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SOME ASPECTS OF SIMULTANEOUS ACCELERATION OF PROTON AND H$^-$ BEAMS IN A LINAC

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When both protons and negative ions are accelerated in the same linac, at the same time new problems arise concerning suppression of beam coherent and incoherent oscillations caused by errors in doublet adjustment. Although the main parameters of the focusing channel do not vary if the sign of particle charge is reversed, the individual beam trajectories of protons and negative ions are different. New techniques for beam position control are suggested and their efficiency is evaluated by computer modelling. It is shown that a transverse artificial displacement of doublet axes can be used as a rather efficient means to shift the positive and the negative beam in the same direction. Such axis displacements of two successive doublets combined with axis displacement of individual lenses in each of these doublets, all produced electrically, make it possible to place the positive and negative beams at any required point in the transverse phase-space plane. Beam axial alignment appeared much less efficient than might be desirable in order to suppress beam oscillations. Production of new beam oscillations with the phases and amplitudes properly chosen is shown to be more efficient than beam alignment along the linac axis.

A global control of focusing field throughout the linac is briefly discussed as a possible method to reduce oscillations through off-resonance tuning.

Simultaneous acceleration of both protons and negative H$^-$ ions has been proposed in high energy linacs (e.g. in the Los Alamos Meson Facility or in the 600 MeV linac now under development at the Radiotechnical Institute of AS USSR). The transverse action of a strong focusing channel on a beam depends on the sign of the beam particles. Alternation of particle sign is equivalent to a global change of polarity in all lenses of the focusing channel or to interchange of the two transverse planes. It does not affect in any way the main constants of a channel such as μ transverse oscillation phase shift per period of a channel, $\nu_{\min}$—the minimum value of normalized oscillation frequency over a period and $\mathcal{N}$-the modulation factor of the beam envelope.

Transverse displacements of strong focusing channel elements give rise to coherent oscillations of both proton and H$^-$ beams. The rms intensities of the oscillations over many linac examples appear to be the same for the two components of a beam as well as the main constants of a channel. However in each particular case the two intensities differ from each other. Therefore new and more complicated demands for beam position control have to be satisfied.

The consideration below is outlined for a side-coupled linac used as the high energy part of the above mentioned linacs.

COHERENT OSCILLATIONS OF A TWO-COMPONENT BEAM

Let us consider a focusing channel of FDO structure as seen by a proton beam. If Δx_{in} and Δx_{ou} are doublet input and output end displacements from the axis of a linac, the beam output displacement and inclination may be determined from the following relation

$$\begin{bmatrix} x_+ \\ x'_+ \end{bmatrix}_{ou} = \begin{bmatrix} \Delta x_{ou} \\ \delta \end{bmatrix} - M_d M_f \begin{bmatrix} \Delta x_{in} \\ \delta \end{bmatrix}, \tag{1}$$

where M_d and M_f are the matrixes of focusing and defocusing lenses and $\delta = (\Delta x_{ou} - \Delta x_{in})/(2\xi)$, ξ being the lens length. Beam displacement at the output of a doublet may be reduced to corresponding displacement at a doublet centre

$$\begin{bmatrix} x_+ \\ x'_+ \end{bmatrix} = M_d^{-1} \begin{bmatrix} x_+ \\ x'_+ \end{bmatrix}_{ou} = M_d^{-1} \begin{bmatrix} \Delta x_{ou} \\ \delta \end{bmatrix} - M_f \begin{bmatrix} \Delta x_{in} \\ \delta \end{bmatrix}. \tag{2}$$

If the matrix of the focusing system between the centres of two subsequent doublets

$$M_n = \begin{bmatrix} \cos\mu + \alpha\sin\mu & \frac{1}{\nu}\sin\mu \\ -\nu(1+\alpha^2)\sin\mu & \cos\mu - \alpha\sin\mu \end{bmatrix} \quad (3)$$

is known†, the maximum displacement of beam centre in the channel following the doublet considered may be determined in terms of equivalent beam displacement at the doublet centre

$$x_{\max} = \frac{\nu}{\nu_{\max}}(1+\alpha^2)x_{+0}^2 + \frac{2\alpha}{\nu_{\min}} x_{+0}x'_{+0} + \frac{1}{\nu\nu_{\min}} x'_{+0}, \quad (4)$$

where $\nu_{\min}$ is the ν value for the period with its origin chosen in such a way to provide $\alpha = 0$ at both of its ends.

