View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by CERN Document Server



# From QCD to nuclear matter saturation

Magda Ericson [1,2,*)] and Guy Chanfray [1,**)]

[1] *Université de Lyon, Univ. Lyon 1, CNRS/IN2P3, IPN Lyon, F-69622 Villeurbanne Cedex*

[2] *Theory division, CERN, CH-12111 Geneva*

arXiv:nucl-th/0703022v1 9 Mar 2007

We discuss a relativistic chiral theory of nuclear matter with $\sigma$ and $\omega$ exchange using a formulation of the $\sigma$ model in which all the chiral constraints are automatically fulfilled. We establish a relation between the nuclear response to the scalar field and the QCD one which includes the nucleonic parts. It allows a comparison between nuclear and QCD information. Going beyond the mean field approach we introduce the effects of the pion loops supplemented by the short-range interaction. The corresponding Landau-Migdal parameters are taken from spin-isospin physics results. The parameters linked to the scalar meson exchange are extracted from lattice QCD results. These inputs lead to a reasonable description of the saturation properties, illustrating the link between QCD and nuclear physics. We also derive from the corresponding equation of state the density dependence of the quark condensate and of the QCD susceptibilities.

## §1. Introduction

The subject of this work is the interplay between nuclear physics and QCD and examples of what each field can bring to the other. Its motivations have been twofold. The first aim is to study QCD related quantities such as the quark condensate in the nuclear medium or the QCD scalar susceptibility which is its derivative with respect to the quark mass, in a way which is fully consistent with the saturation properties of nuclear matter. The second one is to build a relativistic theory of nuclear matter which satisfies all the chiral constraints. It is possible to find a common frame to reach this goal in effective theories. They are built to mimick QCD at low energy, allowing the study of QCD related quantities. Moreover a linear realization of chiral symmetry involves a scalar isoscalar field which can generate the nuclear attraction, *i.e.*, the scalar field of the $\sigma\omega$ model of Walecka-Serot.[1)] Our final conclusion will be that there exists a direct and model independent link between QCD quantities and the parameters which govern the attraction in the $\sigma\omega$ model. In the linear sigma model the explicit symmetry beaking part of the Lagrangian is $\mathcal{L}_{\chi SB} = c\,\sigma$ where $c = f_\pi\, m_\pi^2$ and $\sigma$ is the scalar field, chiral partner of the pion. This quantity plays the role of the symmetry breaking Lagrangian of QCD : $\mathcal{L}_{\chi SB}^{QCD} = -2\, m_q\, \bar{q}\, q$. We have then the equivalence :

$$\frac{\bar{q}q(x)}{\langle\bar{q}q\rangle_{vac}} = \frac{\sigma(x)}{f_\pi}. \tag{1·1}$$

*) e-mail address: magda.ericson@cern.ch

**) e-mail address: g.chanfray@ipnl.in2p3.fr

Thus it is the expectation value of the scalar field which controls the evalution of the condensate :

$$\frac{\langle\bar{q}q(\rho)\rangle}{\langle\bar{q}q\rangle_{vac}} = \frac{\langle\sigma(\rho)\rangle}{f_\pi} \tag{1·2}$$

and the sigma propagator (taken at zero momentum) which controls the susceptibility :

$$\chi_S = \int d^4x \, \langle\bar{q}q(x)\,\bar{q}q(0)\rangle = \frac{\langle\bar{q}q\rangle^2_{vac}}{f^2_\pi}\int d^4x\langle\sigma(x)\,\sigma(0)\rangle = \frac{\langle\bar{q}q\rangle^2_{vac}}{f^2_\pi}D_\sigma(0). \tag{1·3}$$

For the nuclear binding problem we use, as in ref.,[2] a non linear representation, keeping a chiral singlet scalar field which is the radius of the chiral circle, $S$. In the vacuum $S = f_\pi$, and we denote $s$ the fluctuation: $S = f_\pi + s$. In the usual non linear sigma model this fluctuation is ignored. We have shown previously, (ref.[2]) that the mean value of $s$ can be identified with the scalar mean field of the $\sigma\omega$ model. In this way, with the introduction of the non-linear representation, all chiral constraints are fulfilled. This would not be the case if the identification would be made instead with the non chiral invariant sigma field. The two fields $s$ and $\sigma$ are related through the transformation from polar to cartesian coordinates :

$$\sigma = (f_\pi + s)\cos F\left(\frac{\phi}{f_\pi}\right) \simeq f_\pi + s - \frac{\phi^2}{2f_\pi} \tag{1·4}$$

in which terms of order $s\,\phi^2$ have been ignored. The effective nucleon mass is influenced by the mean field $\bar{s}$ :

$$M^*_N(\bar{s}) = M_N\,\left(1 + \frac{\bar{s}}{f_\pi}\right)\,\simeq M_N\,-\,\frac{g^2_S}{m^2_\sigma}\,\rho_S. \tag{1·5}$$

