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RETARDING FORCES AND RUNAWAY EFFECTS ACCOMPANYING THE MOTION OF ELECTRON RINGS ALONG CONDUCTING CYLINDERS†

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Current-carrying electron rings moving along conducting cylinders produce image currents which with finite resistivity, exert retarding forces on the ring. The forces depend on the ratio V of the ring velocity v to the surface resistivity s of the thin-wall cylinder. For low V the force is proportional to V, for large V inversely proportional. For large V a runaway situation exists, which limits the usefulness of conducting cylinders, especially in electron ring accelerators.

INTRODUCTION

In electron ring accelerators (ERA) it is mainly the negative mass instability[1] which prevents the achievement of higher ring qualities. Although the threshold in the number of electrons beyond which the system is unstable can be increased by a larger energy spread of the electrons, this energy spread at the same time leads to larger radial minor dimensions and therefore impairs the holding power of the ring. The threshold of the instability could be further increased by conducting non-resonant walls close to the ring. In Ref. 2 special attention is paid to the case of a ring of relativistic electrons close to and coaxial with a metal cylinder. It was shown that the threshold for low harmonic excitation is largely increased. Conducting walls close to the ring are therefore considered to be advantageous for ERA's. On the other hand, one has to investigate the effect of the finite resistivity of these walls on the ring dynamics. In Ref. 3 this is done for the sudden build-up of a ring close to the walls, "sudden" meaning that the build-up time of the ring is short compared with the penetration time of the self-field of the ring through the surrounding walls. In Ref. 4 the calculations were extended to include ring motion along the walls. The walls are always supposed to be thin compared with the relevant skin depth. Two typical cases were calculated in Ref. 4: radial compression between side walls and axial motion along cylinders. The energy losses accompanying these motions did not seem to be too serious for present day ERA's, but the forces reacting on the ring owing to the decaying image currents in the resistive walls are remarkable in some circumstances. In the case of radial compression between side walls the force leads to an increase of the radius and a decrease of the field index. This part of the calculation shall not be described here. The subject of this report is the runaway situation which could develop during the motion of a ring along a cylinder and could limit the use of these cylinders in the accelerating structure of an ERA. Retarding forces in linear approximation have also been calculated analytically by Merkel.[5] A continuation of this work includes a solution in cylindrical geometry[6] which gives very good agreement with the results presented in Fig. 1.

THE RETARDING FORCES

Since the effects of the electric image charges are negligible as long as the ring velocity is nonrelativistic as has been shown particularly in Ref. 5, this paper only considers the magnetic field effects. If a ring of relativistic electrons encircling the axis moves along a cylinder with perfect conductivity, the image currents in front of and behind the ring are the same and no radial magnetic field—no

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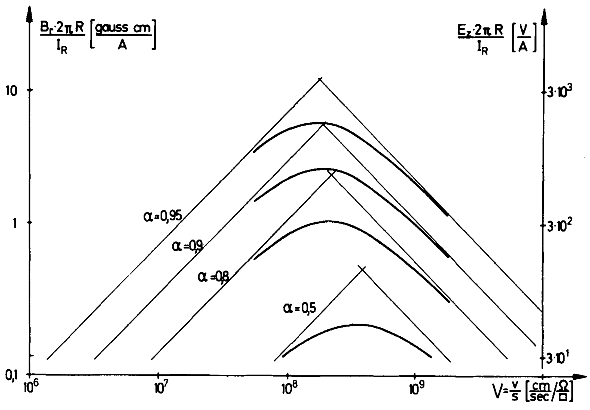

FIGURE 1 Radial magnetic field B_r (left ordinate) and axial electric field E_z (right ordinate) at the position of the ring that are due to the image current distribution on the cylinder as a function of the ratio V of the ring axial velocity v to the surface resistivity s of the cylinder. R = ring radius, I_R = ring current, α = ratio of cylinder radius to ring radius.

axial force—is present at the position of the ring. If, however, the conductivity is finite, the image currents are damped and are therefore smaller or even of opposite sign behind the ring. This gives rise to a radial magnetic field component which, together with the azimuthal ring velocity, produces an axial retarding force on the ring.