If the matrices of focusing and defocusing lenses are represented as

$$M_f = \begin{bmatrix} \cos\Lambda\xi & \frac{1}{\Lambda}\sin\Lambda\xi \\ -\Lambda\sin\Lambda\xi & \cos\Lambda\xi \end{bmatrix}; \quad M_d = \begin{bmatrix} \cosh\Lambda\xi & \frac{1}{\Lambda}\sinh\Lambda\xi \\ \Lambda\sinh\Lambda\xi & \cosh\Lambda\xi \end{bmatrix} \quad (5)$$

and $\Lambda\xi \ll 1$ the following expressions may be derived for equivalent beam displacements

$$x_{+0} = (\Lambda\xi)^2\Delta; \; \xi x'_{+0} = -\tfrac{1}{3}(\Lambda\xi)^4\Delta - (\Lambda\xi)^2\Delta_d, \quad (6)$$

where

$$\Delta = \frac{\Delta x_{ou} + \Delta x_{in}}{2}; \; \Delta_d = \frac{\Delta x_{ou} - \Delta x_{in}}{2}$$

and terms higher than $(\Lambda\xi)^4$ are neglected.

If protons are replaced by H^- ions then the signs must be changed in all $(\Lambda\xi)^2$ terms, leading to the following expressions for corresponding H^- beam displacements:

$$x_{-0} = -(\Lambda\xi)^2\Delta; \; \xi x'_{-0} = -\tfrac{1}{3}(\Lambda\xi)^4\Delta + (\Lambda\xi)^2\Delta_d. \quad (7)$$

In Eq. (4) only the α sign has to be reversed.

† α and ν may be found by derivation of matrix (3) for each specific case.

After substitution of (6) and (7) into (4) one may see that maximum transverse shifts of proton and H^- beams downstream from the displaced doublet are indeed different. If however the displacements of doublet ends are mutually independent (their average values vanish and the dispersions are equal) the rms values $\overline{x_{\max}^2}$ for both beam components are identical:

$$\overline{x_{\max}^2} = \frac{(\Lambda\xi)^4}{2\nu_{\min}}\left[\nu(1+\alpha^2) - \frac{2|\alpha|}{3\xi}(\Lambda\xi)^2 + \frac{1}{\nu\xi^2}\right]\overline{(\Delta x)^2}. \quad (8)$$

Beam behaviour in the y-plane is of similar nature. A doublet displacement gives rise to coherent oscillations of both beams. Amplitudes of the two oscillations differ from each other as well as from the amplitudes of x-plane motion. If $\overline{(\Delta x)^2} = \overline{(\Delta y)^2}$ the rms amplitudes of all four oscillations are just the same.

If all doublets in a channel have random displacements the above derived single-doublet conclusions remain valid.

SUPPRESSION OF COHERENT OSCILLATIONS IN TWO-COMPONENT BEAM

If a certain value of radius is assumed as a critical one, a two-component beam would exceed this value with higher probability then a one-component beam while rms inaccuracy of doublet manufacturing and adjustment is assumed the same for both cases. This is shown in Figure 1, where a certain value of radius is plotted along the horizontal axis while the probability for the beam to exceed this radius is plotted along the vertical axis. The results represented in Figure 1 were obtained by Monte-Carlo modelling of transverse proton and H^- motion in the side-coupled part of the linac.[1] In every realization of the channel, the ends of a doublet were displaced from the linac axis independently of each other with an rms shift of 0,1 mm†. The following procedure of computer modelling was used. First the proton beam was transmitted through both vertical and

† If true tolerances were assumed represented in B. P. Murin[1] and if correlation provided by step method of doublet adjustment[2] were taken into account beam oscillations would be approximately twice weaker than those shown in Figure 1.

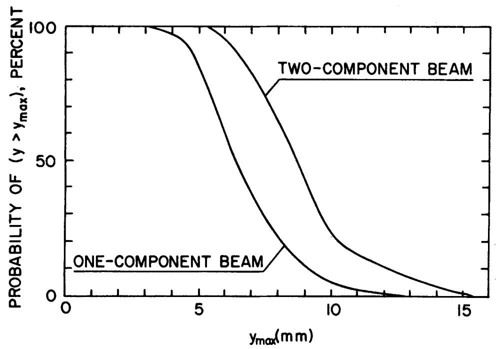

FIGURE 1 Probability of ($y > y_{max}$) vs y_{max} curves for the side-coupled linac without any correction. y is the maximum beam displacement in the linac. The curves were obtained from data received by computer for 140 random realizations.

horizontal channels with randomly displaced doublets and the maximum beam position shift was taken for the two channels. Then all magnetic field polarities in both channels were changed and the procedure was repeated. From the two maximum beam position shifts each corresponding to a certain polarity of the channel the smaller one was chosen as a maximum single-component shift for a given realization, while the greater was taken as a maximum two-component shift for the same realization.