Thus the nucleon mass and the condensate evolutions are linked but not proportional. The term which involves the scalar density of nuclear pions $\phi^2$ is absent in the mass evolution. Similarly the propagator of the $\sigma$ field, which gives the scalar susceptibility and the one of the $s$ field are linked but not identical, with (ref.[3]) :

$$D_\sigma = D_s\,+\,\frac{3}{2\,f^2_\pi}G \tag{1·6}$$

where G is the two-pion propagator. At low momenta the $s$ field is not coupled to two pions in contradistinction with the $\sigma$ one which is strongly coupled (said differently the $s$ field, which is at the origin of the nuclear binding, just decouples from low energy pions the dynamics of which is described by chiral perturbation theory).

In this framework a joint study of the nuclear binding and of the QCD quantities is natural and the compatibility between the two is insured. The question is wether something more can be learned from this common study. In a first step we ignore the pion loops. In this case the condensate evolution is simply given by :

$$\frac{\Delta\langle\bar{q}q(\rho)\rangle}{\langle\bar{q}q\rangle_{vac}} = \frac{\bar{s}}{f_\pi} \tag{1·7}$$

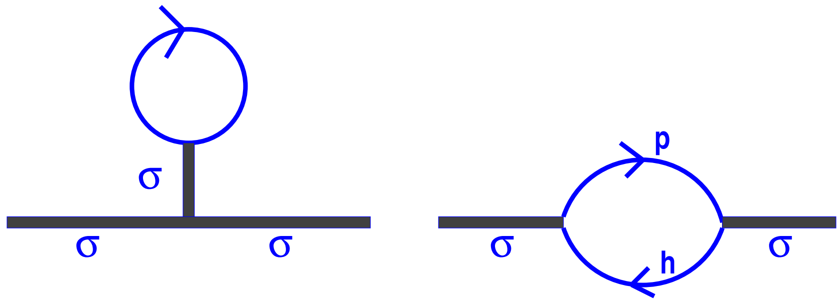


Fig. 1. Sigma self-energy in the nuclear medium.

where the symbol $\Delta$ for the condensate stands for the modification with respect to the vacuum value. Similarly the nuclar susceptibility, vacuum value subtracted, is :

$$\chi_S^A \; = 2\,\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^2}(D_\sigma^* - D_\sigma^0). \tag{1·8}$$

Here $D_\sigma^*$ is the full in-medium sigma propagator while $D_\sigma^0$ is the vacuum one. Both are taken at zero momentum and they are related by :

$$-\,(D_\sigma^* - D_\sigma^0) = \frac{1}{m_\sigma^{*2}} - \frac{1}{m_\sigma^2} \;-\; \frac{g_S^2}{m_\sigma^{*2}}\,\Pi_S(0)\,\frac{1}{m_\sigma^{*2}}. \tag{1·9}$$

In this expression, $g_S$ is the scalar coupling constant and $\Pi_S$ is the full RPA particle-hole polarization propagator in the scalar-isoscalar channel. The mass $m_\sigma^*$ is the effective sigma mass which differs from the free one mostly by the effect of the tadpole term :

$$m_\sigma^{*2} = m_\sigma^2\left(1 + \frac{3\bar{s}}{f_\pi} + \frac{3}{2}\left(\frac{\bar{s}}{f_\pi}\right)^2\right) \simeq m_\sigma^2 - \frac{3\,g_S}{f_\pi}\rho_S \tag{1·10}$$

This represents a large decrease of the mass $\simeq 30\%$ at $\rho_0$. The tadpole contribution and the effect of the p.h. polarization propagator are illustrated in fig. 1. The first part corresponds to the $\sigma$ self-energy from the following $\sigma N$ (non Born) amplitude, $T_{\sigma N} = -3g_s/f_\pi$, while the second part arises from the in medium modified (Pauli blocked) $\sigma N$ Born amplitude. Inserting the expression (1·9) of the sigma propagator into that, eq. (1·8), of the susceptibility we obtain to lowest order in density :

$$\chi_S^A \;\simeq\; 2\,\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^2\,m_\sigma^4}\left[-\frac{3\,g_S\,\rho_S}{f_\pi} \;+\; g_S^2\,\Pi_S(0)\right]. \tag{1·11}$$

