

Next-to-Leading-Order QCD Corrections to $t\bar{t}$ + jet Production at Hadron Colliders

S. Dittmaier,¹ P. Uwer,² and S. Weinzierl³

¹Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), D-80805 München, Germany

²CERN, CH-1211 Geneva 23, Switzerland

³Universität Mainz, D-55099 Mainz, Germany

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We report on the calculation of the next-to-leading-order QCD corrections to the production of top-quark–top-antiquark pairs in association with a hard jet at the Fermilab Tevatron and the CERN Large Hadron Collider. We present results for the $t\bar{t}$ + jet cross section and the forward-backward charge asymmetry. The corrections stabilize the leading-order prediction for the cross section. The charge asymmetry receives large corrections.

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Introduction.—More than ten years after its discovery, the dynamics and many properties of the top quark, such as its electroweak quantum numbers, are not yet precisely measured. Since the top quark is the heaviest of the known elementary particles, it is widely believed that it plays a key role in extensions of the standard model. This renders experimental investigations of the top quark particularly important. Up to now the main (direct) source of information on top quarks are top-quark pairs produced at the Tevatron. Only recently first evidence for single-top production has been found [1]. It is important to note that in the inclusive $t\bar{t}$ sample a significant fraction comprises $t\bar{t}$ + jet events. An investigation of the process of $t\bar{t}$ production in association with a hard jet can, thus, certainly improve our knowledge about the top quark.

In this context, the forward-backward charge asymmetry of the top quark (or top antiquark) [2–5] is of particular interest. In inclusive $t\bar{t}$ production it appears first at one loop, because it results from interferences of C -odd with C -even parts of double-gluon exchange between initial and final states. This means that the available prediction for $t\bar{t}$ production—although of one-loop order—describes this asymmetry only at leading-order (LO) accuracy. In $t\bar{t}$ + jet production the asymmetry appears already in LO. Thus, the next-to-leading-order (NLO) calculation described in the following provides a true NLO prediction for the asymmetry. Our calculation will, therefore, be an important tool in the experimental analysis of this observable at the Tevatron where the asymmetry is measurable as discussed in Ref. [5].

Measuring the cross section of the related process of $t\bar{t}$ + γ production provides direct access to the electric charge of the top quark. Obviously NLO QCD predictions to this process are important for a reliable analysis. They can be obtained from $t\bar{t}$ + jet production presented in this Letter via simple substitutions. We postpone this, however, to a forthcoming publication.

Finally, a signature of $t\bar{t}$ in association with a hard jet represents an important background process for searches at

the LHC, such as the search for the Higgs boson in the weak-vector-boson fusion or $t\bar{t}H$ channels.

The above-mentioned issues clearly underline the case for an NLO calculation for $t\bar{t}$ + jet production at hadron colliders. We report here on a first calculation of this kind; a status report was already given in Ref. [6].

Details of the NLO calculation.—At LO, hadronic $t\bar{t}$ + jet production receives contributions from the partonic processes $q\bar{q} \rightarrow t\bar{t}g$, $qg \rightarrow t\bar{t}q$, $\bar{q}g \rightarrow t\bar{t}\bar{q}$, and $gg \rightarrow t\bar{t}g$. The first three channels are related by crossing symmetry to the amplitude $0 \rightarrow t\bar{t}q\bar{q}g$.

Evaluating $2 \rightarrow 3$ particle processes at the NLO level is nontrivial, both in the analytical and numerical parts of the calculation. In order to prove the correctness of our results we have evaluated each ingredient twice using independent calculations based—as far as possible—on different methods, yielding results in mutual agreement.

Virtual corrections.—The virtual corrections modify the partonic processes that are already present at LO. At NLO these corrections are induced by self-energy, vertex, box (4-point), and pentagon (5-point) corrections. The prototypes of the pentagon graphs, which are the most complicated diagrams, are shown in Fig. 1.

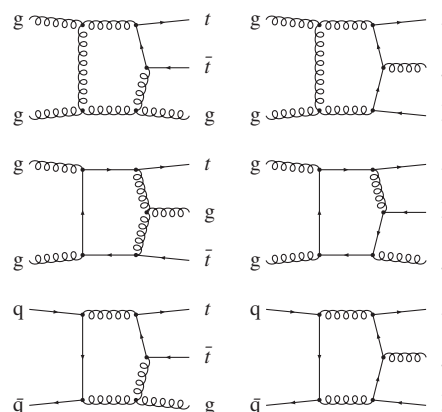


FIG. 1. A representative set of pentagon diagrams for hadronic $t\bar{t}$ + jet production at NLO QCD.