It is obvious from Figure 1 that beam displacements may achieve a rather noticeable value and therefore some special control system is needed to suppress or diminish coherent transverse oscillations.

Correcting beam shift in a transverse plane may be accomplished either by a special dipole steering magnet or by adding a dipole component to the quadrupole lens field. The latter may be accomplished either by disturbing the current balance in the lens coils or by mechanical motion of the relevant lens axis. Beam position monitors can be used in such a system as quite natural sensors of beam displacement from the axis of a linac. For each transverse plane at least four correctors and two monitors† are obviously required to provide complete beam adjustment. While the linac[1] was under design four various methods of correction were studied:

1) Mechanical displacement and tilt of two doublets D1 and D2 (Figure 2a).

2) Magnetic field control in two pairs of steering

† Each of the monitors is supposed to measure position of both proton and H⁻ beams.

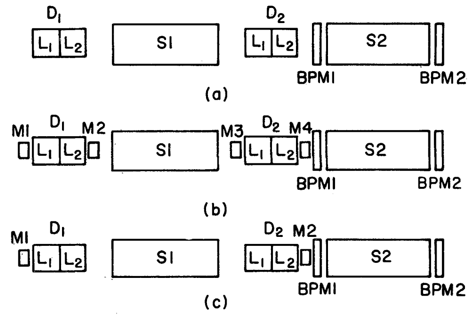

FIGURE 2 Three variants of a beam correcting assembly. D-doublets, L-lenses, M-steering magnets, BPM-beam position monitors, S-accelerating tanks.

magnets M1 – M4, placed at the ends of two adjacent doublets (Figure 2b).

3) Control of dipole components in doublets D1 and D2 (the dipole fields in both lenses of a doublet are the same) and of steering field in magnets M1 and M2 (Figure 2c).

4) Independent control of four dipole components in the four lenses of two adjacent doublets (Figure 2a).

In all cases beam position is measured by the monitors BPM1 and BPM2 located at the input and output ends of the tank just beyond the correcting arrangement. Each of the monitors provides the position measurement of both proton and H⁻ beam.

The first three methods were discarded for various reasons: the first of them contained mechanical arrangements; the second and the third were based on steering magnets and therefore required more space between adjacent tanks than the fourth one which seems to be the most attractive. The fourth method was approved for the project and several modes of operation were tested by computer. The tests revealed a rather satisfactory convergency of beam setting onto the axis of a linac and confirmed the possibility of beam positioning by empirical determination of required dipole components without solving correction Eqs. (12) derived in the following section.

Multipole components of the doublet field that usually accompany the dipole component are a common disadvantage of all correction methods suitable for two-component beams. These components may cause undesirable distortions of particle distributions over transverse phase planes. Computer modelling was used to estimate such distortions. In most cases they turned out to be quite negligible.

A detailed consideration of the fourth beam correction method is presented in the following section.

THE BEAM CORRECTING ASSEMBLY

Let us consider the beam correction assembly shown in Figure 2a and adjusting the beam say in the x-direction. If δ_i $(i = 1, 2, 3, 4)$ are equivalent displacements of lens axes in the first and the second doublets and

$$\Delta_i = \begin{bmatrix} \delta_i \\ 0 \end{bmatrix}, \qquad (i = 1, 2, 3, 4) \tag{9}$$

zero beam coordinates are achieved as soon as the following vector equations are satisfied:

a) for the proton beam

$$(E - M_d)\,\Delta_4 + M_d[(E - M_f)\,\Delta_3 + M_f M_s[(E - M_d)\,\Delta_2 + M_d[(E - M_f)\,\Delta_1 + M_f X_{in}^{+}]]] = 0; \tag{10}$$

b) for the H$^-$ beam

$$(E - M_f)\,\Delta_4 + M_f[(E - M_d)\,\Delta_3 + M_d M_s[(E - M_f)\,\Delta_2 + M_f[(E - M_d)\,\Delta_1 + M_d X_{in}^{-}]]] = 0; \tag{11}$$

where X_{in}^{+} and X_{in}^{-} are beam centre vectors at the input of the beam correcting assembly, E is the unit matrix and M_s is the matrix of a tank, acting as a defocusing lens in both transverse directions.