In this expression the first term linear in the density embeds the scalar susceptibility of the individual nucleons while the second term reflects the effect of the nuclear excitations which affect the QCD susceptibility through the coupling of the quark density fluctuations to the nucleonic ones.[4] All in all the parenthesis on the r.h.s. of eq. (1·11) is the total nuclear response to the nuclear scalar field, $\mathcal{R}^A$. The proportionnality factor between $\mathcal{R}^A$ and $\chi_S^A$ can be expressed in terms of the scalar quark number of the nucleon arising from the $\sigma$ field :

$$Q_S^s = \frac{\sigma_N^s}{2\,m_q} = -\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\int d^3x\langle N|\sigma(x)|N\rangle = -\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\,\frac{g_S}{m_\sigma^2} \qquad (1\cdot12)$$

such that :

$$\chi_S^A = 2\,\frac{(Q_S^s)^2}{g_S^2}\,\mathcal{R}^A. \qquad (1\cdot13)$$

It is remarkable that the full response is reflected in the QCD one (and vice versa), including the nucleon part.[6)]

Coming back to eq. (1·11) the term linear in the density provides the QCD scalar susceptibility of the nucleon from the scalar field :

$$(\chi_S^N)^s \;=\; -2\,\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^3}\,\frac{3\,g_S}{m_\sigma^4}. \qquad (1\cdot14)$$

This component, which to our knowledge has not been signalled before, is an example of a new information that the nuclear problem can bring. The question is wether this component has a reality. In the model certainly, since it can be also be obtained as the derivative of $Q_S^s$ with respect to the quark mass. And in reality? There are indications in favor of a negative component (beside the pionic one discussed in ref.[5)]) in the lattice results on the evolution of the nucleon mass with the quark mass, (equivalently the pion squared mass). They are available only above $m_\pi \simeq 400\,MeV$ and in order to extract the physical nucleon mass an extrapolation has to be performed. Thomas et al.[7)] have separated out the pion which introduces a non analytical behavior in $m_q$. For this they evaluate the pionic part of the self-energy, $\Sigma_\pi$, introducing different shapes of the form factor at the $\pi NN$ vertex with an adjustable cut-off parameter. They expand the rest of the nucleon mass in powers of $m_\pi^2$ as follows :

$$M_N(m_\pi^2) = a_0 \,+\, a_2\,m_\pi^2 \,+\, a_4\,m_\pi^4 \,+\, \Sigma_\pi(m_\pi). \qquad (1\cdot15)$$

The parameters $a_i$ show little sensitivity to the shape of the form factor, with values $a_2 \simeq 1.5\,GeV^{-1}$ while $a_4 \simeq -0.5\,GeV^{-3}$, (see ref.[7)]). From this we can infer the non-pionic pieces of the sigma commutator using the Feynman-Hellman theorem :

$$\sigma_N^{non-pion} = m_\pi^2\,\frac{\partial M}{\partial m_\pi^2} = a_2\,m_\pi^2 \,+\, 2\,a_4\,m_\pi^4 \simeq 29\,MeV\,. \qquad (1\cdot16)$$

It is dominated by the $a_2$ term. Its value indicates the existence of a large component in $\sigma_N$ beside the pion one, that it is natural to attribute to the scalar field. The next derivative provides the non pionic susceptibility :

$$\chi_{NS}^{non-pion} \;= 2\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\,\frac{\partial}{\partial m_\pi^2}\left(\frac{\sigma_N^{non-pion}}{m_\pi^2}\right) = \frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\,4\,a_4. \qquad (1\cdot17)$$

