Neutrinos from spin dynamics

Steven D. Bass

Particle Theory Unit, Physics Department, CERN, CH 1211 Geneve 23, Switzerland

Institute for Theoretical Physics,
Universitat Innsbruck, Technikerstrasse 25, Innsbruck, A 6020 Austria

Abstract

We conjecture that neutrino physics might correspond to the spontaneous magnetisation phase of an Ising-like spin model interaction coupled to neutrino chirality which operates at scales close to the Planck mass. We argue that this scenario leads to a simple extension of the Standard Model with no additional parameters that dynamically generates parity violation and spontaneous symmetry breaking for the gauge bosons which couple to the neutrino. The neutrino mass in the model is $m \quad {}^2_{ew}=2M$ where ${}_{ew}$ is the electroweak scale and M is the scale of the "spin-spin" interaction. For the ground state of the model the free energy density corresponding to the cosmological constant is ${}^{\frac{1}{4}} \quad m$, consistent with observation.

steven.bass@cern.ch

1 Introduction

The neutrino sector is one of the most puzzling in particle physics. Only left-handed neutrinos participate in weak interactions. Recent experiments have revealed oscillations between the three families of neutrino plus small neutrino masses, with the heaviest neutrino mass ~ 0.05 eV, much less than the masses of the charged leptons and quarks. The mass of the lightest neutrino is presently not well constrained although for a normal hierarchy of neutrino masses with $m_1 \ll m_2 \ll m_3$ one finds $m_1 \ll m_2 \sim 0.008$ eV — for recent reviews see Refs.[1, 2, 3, 4, 5]. Possible explanations for the small neutrino mass involve either right-handed sterile neutrinos (together with some additional "new physics" to suppress the mass relative to the charged leptons) or Majorana mass terms with local coupling to scalar Higgs fields using the see-saw mechanism [6] to push the mass of the right-handed neutrinos to a very high scale, thus connecting the neutrino sector with new physics at much higher mass scales.

It is interesting to ask whether there might be an alternative explanation: Can we construct a dynamical mechanism which yields observed neutrino physics ? Is there an analogue situation in other branches of physics that one might hope to learn from ?

Here we investigate a possible analogy with the Ising model of statistical mechanics and, more generally, spin glass. Suppose that we associate the "spins" in the Ising model with neutrino chirality and the "internal energy per spin" with the neutrino mass. The ground state of the Ising model exhibits spontaneous magnetisation where all the "spins" line up; the "internal energy per spin" and the free energy density of the spin system go to zero.

This observation suggests the possibility that, perhaps, neutrino physics might arise from collective spin model phenomena at a large scale, logarithmically close to the Planck mass. Electroweak interactions might be included as an "impurity" in the model: the $\nu \to eW$ process corresponds to a small but finite probability for one of the "spins" to turn off if the Ising interaction couples just to neutrinos. The spontaneous magnetisation phase then exhibits parity violation as the right-handed neutrino decouples from the physics. We argue that it also exhibits spontaneous breaking of the SU(2) gauge symmetry coupled to the neutrino. The neutrino mass m_ν in this picture is expected to be about $\Lambda_{ew}^2/2M$ where Λ_{ew} is the electroweak scale and M is the scale of the spin model interaction. The free energy density of the spin system behaves like a cosmological constant and for the low energy spontaneous magnetisation phase is $\lambda^{\frac{1}{4}} \sim m_\nu + O(kT)$, where $kT \sim 0.0002$ eV for the CMB temperature of free space.