Using half sum and half difference of Eqs. (10) and (11) and vectors

$$X_{ou}^{+} = \begin{bmatrix} x_{ou}^{+} \\ x_{ou}^{+\prime} \end{bmatrix} \quad \text{and} \quad X_{ou}^{-} = \begin{bmatrix} x_{ou}^{-} \\ x_{ou}^{-\prime} \end{bmatrix}$$

of precorrection beam centres at the first beam position monitor BPM1 one may derive the following system of four linear equations for lens axis displacements δ_i $(i = 1, 2, 3, 4)$

$$\begin{aligned}
(x_{ou}^{+} + x_{ou}^{-})/2 &= a_{11}\delta_1 + a_{12}\delta_2 + a_{13}\delta_3 + a_{14}\delta_4;\\
(x_{ou}^{+\prime} + x_{ou}^{-\prime})/2 &= a_{21}\delta_1 + a_{22}\delta_2 + a_{23}\delta_3 + a_{24}\delta_4;\\
(x_{ou}^{+} - x_{ou}^{-})/2 &= a_{31}\delta_1 + a_{32}\delta_2 + a_{33}\delta_3 + a_{34}\delta_4;\\
(x_{ou}^{+\prime} + x_{ou}^{-\prime})/2 &= a_{41}\delta_1 + a_{42}\delta_2 + a_{43}\delta_3 + a_{44}\delta_4.
\end{aligned} \tag{12}$$

Constants a_{ij} are expressed as (terms higher than $(\Lambda\xi)^4$ neglected)

$$a_{11} = (\Lambda\xi)^4\left\{-\frac{3C_1}{8} + \frac{C_2}{6\xi}\left[1 + \frac{2(\Lambda\xi)^4}{3}\right] - \frac{C_3\xi}{4}\right\};$$

$$a_{12} = -(\Lambda\xi)^4\left(\frac{C_1}{8} + \frac{5C_2}{6\xi} + \frac{C_3\xi}{12}\right);$$

$$a_{13} = -\frac{5(\Lambda\xi)^4}{24}; \qquad a_{14} = \frac{(\Lambda\xi)^4}{24};$$

$$a_{21} = (\Lambda\xi)^4\left\{-\frac{11C_1}{6\xi}\left[1 - \frac{5(\Lambda\xi)^4}{33}\right] + \frac{5C_2(\Lambda\xi)^4}{9\xi^2} - \frac{41C_3}{24}\right\};$$

$$a_{22} = (\Lambda\xi)^4\left\{\frac{7C_1}{6\xi}\left[1 - \frac{(\Lambda\xi)^4}{21}\right] - \frac{C_2(\Lambda\xi)^4}{9\xi^2} + \frac{13C_3}{24}\right\};$$

$$a_{23} = -\frac{5(\Lambda\xi)^4}{6\xi}; \qquad a_{24} = \frac{(\Lambda\xi)^4}{6\xi};$$

$$a_{31} = -(\Lambda\xi)^2\left\{\frac{7C_1}{2}\left[1 - \frac{11(\Lambda\xi)^4}{84}\right] + \frac{C_2}{3}[1 - (\Lambda\xi)^4] + 3C_3\xi\right\};$$

$$a_{32} = (\Lambda\xi)^2\left\{\frac{5C_1}{2}\left[1 - \frac{(\Lambda\xi)^4}{20}\right] + \frac{C_2}{\xi}\left[1 - \frac{(\Lambda\xi)^4}{3}\right] + C_3\xi\right\};$$

$$a_{33} = -\frac{3(\Lambda\xi)^2}{2}; \qquad a_{34} = \frac{(\Lambda\xi)^2}{2};$$

$$a_{41} = -\Lambda^2\xi\left\{C_1\left[1 - \frac{(\Lambda\xi)^4}{3}\right] - \frac{2C_2(\Lambda\xi)^4}{3\xi} + \frac{3C_2\xi}{2}\right\};$$

$$a_{42} = \Lambda^2\xi\left\{C_1\left[1 - \frac{(\Lambda\xi)^4}{3}\right] - \frac{2C_2(\Lambda\xi)^4}{3\xi} + \frac{C_3\xi}{2}\right\};$$

$$a_{43} = -\Lambda^2\xi; \qquad a_{44} = \Lambda^2\xi;$$

where C_1, C_2 and C_3 are the elements of the M_s matrix:

$$M_s = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_1 \end{bmatrix}.$$

If all four lens axes are shifted by δ_i $(i = 1, 2, 3, 4)$, where δ_i are the roots of the system (12), then coherent oscillations are completely suppressed at the output of the beam correcting assembly. However inside the assembly, beam displacements may be even greater than without correction.