It has a negative sign, as expected from the scalar contribution. If the signs are right, are the numerical values also compatible with the sigma model? If we identify the two quantities, $\sigma_N^{non-pion}$ and $\chi_{NS}^{non-pion}$ with the sigma model values, we first have :

$$\sigma_N^{non-pion} \simeq a_2\, m_\pi^2 \;= \sigma_N^s = f_\pi m_\pi^2\, \frac{g_S}{m_\sigma^2} \tag{1·18}$$

which leads to $m_\sigma = 800\, MeV$ (for $g_S = M_N/f_\pi$). On the other hand the identification of the two expressions 1·14 and 1·17 for $\chi_{NS}^{non-pion}$ and the elimination of $m_\sigma$ using 1·18, leads to the relation :

$$a_4 \simeq -\frac{3}{2M} a_2^2 \simeq -3.4\, GeV^{-3} \tag{1·19}$$

while the value found in the expansion is only $-0.5\, GeV^{-3}$. The model as such fails to pass the QCD test. In fact this is to be expected and even gratifying because it also fails the nuclear physics test. Indeed the effect of the tadpole term on the $\sigma$ propagator is too large. The softening of the sigma mass makes nuclear matter collapse and prevents saturation.[8)] We have seen that the QCD susceptibility and the $\sigma N$ amplitude are related. Both values are too large in magnitude in the model which is incomplete and has to be improved. Indeed an important effect is missing, namely the scalar response of the nucleon, $\kappa_{NS}$, to the scalar nuclear field, which is the basis of the quark-meson coupling model, (QMC), introduced in ref.[9)] The crucial point is that its origin lies in confinement in such a way that its sign is positive, *i.e,*, it opposes an increase of the scalar field. In QMC the response is calculated in a pure bag model. The quantity $\kappa_{NS}$ is then related to the QCD scalar susceptibility of the bag through a relation similar to ours (1·13) but in which only bag quantities enter. The positive sign follows as the confined quarks become less relativistic with increasing quark mass and their scalar number increases. In order to incorporate all aspects, it will be interesting to explore the full nucleonic response to the scalar field in a model of the nucleon, such as the one of Shen and Toki,[10)] which incorporates both aspects, *i.e.*, where the nucleon mass originates from both the coupling to the condensate and the confinement.

It is possible[11)] to improve our previous model described above with the phenomenological introduction of the scalar nucleon response, $\kappa_{NS}$. It modifies the nucleon mass evolution as follows,

$$M_N^*(\bar{s}) = M_N\, \left(1 + \frac{\bar{s}}{f_\pi}\right) + \frac{1}{2}\, \kappa_{NS}\, \bar{s}^2. \tag{1·20}$$

The sigma mass is also affected :

$$m_\sigma^{*2} \simeq m_\sigma^2 - (\frac{3\, g_S}{f_\pi} \,-\, \kappa_{NS})\, \rho_S, \tag{1·21}$$

and the nucleonic QCD susceptibility as well, in such a way that the relation between $a_4$ and $a_2$ becomes :