2 Ising model dynamics

We first briefly outline the basics of the Ising model (for reviews see [7, 8]) and then discuss similarities and possible application to neutrino physics. The Ising model uses a spin lattice to study ferromagnetism for a spin system in thermal equilibrium. Applications include crystals, lattice gases and spin glass. One assigns a "spin" σ ($=\pm 1$) to each site and introduces a nearest neighbour "spin-spin" interaction. In two dimensions the Hamiltonian reads

$$H = -J \sum_{i,j} (\sigma_{i,j}\sigma_{i+1,j} + \sigma_{i,j+1}\sigma_{i,j}) - h \sum_{i,j} \sigma_{i,j}. \tag{1}$$

Here J is the bond energy and $h = \mu B$ where B denotes any external magnetic field and μ is the magnetic moment. (In this paper we take $h = \mu B = 0$.) One sums over the possible spins σ_{ij}. Physical observables are calculated through the partition function

$$Z = \sum_{\sigma_{ij}} \exp(-\beta H). \tag{2}$$

Here $\beta = 1/kT$ where k is Boltzmann's constant and T is the temperature; $k = 1.38 \times 10^{-23}\,JK^{-1} = 8.617 \times 10^{-5}\,eV\,K^{-1}$ and $kT|_{300K} = [38.68]^{-1}eV$. We can normalise the energy by adding a constant so that neighbouring parallel spins give zero contribution. Then, the only positive contribution to the energy will be from neighbouring disjoint spins of $2J$ and the probability for that will be $\exp(-2\beta J)$. Once a magnetisation direction is selected, it remains stable because of the infinite number of degrees of freedom in the thermodynamic limit.

The "internal energy per spin" corresponding to the Hamiltonian in Eq.(1) is

$$u(\beta J) = -2J\tanh(2\beta J) + K\frac{dK}{d\beta}\int_0^{\frac{\pi}{2}} d\theta\, \frac{\sin^2\theta}{\Delta(1+\Delta)} \tag{3}$$

where

$$K = \frac{2}{\cosh(2\beta J)\coth(2\beta J)} \tag{4}$$

and

$$\Delta = \sqrt{1 - K^2\sin^2\theta}. \tag{5}$$

The "free energy per spin" or free energy density for the system is

$$F(\beta) = -\frac{1}{\beta}\ln(2\cosh 2\beta J) + \frac{1}{2\pi}\int_0^{\frac{\pi}{2}} d\theta\, \ln\frac{1}{2}\left(1 + \sqrt{1 - K^2\sin^2\theta}\right). \tag{6}$$

The internal energy and the free energy are related through

$$\begin{aligned} u &= \frac{\partial}{\partial\beta}(\beta F) \\ &= F + TS \end{aligned} \tag{7}$$

where S is the entropy. The pressure for the spin system is $p = -F$. The Ising model has a second order phase transition and exhibits spontaneous magnetisation. In two dimensions the critical coupling $(\beta J)_c$ is determined through the equation $\{\sinh 2(\beta J)_c = 1\}$. For values of $(\beta J) \geq (\beta J)_c$ the magnetisation per spin is $\langle M \rangle$ where

$$\langle M \rangle = \left\{1 - [\sinh(2\beta J)]^{-4}\right\}^{\frac{1}{8}}: \tag{8}$$

The magnetisation vanishes for $(\beta J) < (\beta J)_c$. In the limit $\beta J \to \infty$, one finds

$$\epsilon(\beta J) = -2J - 24J \exp(-8\beta J) + \ldots \tag{9}$$

$$F = -2J + 3kT \exp(-8\beta J) + \ldots \tag{10}$$

and

$$\langle M \rangle = 1 - 2\exp(-8\beta J) + \ldots \tag{11}$$

As advertised above, the zero-point energy is then renormalised by adding $+2J$ to $\epsilon(\beta J)$ and F (or $2JN$ to the Hamiltonian where N is the number of spin sites) so that the "internal energy per spin" and free energy density vanish in the ground state with spontaneous magnetisation: $\epsilon(\infty) = 0$. In general, the critical coupling depends on the number of dimensions. For 1, 2, 3 and 4 Euclidean dimensions the critical coupling $(\beta J)_c$ is ∞, 0.441, 0.167 and 0.150 respectively.

One can extend the model to spin glasses [9] by introducing a probability distribution over the parameters of the model, e.g. the bond energies J and the magnetic moments.