In practice solution of the system (12) may be replaced by iterative adjustment of the two beams on the axis of a linac at both beam position monitors. An estimation of a_{ij} values has been made in order to find a convenient correction procedure. As shown by the estimation the most convenient controlling combination of δ_i seems to be (δ_1 or δ_2), (δ_3 or δ_4), (δ_1 and δ_2 simultaneously) and (δ_3 and δ_4 simultaneously). In other words there are axes of lenses L1, L4 and of doublets D1 and D2 that are to be shifted for beam position correction. Such a correction may be explained in a following way. The dipole component in a single lens of a doublet acts essentially as a steering magnetic field, i.e., causes noticeable displacements of two beams in opposite directions and leaves the centre of the two beams almost unchanged. On the other hand a dipole component applied in both lenses of a doublet acts on the centre of the two beams leaving the mutual position of the beams only slightly changed. Combining doublet dipole component change with that of a single lens it is possible to provide an independent control of both proton and H$^-$ beam positions.

The procedure of two-component beam setting onto the axis of a linac may be realized in a following way. First the beams are matched to each other at the monitor BPM1 by means of the lens L1. Then the centre of the two beams is matched to the axis of a linac at the monitor BPM1 by moving the axes of the first doublet (i.e., by simultaneous positioning of both lenses L1 and L2). If relative divergence of two beams reappears during the second operation then both operations have to be repeated. As soon as both beams coincide with the linac axis at BPM1 both procedures have to be transferred to the doublet D2 and the monitor BPM2. All operations are repeated until complete suppression of beam oscillations is achieved. As shown by computer tests the overall number of operations is rather moderate.

NUMBER OF CORRECTING ASSEMBLIES REQUIRED IN THE LINAC

In order to find the required number of beam correcting assemblies in the linac an investigation of beam dynamics in the linac was carried out with errors of doublets adjustment taken into account. Rms values of alignment inaccuracies were assumed 0,1 mm as was mentioned above. The results of such a computer study for one, three and six correcting assemblies are presented in Figure 3. The linac was separated into 1, 3 and

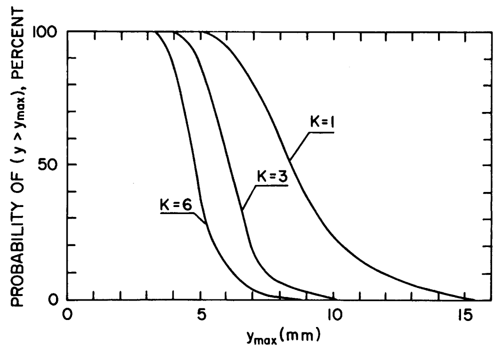

FIGURE 3 Probability of ($y > y_{max}$) vs y_{max} curves for the side-coupled linac with proton and H$^-$ beams. y is the maximum beam displacement in the linac. K is the number of beam correcting assemblies.

6 equal parts correspondingly and coherent oscillations were suppressed at the beginning of each part. A rather low efficiency of such a control is readily seen from the figure. It was even noticed that for some realizations beam matching to the axis of a linac not only does not reduce the maximum amplitude of oscillations but induces its growth. This may be explained in the following way. Oscillations of the beam centre at each part of a linac are excited by doublet shifts and tilts distributed throughout the whole upstream part of the linac. Disturbances induced by various parts of a linac are superposed in arbitrary phase. Therefore in some cases an artificial excitation of new oscillations may prove to be more effective than suppression of natural ones. Such a method seems to be useful enough if the phase of new oscillations provides that natural oscillations cancel at the most critical points of a linac.