$$a_4 = -\frac{a_2^2}{2\,M}(3 \,-\, 2\,C). \tag{1·22}$$

where $C$ is the dimensionless parameter $C \,=\, \left(f_\pi^2/2\,M\right)\kappa_{NS}$. Numerically $a_4 \,=\, -0.5\, GeV^{-3}$ gives $C \,=\, +1.25$. A large cancellation of the tadpole effect is indeed required by the lattice expansion, total cancellation occuring for $C = 1.5$. Our approach has then consisted in going from QCD to nuclear physics and use the lattice expansion to fix the parameters of the $\sigma\omega$ model.[12] The question is wether this procedure makes sense and wether it leads to a possible description of the nuclear binding. We have explored this, first in the mean field approach, but also with the introduction of the pion loops,[12] as is described below. In the latter case the condensate acquires a new component linked to the nuclear pion scalar density, $\langle \phi^2 \rangle$. However the nucleon mass is blind to this component.[2] This is also true for the sigma mass and the eq. (1·20) and (1·21) remain valid. Pion loops nevertheless affect the energy. They do not enter at the mean field level but contribute through the Fock term and through the correlation energy schematically depicted in fig. 2. It includes iterated pion exchange and also the part of the NN potential from the two-pion exchange with $\Delta$ excitation. In the correlation term we have introduced beside pion exchange the short-range components *via* the Landau-Migdal parameters, $g'_{NN}$, $g'_{N\Delta}$, $g'_{\Delta\Delta}$. We have taken their values from a systematic survey of the data on spin-isospin physics by Ichimura *et al.*[13] For the parameters $g'$ they indicate large deviations from universality with : $g'_{NN} = 0.7$, $g'_{N\Delta} = 0.3$, $g'_{\Delta\Delta} = 0.5$. Beside pion exchange we have also introduced the transverse channel, dominated by $\rho$ exchange together with the short-range component. The other parameters of the model are chosen as follows. The form factor at the $\pi NN$ vertex is taken as a dipole with a cutoff parameter $\Lambda = 7\, m_\pi$, which is guided by the following considerations : it leads to a pionic contribution to $\sigma_N$ of $22\, MeV$. Adding the lattice value of the non-pionic part, $\sigma^{non-pion} = 29\, MeV$, the sum takes the value $\sigma_N = 51\, MeV$, in the well accepted range. The scalar coupling constant is the one of the model $g_S = M/f_\pi = 10$ and for the sigma mass we have followed the lattice indications, allowing a small readjustment around the lattice value, which is $m_\sigma = 800\, MeV$. We have found a better fit with $m_\sigma = 850\, MeV$ which corresponds to $\sigma^{non-pion} = 26\, MeV$. The omega mass is known and the $\omega NN$ coupling constant totally free. For the nucleon scalar response we have followed the indications of the lattice data but not strictly in view of the uncertainties attached to the higher derivatives. The value which fits the saturation properties is found to be $C \simeq 1$, not far from the lattice value $C = 1.25$. With these inputs we have obtained a satisfactory description of the nuclear binding. The binding energy per particle is shown in fig. 3, with its different components. We like to comment on the correlation energy which has a value of $-17.4\, MeV$ . The longitudinal channel is strongly suppressed by the short-range component, (with less cancellation in its $N\Delta$ component). This leaves the transverse part as the dominant contribution ($-9.5\, MeV$ *versus* $-7.9\, MeV$ for the longitudinal one). From the equation of state and using the Feynman-Hellmann theorem we have deduced the quark condensate. The next derivative with respect to $m_q$ provides the scalar susceptibility. The two QCD susceptibilities, scalar and pseudoscalar are depicted in fig 4 . The second one follows the evolution of the condensate[4] and it is remarkably linear in the density in spite of the interaction. The scalar one shows a strong increase with respect to the free value, surpassing even the pseudoscalar one around $2\rho_0$. This behavior is largely due to the mixing with the nuclear excitations.

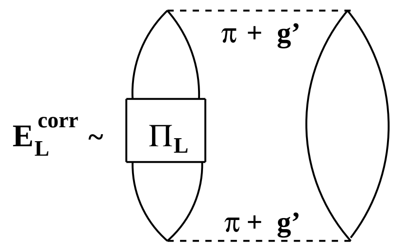


Fig. 2. Schematic representation of the longitudinal spin-isospin contribution to the correlation energy.

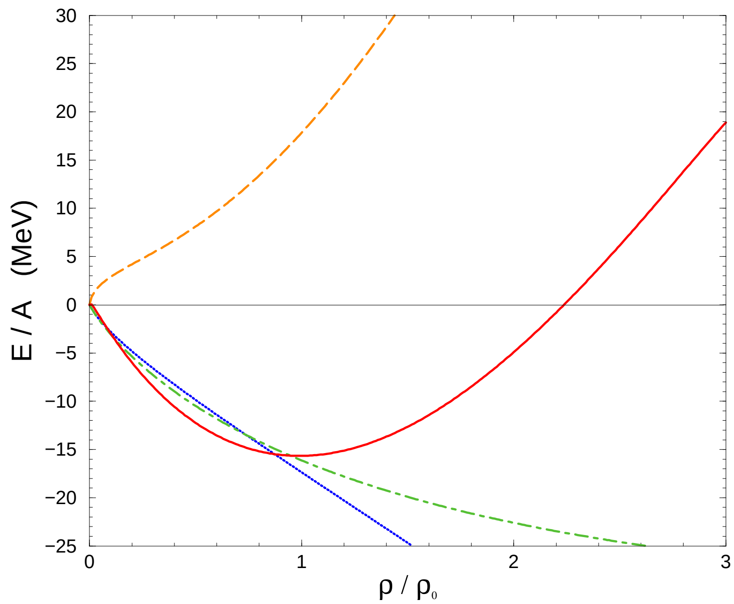


Fig. 3. Binding energy of nuclear matter with $g_\omega = 8.0$, $m_\sigma = 850\,MeV$ and $C = 0.985$. The full line corresponds to the full result, the dotted line represents the binding energy without the Fock and correlation energies and the dot-dahed line corresponds to the contribution of the Fock terms. The decreasing dotted line (always negative) represents the correlation energy.

