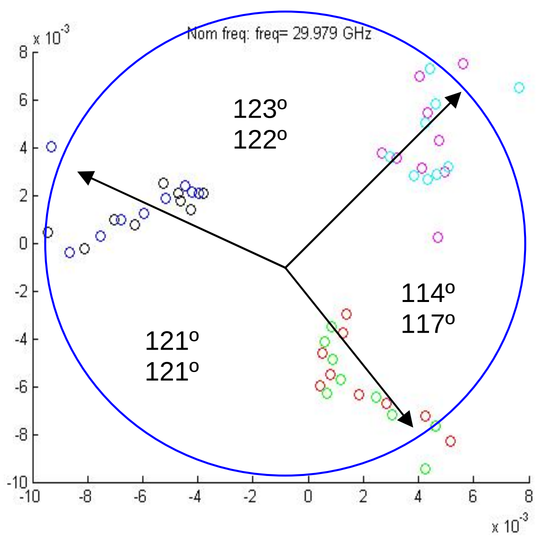

Figure 15: Deviation from the reference of the input reflection coefficient of the HDS60. Since it is a 2pi/6 structure, a 6 cell periodicity is expected. The 6 different colors correspond to this pattern.