These ideas were realized in the new correction procedure that may be called "interferentious" and has been tested by computer. The procedure was reduced to the following operations. First beam position is measured by many monitors distributed between the given control unit and the next downstream correcting assembly. The number of the period N is found where beam displacement reaches its maximum value. Then controlling dipole fields are to be adjusted in such a way to provide reduction to half of the vector $\left[\begin{smallmatrix} x \\ x' \end{smallmatrix}\right]$ at the Nth period.

The physical nature of interferentious correction may be clearly seen from comparing of Figures 4a and 4b which show beam trajectory envelopes for a random realization of focusing channel first without correction and then with "interferentious" control acting at only one point—at the very beginning of the side-coupled linac.

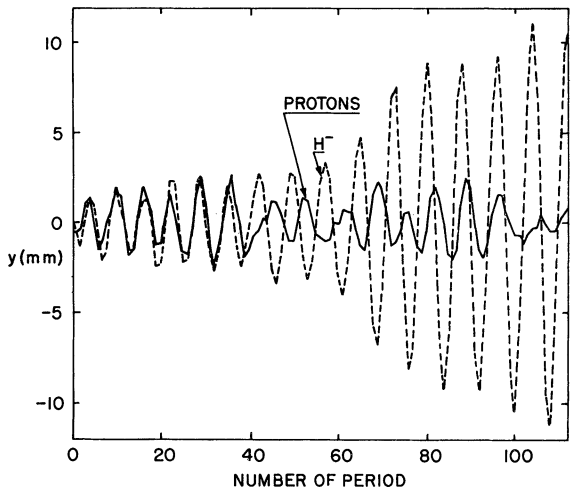

FIG. 4A

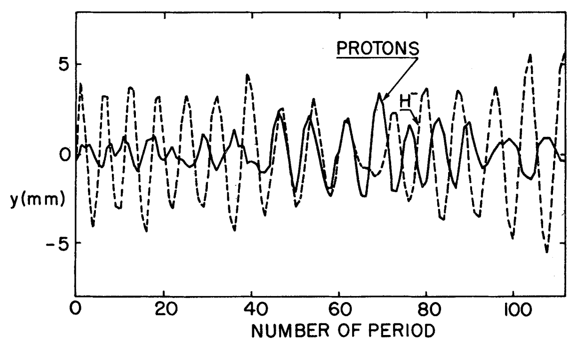

FIG. 4B

FIGURE 4 Beam trajectories for a certain realization of the side-coupled linac before (Figure 4a) and after (Figure 4b) "interferentious" correction. Transverse beam position at each period is plotted only for the input of focusing lens. Solid lines refer to protons and dashed lines to H^- ions.

The results of "interferentious" control obtained by computer tests for many random realizations are shown in Figure 5. The advantage of "interferentious" correction as compared to usual "zero" control is rather obvious.

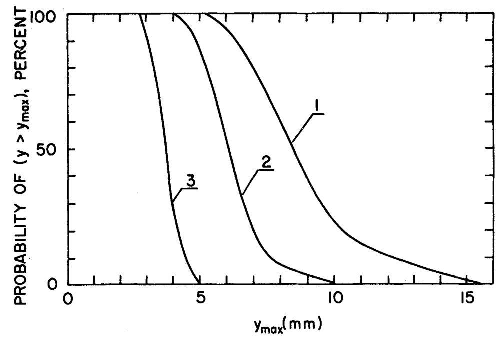

FIGURE 5 Probability of $(y > y_{max})$ vs y_{max} curves for the side-coupled linac with protons and H^- ions. y is the maximum beam displacement in the linac. 1) without any correction; 2) "zero" correction by means of three control assemblies; 3) "interferentious" correction by the same three control assemblies.

The "interferentious" control applied as described above has a substantial disadvantage: beam position is to be measured along the whole linac. It means for instance that the total number of beam position monitors in the side-coupled part of the linac[1] must be as great as 112. Since the amplitude of oscillations is distributed along a linac in a rather gradual and smooth way the total number of beam position monitors may be considerably reduced. The efficiency of correction seems to remain good enough with only a few monitors left to measure each amplitude of oscillation or at least some quantity proportional to the amplitude. The beam displacement vector on the transverse phase plane may be taken as such a quantity. The vector may be measured by a pair of beam position monitors separated by a distance (e.g., by an accelerating tank). The beam position control must provide reduction to half of beam displacements at both monitors of the pair. This method of control also was checked by computer. The results were rather optimistic: reduction of the number of beam position monitors from 36 per correcting assembly (one monitor per tank) to only one pair per 36 tanks caused growth of maximum corrected beam displacement by only 10 to 20%.