In summary we have studied a chiral relativistic theory of nuclear matter based on the sigma model and its interplay with QCD. We have shown that the softening of the sigma mass arising from the tadpole term, (from $3\sigma$ interaction) has a counterpart in QCD in the form of a negative contribution to the nucleonic QCD scalar susceptibility from the scalar field. We have investigated the presence of this component in the lattice expansion of the nucleon mass with respect to the quark mass. There is indeed an indication in favor of a negative component but its magnitude is much too small. This is in fact totally consistent with nuclear physics. The only effect of the tadpole with the softening of the $\sigma$ mass prevents saturation. In both cases a canceling effect must occur. For this we have introduced, as in QMC, the scalar response of the nucleon which is a reflect of the QCD scalar susceptibility of the nucleon which is due to confinement. Our approach has then be to utilize the QCD information on the nucleon mass evolution with the quark mass to fix or

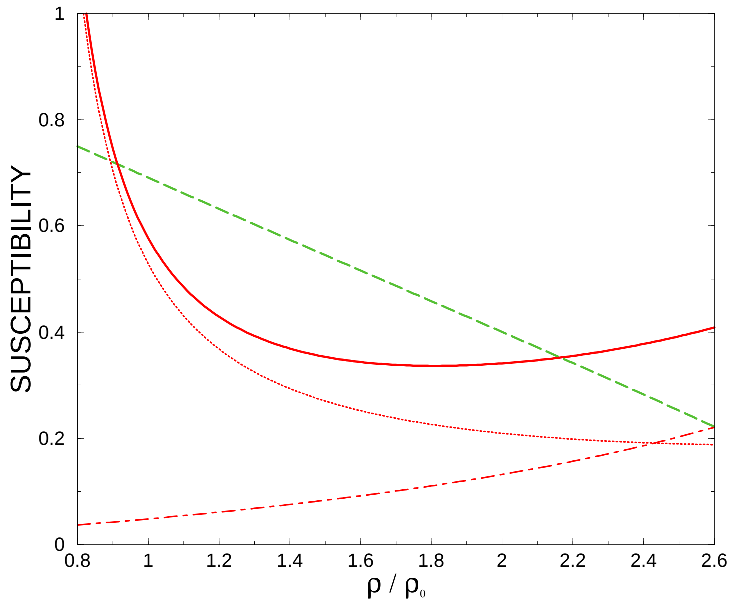


Fig. 4. Density evolution of the QCD susceptibilities (normalized to the vacuum value of the pseudoscalar one) with $g_\omega = 8.$, $m_\sigma = 850\, MeV$ and $C = 0.985$. Dashed curve: pseudoscalar susceptibility. Full curve: Scalar susceptibility. Dotted curve: nuclear contribution to the scalar susceptibility. Dot-dashed curve: pion loop contribution to the scalar susceptibility.

constrain the parameters of the $\sigma\omega$ model. This, together with the information from spin-isospin physics, has allowed a successfull description of the binding properties of nuclear matter.

## Acknowledgments

One of the authors (M. E.) thanks the Yukawa Institute for Theoretical Physics at Kyoto University, for the hospitality during the YKIS2006 on "New Frontiers on QCD".

### References

1) B.D. Serot, J.D. Walecka, Adv. Nucl. Phys. 16(1986) 1; Int. J. Mod. Phys. E16 (1997) 15.
2) G. Chanfray, M. Ericson and P.A.M. Guichon, Phys. Rev C63 (2001)055202.
3) G. Chanfray, D. Davesne, M. Ericson and M. Martini, EPJA 27 (2006) 191.
4) G. Chanfray and M. Ericson, EPJA 16 (2003) 291.
5) G. Chanfray, M. Ericson and P.A.M. Guichon, Phys. Rev C61 (2003) 035209.
6) G. Chanfray and M. Ericson, nucl-th/0606057
7) A. W. Thomas, P. A. M. Guichon, D. B. Leinweber and R. D. Young,Progr. Theor. Phys. Suppl. 156 (2004) 124; nucl-th/0411014.
8) A.K. Kerman and L.D. Miller in "Second High Energy Heavy Ion Summer Study", LBL-3675, 1974.
9) P.A.M. Guichon, Phys. Lett. B200 (1988) 235.
10) H. Shen and H. Toki, Phys.Rev. C61 (2000) 045205.
11) G. Chanfray and M. Ericson, EPJA 25 (2005) 151.
12) G. Chanfray and M. Ericson, Phys.Rev. C75 (2007) 015206, , nucl-th/0611042.
13) M. Ichimura, H. Sakai and T. Wakasa, Prog. Part. Nucl. Phys., 56, 446 (2006).