Version 1 of the virtual corrections is essentially obtained following the method described in Ref. [7], where $t\bar{t}H$ production at hadron colliders was considered. Feynman diagrams and amplitudes have been generated with the FEYNARTS package [8,9] and further processed with in-house MATHEMATICA routines, which automatically create an output in FORTRAN. The IR (soft and collinear) singularities are analytically separated from the finite remainder as described in Refs. [7,10]. The tensor integrals appearing in the pentagon diagrams are directly reduced to box integrals following Ref. [11]. This method does not introduce inverse Gram determinants in this step, thereby avoiding notorious numerical instabilities in regions where these determinants become small. Box and lower-point integrals are reduced in the manner of Passarino and Veltman [12] to scalar integrals, which are either calculated analytically or using the results of Refs. [13–15]. Sufficient numerical stability is already achieved in this way. Nevertheless the integral evaluation is currently further refined by employing the more sophisticated methods described in Ref. [16] in order to numerically stabilize the tensor integrals in exceptional phase-space regions.

Version 2 of the evaluation of loop diagrams starts with the generation of diagrams and amplitudes via QGRAF [17], which are then further manipulated with FORM [18] and eventually automatically translated into C++ code. The reduction of the 5-point tensor integrals to scalar integrals is performed with an extension of the method described in Ref. [19]. In this procedure also inverse Gram determinants of four four-momenta are avoided. The lower-point tensor integrals are reduced using an independent implementation of the Passarino-Veltman procedure. The IR-finite scalar integrals are evaluated using the FF package [20,21]. Although the entire procedure is sufficiently stable, further numerical stabilization of the tensor reduction is planned following the expansion techniques suggested in Ref. [22] for exceptional phase-space regions.

Real corrections.—The matrix elements for the real corrections are given by $0 \rightarrow t\bar{t}g g g g$, $0 \rightarrow t\bar{t}q\bar{q}g g$, $0 \rightarrow t\bar{t}q\bar{q}q'\bar{q}'$, and $0 \rightarrow t\bar{t}q\bar{q}q\bar{q}$. The various partonic processes are obtained from these matrix elements by all possible crossings of light particles into the initial state.

The evaluation of the real-emission amplitudes is performed in two independent ways. Both evaluations employ (independent implementations of) the dipole subtraction formalism [23–25] for the extraction of IR singularities and for their combination with the virtual corrections.

Version 1 results from a fully automated calculation based on helicity amplitudes, as described in Ref. [26]. Individual helicity amplitudes are computed with the help of Berends-Giele [27] recurrence relations. The evaluation of color factors and the generation of subtraction terms is automated. For the channel $g g \rightarrow t\bar{t}g g$ a dedicated soft-insertion routine [28] is used for the generation of the phase space.

Version 2 uses for the LO $2 \rightarrow 3$ processes and the $g g \rightarrow t\bar{t}g g$ process optimized code obtained from a Feynman diagrammatic approach. As in version 1, standard techniques like color decomposition and the use of helicity amplitudes are employed. For the $2 \rightarrow 4$ processes including light quarks, MADGRAPH [29] has been used. The subtraction terms according to Ref. [25] are obtained in a semiautomatized manner based on an in-house library written in C++.

Numerical results.—In the following we consistently use the CTEQ6 [30,31] set of parton distribution functions (PDFs). In detail, we take CTEQ6L1 PDFs with a 1-loop running α_s in LO and CTEQ6M PDFs with a 2-loop running α_s in NLO. The number of active flavors is $N_F = 5$, and the respective QCD parameters are $\Lambda_5^{\text{LO}} = 165$ MeV and $\Lambda_5^{\overline{\text{MS}}} = 226$ MeV. Note that the top-quark loop in the gluon self-energy is subtracted at zero momentum. In this scheme the running of α_s is generated solely by the contributions of the light quark and gluon loops. The top-quark mass is renormalized in the on-shell scheme; as the numerical value we take $m_t = 174$ GeV.

We apply the jet algorithm of Ref. [32] with $R = 1$ for the definition of the tagged hard jet and require a transverse momentum of $p_{T,\text{jet}} > 20$ GeV for the hardest jet [33]. The outgoing (anti-)top quarks are neither affected by the jet algorithm nor by the phase-space cut. Note that the LO prediction and the virtual corrections are not influenced by the jet algorithm, but the real corrections are.

Figure 2 shows the dependence of the integrated LO cross sections on the renormalization and factorization scales, which are identified here ($\mu = \mu_{\text{ren}} = \mu_{\text{fact}}$). The variation ranges from $\mu = 0.1m_t$ to $\mu = 10m_t$. The dependence is rather large, illustrating the well-known fact that the LO predictions can only provide a rough estimate. At the Tevatron the $q\bar{q}$ channel dominates the total $p\bar{p}$ cross section by about 85%, followed by the $g g$ channel with about 7%. At the LHC, the $g g$ channel comprises about 70%, followed by $q g$ with about 22%.