Details of the calculation of the radial field component B_r are given in Ref. 4. The main results are contained in Figure 1, which presents the axial retarding force as a function of the ratio V of the ring axial velocity v to the surface resistivity s of the cylinder. The ordinate at the left side gives the force in terms of B_r at the position of the ring. At the right-hand side the forces are converted to the equivalent electric field E_z assuming that the azimuthal velocity of the electrons is the velocity of light. Curves are displayed for different ratios α of the cylinder radius to ring radius. Very similar curves are obtained when the ring moves inside a cylinder.

The curves go through a maximum as a function of V whose position depends only slightly on α. For low V the retarding forces are proportional to V, for high V they are inversely proportional to V. This can easily be understood. For low velocities or high resistivities the image current density $j(z)$ is at each instant and location proportional to the time derivative of the vector potential $\dot{A}$ divided by the resistivity which is assumed to be constant along the cylinder:

$$j(z) \sim \frac{\dot{A}}{s}.$$

If the only time dependence of A is via the axial ring velocity v, $\dot{A}$ is proportional to v and

$$j(z) \sim \frac{v}{s}.$$

The energy loss E_t per unit time—the retarding power—is

$$E_t = s \int j^2(z)\,\mathrm{d}z \sim \frac{v^2}{s}.$$

The energy loss E_c per m or the equivalent retarding force is

$$E_c = \frac{1}{v} E_t \sim \frac{v}{s} = V$$

In the limit for large V, the induced image currents cease to depend on velocity or resistivity. They are then equal to the ring current I_R. One obtains

$$E_t = s \int j^2(z)\,\mathrm{d}z \sim s I_R^2$$

$$E_c \sim \frac{s}{v} \cdot I_R^2 \sim \frac{1}{V}.$$

The straight lines in Figure 1 result from these approximations. For very large V the calculation breaks down because the wall thickness is then no longer thin relative to the skin depth. For more details see Ref. 4. The full curves in Figure 1 can be described in the form

$$F_i = \frac{2B \cdot A_i \cdot I_R}{R} \cdot \frac{V}{B^2 + V^2}.$$

Here B is the value of V at which the maximum occurs, F_i represents B_r or E_z, and A_i the maximum value of B_r or E_z. For the curves displayed one finds the following numbers for A_i and B (with $A_{E_z} = 300 A_{B_r}$):

α	$B\left[\frac{\text{cm}}{\text{sec}}/\Omega\right]$	$A_{B_r}\left[\frac{\text{gauss cm}}{A}\right]$	$A_{E_z}\left[\frac{V}{A}\right]$
0.95	$1.8 \cdot 10^8$	0.923	276.9
0.9	$1.9 \cdot 10^8$	0.429	128.9
0.8	$2.2 \cdot 10^8$	0.175	52.52
0.5	$3.4 \cdot 10^8$	0.029	8.59

For $\alpha = 0.8$ one finds $B_{r\,\max} = 117$ G for a ring with $I_R = 2000\,A$ and a radius $R = 3$ cm.

THE RUNAWAY EFFECT

The retarding forces just calculated are much larger than the accelerating forces tolerable in present day ERA's. In principle, these forces can be compensated by external fields. But because these forces depend on properties of the ring, a jitter in the ring quality might create severe difficulties. An additional problem arises if one tries to accelerate the ring to a velocity beyond the maximum of the retarding force. There the slope of the curve is negative, which causes a runaway effect: If a ring is accelerated in an external field just slightly larger than the maximum retarding field, the difference of the external field and the retarding field—the effective acceleration field—becomes the larger the faster the ring is, once the velocity exceeds the velocity at the peak of B_r.