3 Neutrinos

How can we construct an analogy with neutrinos and particle physics?

First, the Ising-like interaction itself must be non-gauged otherwise it will average to zero and there will be no spontaneous symmetry breaking and no spontaneous magnetisation (see e.g. page 51 of [7]).

Second, it is necessary to set a mass scale for J. If the Ising model analogy is to have connection with particle physics it is important to note that the coupling constant for the "spin-spin" interaction is proportional to the mass scale J. It therefore cannot correspond to a renormalisable interaction suggesting that fluctuations around the scale J occur only near the extreme high-energy or high-temperature limit of particle physics near the Planck mass M_P. We consider the effect of taking $J \sim +M_P$. The combination βJ is then very large making it almost certain that, if the analogy is applicable, the spontaneous magnetisation phase is the one relevant to particle physics phenomena. (Fermion generations in particle physics might be a further hint at some kind of spin related dynamics at a very high mass scale.) The exponential suppression factor $e^{-2\beta J}$ ensures that fluctuations associated with the Ising-like interaction are negligible, thus preserving renormalisability for all practical purposes.

Motivated by these observations, suppose we start with a gauge theory based on

$$SU(3) \otimes SU(2) \otimes U(1) \tag{12}$$

coupled to quarks and leptons with no chiral dependent couplings and unbroken local gauge invariance. We then turn on the non-gauged Ising interaction coupled just to the neutrino in the upper component of the SU(2) isodoublet with the coupling $J \gg M \gg \alpha_s; \alpha_{ew};$ (the QCD, SU(2) weak and QED couplings). It seems reasonable that the Ising interaction here exhibits the same two-phase picture with spontaneous magnetisation. Then, in the symmetric phase where $\beta J < (\beta J)_c$ the theory is symmetric under exchange of left and right handed neutrino chiralities and we have unbroken local gauge invariance. In the spontaneous magnetisation phase the neutrino vacuum is "spin"-polarised, a choice of chirality is made and the right-handed neutrino decouples from the physics. Parity is spontaneously broken and the gauge theory coupled to the leptons becomes

$$SU(2)_L \otimes U(1) \tag{13}$$

It is reasonable to believe that the SU(2) gauge symmetry coupled to the neutrino is now spontaneously broken. To understand how the $W^{\pm}$ and Z^0 gauge bosons might acquire mass, first consider the issue of confinement. Before we turn on the Ising interaction we have QCD quark and gluon confinement plus a vector SU(2) gauge theory with electron and neutrino confinement. Confinement is intimately connected with dynamical chiral symmetry breaking and the generation of Dirac mass terms for constituent quarks and (at this stage) analogue constituent leptons. (See Ref. [10] for a recent discussion in QCD [1] and Ref. [11] for an overview how this connection is implemented in the phenomenological Bag model of confinement.) Next turn on the Ising interaction and go to the spontaneous magnetisation phase. The right-handed neutrino decouples from the physics: the scalar chiral condensate for the neutrino "melts" and the confining solution for the neutrino should disappear. The left-handed neutrino will want to escape the confinement radius whereas the SU(2) gauge bosons will want to remain confined – in contradiction to the fundamental SU(2) symmetry. Some modification of the propagators must happen, viz. mass generation for the gauge bosons and the transition from the confinement to Higgs phases of the model so that the Coulomb force is replaced by a force of finite range

[1] Pure Yang-Mills theory and Yang-Mills theory coupled to fermions are both confining theories but the mechanism is different for each. Recent calculations [10] of quenched QCD in the covariant Landau gauge suggest that if chiral symmetry is restored, then the quark confinement solution disappears.

with finite mass scale and the issues associated with infrared slavery are avoided [2]. In this scenario experimental investigations of electroweak symmetry breaking will be probing fundamental properties of strong coupling dynamics. The $W^{\pm}$ and Z^0 gauge bosons which couple to the neutrino are massive and the QED photon and QCD gluons are massless.