In Fig. 3 the scale dependence of the NLO cross sections is shown. For comparison, the LO results are included as well. As expected, the NLO corrections significantly reduce the scale dependence. We observe that around $\mu \approx m_t$ the NLO corrections are of moderate size for the chosen setup.

Finally, we have performed a first study of the forward-backward charge asymmetry of the top quark at the Tevatron. In LO the asymmetry is defined by

$$A'_{\text{FB,LO}} = \frac{\sigma_{\text{LO}}^-}{\sigma_{\text{LO}}^+}, \quad \sigma_{\text{LO}}^\pm = \sigma_{\text{LO}}(y_t > 0) \pm \sigma_{\text{LO}}(y_t < 0), \quad (1)$$

where y_t denotes the rapidity of the top quark. Cross section contributions $\sigma(y_t \geq 0)$ correspond to top quarks in the forward or backward hemispheres, respectively, where incoming protons fly into the forward direction by

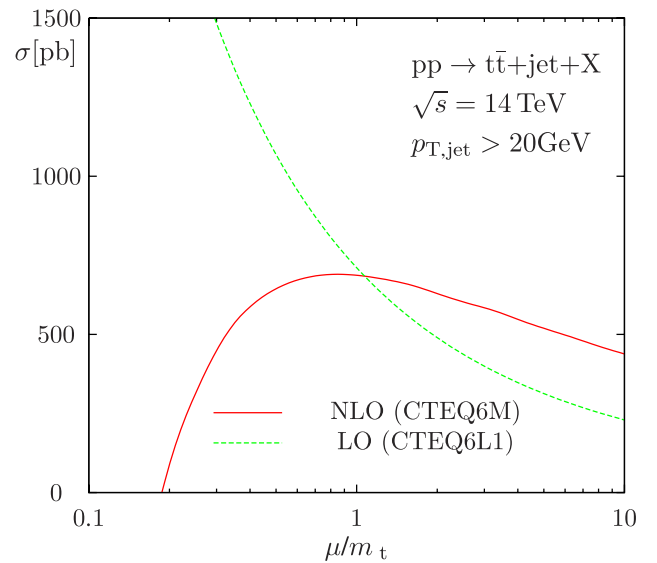
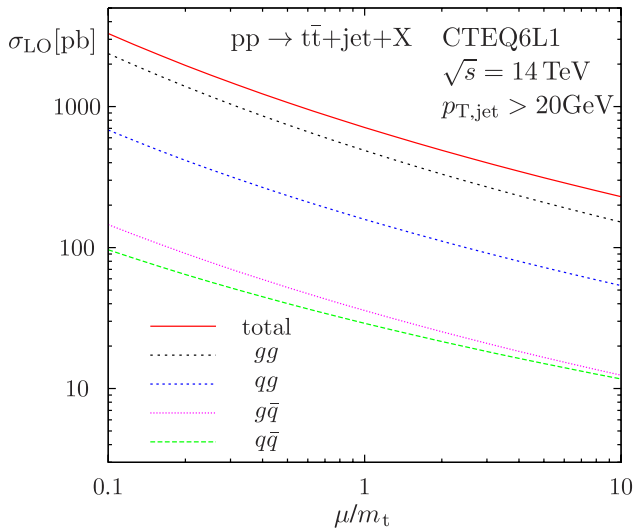
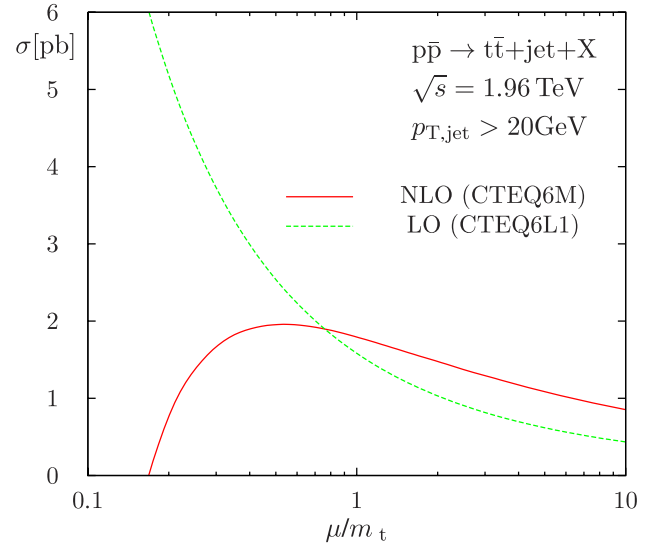
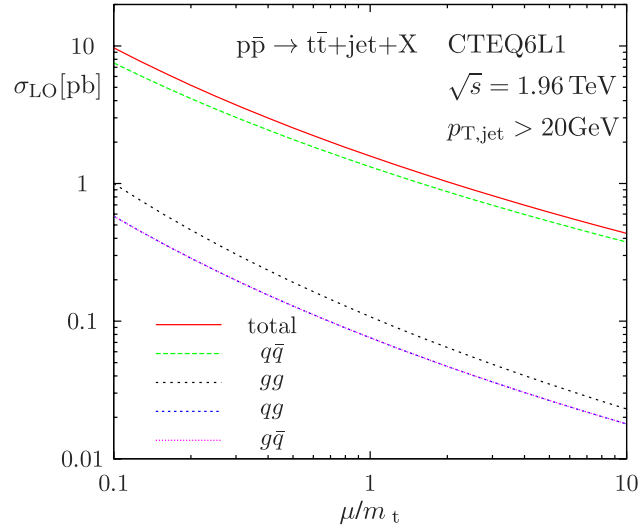