In principle, it seems possible to shape the external field in such a way that it controls the motion even for large V. This would be the case if the external field had a very steep negative gradient at the position of the ring. But because the time and space when and where the maximum of the retarding force occurs depend on the imperfectly reproducible parameters of the ring, a general solution is practically impossible. The runaway effect is the more pronounced, the lower the velocity at which the maximum of B_r occurs. A small force then already results in a large relative change of the velocity.

Figure 2 illustrates the runaway effect. It gives as a function of time the radial magnetic field $B_{r\,\mathrm{eff}}$ seen by a ring which moves under the influence of an external magnetic field B_{ex} and the retarding field B_r. The external field is composed of a constant axial component $B_{z\,\mathrm{ex}}$, to which a radial component $B_{r\,\mathrm{ex}}$ is added. $B_{r\,\mathrm{ex}}$ increases in time to a certain level, whose value is indicated at the curves in gauss, at which it remains constant in time.

The surface resistivity chosen is $s = 1\ \Omega$. The corresponding velocity for the peak retarding force is 2.2×10^8 cm/sec and the maximum retarding field is 112.5 G. The different curves belong to different levels of the external radial field $B_{r\,\mathrm{ex}}$. As long as these levels are lower than the maximum retarding field, the actual field $B_{r\,\mathrm{eff}}$ seen by the ring stays rather small and goes to zero after the external field becomes constant. That is, at a certain velocity the retarding field just cancels the external field and the ring moves with constant velocity. If, however, the level of the external field is only slightly larger than the peak retarding field (see the curve for 113 G), the accelerating field seen by the ring grows rather fast towards the level of the external field.

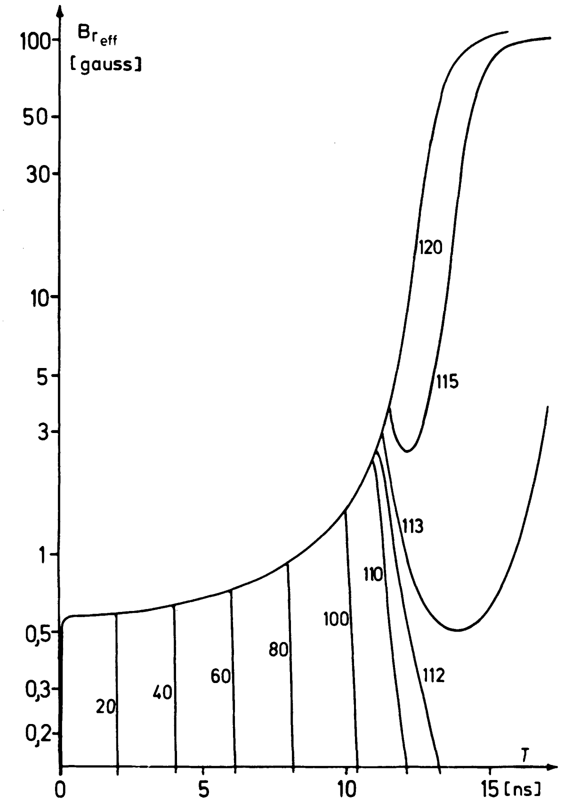

FIGURE 2 Effective radial field $B_{r\,\mathrm{eff}}$ as a function of time, seen by a ring moving in an external magnetic field whose radial component increases linearly in time to certain levels, indicated in the curves in gauss.

Figure 3 shows as a function of time the velocity v of the ring for the same conditions as in Figure 2. Here the curves for 112.5 and for 112.6 G are particularly interesting. The one is just barely below, the other above the peak value of the retarding field. At 112.5 G the velocity is bound to a fixed value, whereas for 112.6 G the velocity grows rapidly as soon as the velocity is larger than the peak velocity, owing to the runaway effect.

These figures demonstrate that acceleration along conducting cylinders should be avoided as long as the holding powers in the rings do not tolerate radial magnetic fields as large as the peak retarding fields.