What about the electroweak scale? Is there anything which stabilises it? First, in this scenario the SU(2) gauge symmetry coupled to the neutrino is dynamically broken. Second, dynamics associated with the Ising "spin" interaction are suppressed in the spontaneous magnetisation phase by the exponential factor $e^{-\beta M}$ where β is a finite inverse mass-scale like an inverse small-temperature factor or a mass scale typical of laboratory experiments, suggesting a natural hierarchy of scales. Further, the enormous excitation energy for right-handed neutrinos $\sim M$ reminds us of the infinite energy required to excite a free-quark in QCD because of confinement. The infinite free-quark excitation energy compares with the finite chiral symmetry breaking scale $F_\pi \sim 100$ MeV. Likewise, the electroweak mass-scale ~ 250 GeV is much less than the right-handed neutrino excitation energy.

What about finite neutrino masses? Weak interactions mean that we have two basic scales in the problem: $J \sim M$ and the electroweak scale μ_{ew} induced by spontaneous symmetry breaking. (Realistic accessible neutrino kinetic energies will be much less than $J \sim M$ and hence a minor correction to the total energy and Hamiltonian.) First, make the usual assumption that electroweak symmetry breaking generates Dirac mass terms for fermions which participate in electroweak interactions, e.g. as "impurities" in the "spin" system. Next, suppose that we approximate the two-phase spin-magnetisation system by left-handed and right-handed neutrinos with Majorana mass terms so that the right-handed neutrino appears only at scales $\sim O(M)$ and electroweak processes contribute a regular Dirac mass term. Then the "internal energies per spin" read in matrix form as

$$\sim M \quad \begin{bmatrix} 0 & \mu_{ew} \\ \mu_{ew} & 2M \end{bmatrix} \tag{14}$$

where the first row and first column refer to the left-handed states of the neutrino and the second row and second column refer to the right-handed states. Diagonalising this matrix for $M \gg \mu_{ew}$ gives the light mass eigenvalue

$$m_\nu \sim \mu_{ew}^2 / 2M \tag{15}$$

after the usual chiral rotation. Substituting the values of the Planck mass $M_{Pl} \sim \sqrt{\hbar c / G_N} \sim 1.2 \times 10^{19}$ GeV and the electroweak scale $\mu_{ew} \sim 250$ GeV into Eq.(15)

[2] A similar effect occurs in 1+1 dimensional gauge theories such as the Schwinger model coupled to dynamical fermions: confinement gives way to Higgs phenomena if the fermion mass is set exactly to zero [12].

gives $m \quad 3 \quad 10^{6}$ eV, which is plausible for the mass of the lightest neutrino and respectable given the simple approximations used above.

The matrix in Eq.(14) looks like the see-saw mechanism result [6] although the fundamental physics is quite different. In the see-saw picture the left-handed and right-handed components of a four-state Dirac neutrino are split by Majorana mass terms involving coupling to scalar Higgs fields into a pair of two-state Majorana neutrinos with different masses. The masses of these left-handed light and right-handed heavy Majorana neutrinos are related through $m \quad {}^{2}_{ew}=M_{D}$ where m is the mass of the light neutrino and M_{D} is both the "new physics" scale and the mass of the heavy neutrino. For a light neutrino mass $0{:}05$ eV this relation gives $M_{D} \quad 10^{15}$ GeV, which is not so far (on a logarithmic scale) from the Planck mass $M_{Pl} = 1{:}2 \quad 10^{19}$ GeV. In the two-phase picture suggested here the tiny mass for the neutrino originates from collective "spin dynamics" near the Planck scale instead of through local Yukawa couplings to elementary scalar Higgs fields. The connection to the see-saw matrix (14) suggests that, perhaps, the neutrino in this picture should be Majorana and therefore have no vector current: its electric charge should vanish.