Thus six correcting units (three units for each transverse plane) give noticeable reduction of the rms value of maximum beam displacement in the side-coupled linac. The maximum value of displacement exceeded by two beams with probability say of 10% is reduced by the correction from 12 mm down to 4.5 mm i.e., 2.7 times.

REDUCTION OF EFFECTIVE BEAM RADIUS

Three different kinds of original errors are mainly responsible for effective† beam radius increase in an ion linac. These are displacements and tilts of quadrupole lens axes, rotation of lens median plane about longitudinal axis and magnetic field gradient dispersion around its prescribed value.

As shown above, the first of these is responsible for coherent beam oscillations. The last two do not cause any displacement of beam centre but are responsible for beam radius increase. At the same time beam emittance growth may be observed as a result of lens random rotations about their axes.

As has been shown in,[3,4] beam radius increases along a linac more rapidly than may be expected from previous theories. Simple expressions are derived in[3,4] for mean value (averaged over many realizations) of radius increase $\bar{R}_b/R_0$. These are

a) for random gradient dispersion:

$$\frac{\bar{R}_b}{R_0} = [e^{n\Delta_g} + (e^{2n\Delta_g} - 1)^{1/2}]^{1/2}, \tag{14}$$

where

$$\Delta_g = \frac{1}{2}\left(\frac{\Delta^2\xi}{2v}\right)^2 (1 + \mathcal{N}^{-2})\overline{\left(\frac{\Delta G}{G}\right)^2},$$

ΔG is gradient deviation from its prescribed value, n is the number of magnetic system periods passed by the beam and $\mathcal{N}$-relation of maximum dimension of a beam over a period of focusing system to its minimum value over the same period.

b) for small and random lens rotations:

$$\frac{\bar{R}_b}{R_0} = [(1 + \mathcal{N}^{-2})(2e^{n\Delta\chi} - 1)]^{1/2}, \tag{15}$$

where

$$\Delta_\chi = \left(\frac{\Lambda^2\xi}{v_{\min}\mathcal{N}}\right)^2 \overline{\chi^2}$$

and $\overline{\chi^2}$ is rms value of lens rotation χ.

Beam emittance increase E_b/E_0 caused by random lens rotations is determined as

$$\frac{E_b}{E_0} = e^{n\Delta_\chi}. \tag{16}$$

As can be shown from Eqs. (14–16) beam blow-up in a high energy linac (e.g., LAMPF or the linac[1]) for any assumed tolerances may occur to much greater radius than predicted by previous theories. Since particle loss in such a linac must be less than 0.1% reduction of tolerances is required as well as improvement of the correction system.

As has been mentioned in a previous section a certain number of beam correcting assemblies acting in accordance with a suitable control procedure provide for a substantial decrease of oscillation amplitude. Further suppression of coherent oscillations would be valuable only together with corresponding beam radius reduction.

Finally one more way of coherent oscillation suppression will be pointed out. The doublet shift distribution along the axis of the linac may be represented as a Fourier expansion. Therefore oscillations do not appear in any given realization unless the Fourier expansion contains some components that are multiples of the beam resonant frequency. Thus if oscillations do appear it is possible to go away from resonance by means of a global change in magnetic fields throughout the linac or over a considerable part thereof. This type of control may be of practical importance unless the field change is accompanied by a beam radius increase. This problem seems to be soluble more readily if, prior to the global field control procedure, the beam radius reduction procedure is made locally in few separate parts of the linac by varying the gradients in any sequence of four adjacent lenses.

† The term "effective" is used for the minimum linac circular aperture enveloping beam with both coherent and incoherent oscillations taken into account.

REFERENCES

1. B. P. Murin, *Proc. 8th Int. Conf. on High Energy Accelerators, CERN, 1971*, pp. 540–547.
2. E. A. Khesed and E. A. Ponomarenko, *Trudy Vsesoyuznogo Soveshchaniya po Uskoritelyam Zaryazhennykh Chastits*, **2**, 244, M. VINITI (1970).
3. B. I. Bondarev, A. P. Durkin, and L. Yu. Soloviev, *Trudy Radiotekhnicheskogo Instituta AN SSSR*, **9**, 3 (1972).
4. B. P. Murin, B. I. Bondarev, A. P. Durkin, and L. Yu. Soloviev, *Particle Accelerators*, **6**, 1 (1974).