FIG. 2 (color online). Scale dependence of the LO cross sections for $t\bar{t}$ + jet production at the Tevatron (upper plot) and at the LHC (lower plot), where the renormalization and factorization scales are set equal to μ . The individual contributions of the various partonic channels are shown also separately.

FIG. 3 (color online). Scale dependence of the LO and NLO cross sections for $t\bar{t}$ + jet production at the Tevatron (upper plot) and at the LHC (lower plot), where the renormalization and factorization scales are set equal to μ .

definition. Denoting the corresponding NLO contributions to the cross sections by $\delta\sigma_{\text{NLO}}^{\pm}$, we define the asymmetry at NLO by

$$A_{\text{FB,NLO}}^t = \frac{\sigma_{\text{LO}}^-}{\sigma_{\text{LO}}^+} \left(1 + \frac{\delta\sigma_{\text{NLO}}^-}{\sigma_{\text{LO}}^-} - \frac{\delta\sigma_{\text{NLO}}^+}{\sigma_{\text{LO}}^+} \right), \quad (2)$$

i.e., via a consistent expansion in α_s . Note, however, that the LO cross sections in Eq. (2) are evaluated in the NLO setup (PDFs, α_s).

Figure 4 shows the scale dependence of the asymmetry at LO and NLO. The LO result for the asymmetry is of order α_s^0 and does therefore not depend on the renormalization scale. The plot for the LO result shows a mild residual dependence on the factorization scale, but the

size of this variation does not reflect the theoretical uncertainty, which is much larger. The NLO corrections to the asymmetry are of order α_s^1 and depend on the renormalization scale. It is therefore natural to expect a stronger scale dependence of the asymmetry at NLO than at LO, as seen in the plot. The size of the asymmetry, which is about -7% at LO, is drastically reduced by the NLO corrections. The residual scale dependence suggests that the asymmetry is $-1.5(\pm 1.5)\%$ for the chosen setup. Before closing, we comment on an alternative possibility to define the asymmetry via the unexpanded ratio of the NLO-corrected cross sections $\sigma_{\text{LO}}^{\pm} + \delta\sigma_{\text{NLO}}^{\pm}$. With this definition (not shown in the plot), one observes an even larger scale dependence with a dramatic rise for low scales μ where the NLO cross section (the denominator in the asymmetry) becomes per-

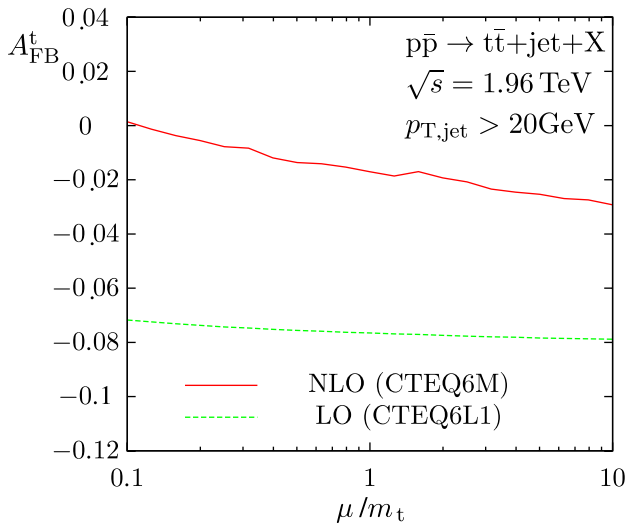


FIG. 4 (color online). Scale dependence of the LO and NLO forward-backward charge asymmetry of the top quark in $p\bar{p} \rightarrow t\bar{t} + \text{jet} + X$ at the Tevatron, where the renormalization and factorization scales are set equal to μ .

turbatively unstable and turns to negative values (see Fig. 3).

Conclusions.—First predictions for $t\bar{t} + \text{jet}$ production at hadron colliders have been presented at NLO QCD. For the cross section the NLO corrections drastically reduce the scale dependence of the LO predictions, which is of the order of 100%. The charge asymmetry of the top quarks, which is going to be measured at the Tevatron, is significantly decreased at NLO and is almost washed out by the residual scale dependence.

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