It is interesting that the vacuum energy density corresponding to the cosmological constant $^{\frac{1}{4}} \quad 0{:}002$ eV [13] has a similar numerical value to the range of possible light neutrino masses [2], prompting the question whether related underlying dynamics might be at work ? In the Ising model the free energy density for the spin system (or energy available for work) is related to the internal energy per spin through Eq.(7): $= @(F)=@$. It follows that $F \quad {}^{2}_{ew}=2M + O(kT)$ where T is the temperature of the system, which for the present Universe is the CMB temperature 2.73K or $kT \quad 0{:}0002$ eV. The free energy density is suppressed by the "spin dynamics" which generate the spontaneous magnetisation and which "spin"-polarise the neutrino vacuum. The small finite value reflects the relatively small scalar component, $_{ew}=2M \; |scalar\rangle_{ew,qcd}$, associated with the electroweak and QCD scales which is induced in an otherwise "spin"-polarised total vacuum, viz.

$$|vacuum\rangle_{total} \quad (_{ew}=2M)\,|scalar\rangle_{ew,qcd} + \; |polarised\rangle \tag{16}$$

Electroweak and QCD interactions couple to the scalar condensates associated with the scales $_{ew}$ and $_{qcd}$ whereas the total vacuum in this picture is dominated by the "spin"-polarised neutrino component with zero free-energy density. Without the "spin-polarised" component the free-energy density would be $^{\frac{1}{4}} \quad _{ew}$. The positive sign for the free-energy density corresponds to negative pressure.

In conclusion, there are clear similarities between neutrino phenomenology and spontaneous magnetization in the Ising model. This suggests the conjecture that, perhaps, neutrinos are associated with the spontaneous magnetisation phase of a spin model interaction which operates at scales close to the Planck mass. The phase transition associated with the spontaneous magnetisation of neutrino chirality in the spin system would, in turn, lead to parity violation and, we have argued, also spontaneous breaking of the SU(2) gauge symmetry coupled to the neutrino. Further, there would be a rapid drop in the potential governing the effective neutrino mass or "energy per spin" from a value close to M at temperatures $kT \quad M$ to a value $\quad {}^2_{ew}/2M$ in the ground state which might be connected to the potential needed for inflation. There is no elementary scalar field in the model. It is interesting that the observed cosmological constant corresponds to a vacuum energy density $^{\frac{1}{4}}$ comparable with the expected value of the light neutrino mass. For the spin system the corresponding free energy density goes as $F \quad m + O(kT)$.

Acknowledgements

I thank the Austrian Science Fund for financial support (grant P17778-N08), the CERN particle theory unit for hospitality and T. Ericson, B. Kayser and G. Ross for helpful conversations.

References

[1] J.M. Conrad, hep-ex/9811009.

[2] G. Altarelli, Nucl. Phys. B (Proc. Suppl.) 143 (2005) 470, hep-ph/0410101.

[3] R.N. Mohapatra et al., hep-ph/0412099.

[4] B. Kayser, hep-ph/0506165.

[5] A. Blondel, Nucl. Phys. B (Proc. Suppl.) 155 (2006) 131, hep-ph/0601158.

[6] P. Minkowski, Phys. Lett. B 67 (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, edited by D. Freedman and P. van Nieuwenhuizen (North Holland, Amsterdam, 1979), p. 315; T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979); R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912 and Phys. Rev. D 23 (1981) 165.

[7] M. Creutz, Quarks, gluons and lattices (Cambridge U.P., 1983)

[8] B.M. McCoy and T.T. Wu, The two-dimensional Ising Model (Harvard University Press, 1973).

[9] S.F. Edwards and P.W. Anderson, J. Phys. F 5 (1975) 965.

[10] R. Alkofer, C.S. Fischer and F.J. Llanes-Estrada, hep-ph/0607293.

[11] A.W. Thomas, Adv. Nucl. Phys. 13 (1984) 1.

[12] D.J. Gross, I.R. Klebanov, A.V. Matytsin and A.V. Smilga, Nucl. Phys. B 461 (1996) 109.

[13] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1; V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D 9 (2000) 373; D.N. Spergel et al. (WMAP Collaboration), astro-ph/0603449.