Suppression of the negative mass instability then seems difficult. If a surface resistivity of 20 Ω is considered to be tolerable for some experiments, velocities of 4×10^9 cm/sec or ion energies of

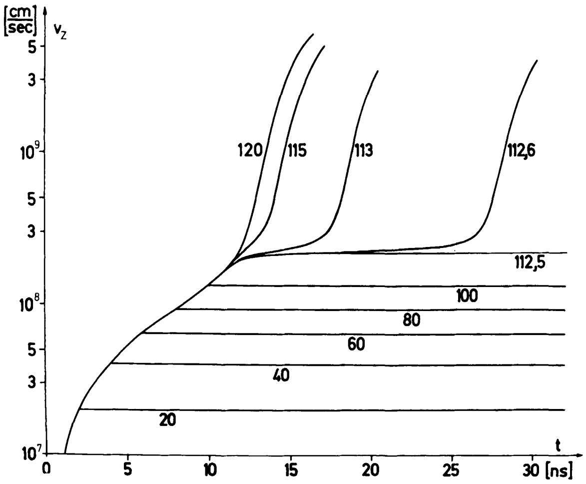

FIGURE 3 Axial velocity v_z versus time of a ring moving in an external magnetic field whose radial component increases linearly in time to certain levels, indicated in the curves in gauss.

10 MeV/nucleon can be achieved without crossing the peak of the retarding forces. For some types of heavy ion accelerators this is sufficient. In this case one would avoid the runaway effect but would have jitter in the energy if jitter in the ring current or radius occurs.

In addition, if an accelerator is operated just below the peak retarding field the energy loss to the walls becomes important. This effect can easily be estimated if the reaction of the losses on the ring properties is neglected. If a constant effective acceleration b is assumed, then v_z is: $(2 \cdot b \cdot x)^{1/2}$, where x is the distance of motion. For the retarding electric field one has (Eq. 1)

$$E_z = \frac{2B \cdot A_{E_z} \cdot I_R}{R} \cdot \frac{\dfrac{(2bx)^{1/2}}{s}}{B^2 + \dfrac{2bx}{s^2}}$$

and for the corresponding energy loss one gets

$$E_L = \int_0^{x_0} eE_z(x)\,\mathrm{d}x$$

$x_0 = v_E/2b$ is the length of the acceleration and v_E the desired velocity. Integration yields

$$E_L = \frac{2A_{E_z} \cdot I_R}{R} \sqrt{C} \left\{ \sqrt{x_0} - \sqrt{C} \cdot \operatorname{arctg} \sqrt{\frac{x_0}{C}} \right\}$$

$$C = \frac{B^2 \cdot s^2}{2b}.$$

If the acceleration b is calculated for a constant radial magnetic field of 5 G, an electron number of $N_e = 5.0 \times 10^{12}$, an ion-to-electron charge ratio of 1% and 30 times ionized xenon, and if $R =$ 2.5 cm, $\alpha = 0.8$ and $s = 20\ \Omega$ are assumed, one gets an energy loss per electron of 5.3 MeV. This is about one-third of the original kinetic energy of a ring in a 20 kG field. If the acceleration is lower, the losses are even higher.

Although the estimate is rather crude, it at least states that the energy loss and its effect on the ring dynamics have to be calculated carefully when a ring of electrons is moved along a cylinder with a velocity close to that corresponding to the peak of the retarding force.

CONCLUSION

Although conducting cylinders close to rings are very desirable for suppressing the negative mass instability, the retarding forces connected with the decaying image current impose severe limitations on their usefulness. Only if it is possible to operate in a V-regime well below the peak of the retarding force is stable motion with only minor energy losses possible.

If higher energies should be obtained without crossing the peak of the retarding force, the energy losses connected with a motion close to the peak have to be taken into account in the calculation of the ring dynamics. One nevertheless has to expect jitter in the energy, which depends on the jitter in the ring properties.

Acceleration beyond the peak of the retarding force has to be avoided because a runaway effect leads to acceleration which is not tolerable in present-day ERA's.

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