

Elements of Quantum Information

Edited by

Wolfgang P. Schleich and Herbert Walther



WILEY-VCH Verlag GmbH & Co. KGaA

Contents

	Preface to the Book	<i>XVII</i>
	Preface to the Journal	<i>XIX</i>
	List of Contributors	<i>XXI</i>
1	The Deterministic Generation of Photons by Cavity Quantum Electrodynamics	1
	<i>H. Walther</i>	
1.1	Introduction	1
1.2	Oscillatory Exchange of Photons Between an Atom and a Cavity Field	1
1.2.1	Experimental Set-up of the One-atom Maser	3
1.2.2	One-atom Maser as a Source of Non-classical Light	5
1.2.3	Review of Experiments on Basic Properties of the One-atom Maser	8
1.2.4	Statistics of Detector Clicks	12
1.2.5	Trapping States	13
1.2.6	Trapping State Stabilization	17
1.2.7	Fock States on Demand	17
1.2.8	Dynamical Preparation of n -photon States in a Cavity	18
1.2.9	The One-atom Maser Spectrum	24
1.3	Other Microwave Cavity Experiments	26
1.3.1	Collapse-and-revival of the Rabi Oscillations in an Injected Coherent Field	26
1.3.2	Atom-photon and Atom-atom Entanglement	27
1.3.3	Atom-photon Phase Gate	28
1.3.4	Quantum Nondestructive-measurement of a Photon	28
1.3.5	Wigner-function of a One-photon State	29
1.3.6	Multiparticle Entanglement	29
1.3.7	Schrödinger Cats and Decoherence	29

1.4	Cavity QED Experiments in the Visible Spectral Region	30
1.4.1	The One-atom Laser	30
1.4.2	Atoms Pushed by a Few Photons	31
1.4.3	Single-photon Sources	33
1.4.4	Single-atom Laser using an Ion Trap	34
1.5	Conclusions and Outlook	38
	References	39
2	Optimization of Segmented Linear Paul Traps and Transport of Stored Particles	45
	<i>Stephan Schulz, Ulrich Poschinger, Kilian Singer, and Ferdinand Schmidt-Kaler</i>	
2.1	Introduction	45
2.2	Optimization of a Two-layer Microstructured Ion Trap	46
2.2.1	Design Objectives	47
2.2.2	Operating Mode and Modeling of the Segmented Linear Paul Trap	49
2.2.3	Optimization of the Radial Potential	51
2.2.4	Optimization of the Axial Potential	52
2.3	Open Loop Control of Ion Transport	54
2.3.1	Non-adiabatic Heating Sources	54
2.3.2	Overview of the Applied Optimization Strategies	55
2.3.3	The Optimal Control Method	55
2.3.4	Optimization Results	58
2.3.5	Ion Heating due to Anharmonic Dispersion	59
2.3.6	Quantum Mechanical Estimate of Non-adiabatic Parametric Heating	59
2.3.7	Improved Initial Guess Function and Ultra-fast Transport	60
2.3.8	Discussion of the Open-loop Result	62
2.4	Outlook	64
A	Appendix	65
	References	66
3	Transport Dynamics of Single Ions in Segmented Microstructured Paul Trap Arrays	69
	<i>R. Reichle, D. Leibfried, R. B. Blakestad, J. Britton, J. D. Jost, E. Knill, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland</i>	
3.1	Introduction	69
3.2	Classical Equations of Motion	71
3.3	Classical Dynamics of Ion Transport	72
3.3.1	Homogeneous Solution	73
3.3.2	Green's Function and General Solution	74

3.3.3	Adiabatic Limit	75
3.4	Quantum and Classical, Dragged Harmonic Oscillators with Constant Frequency	76
3.5	The Dragged Quantum Harmonic Oscillator	78
3.6	Transport Dynamics in a Well-controlled Regime	81
3.6.1	Two Analytical Examples	82
3.6.2	Near-optimum Transport Functions	86
3.6.3	High-frequency Limit, Adiabatic Transport, and Approximate Trajectories	86
3.7	Please supply a short title	87
3.7.1	Determination of Waveforms	87
3.7.2	Potential Fluctuations and Aspect-ratio Rule	90
3.8	Conclusions	95
A	Appendix	96
	References	96
4	Ensemble Quantum Computation and Algorithmic Cooling in Optical Lattices	99
	<i>M. Popp, K. G. H. Vollbrecht, and J. I. Cirac</i>	
4.1	Introduction	99
4.2	Physical System	102
4.2.1	Bose-Hubbard Model	102
4.2.2	Initial State Properties	103
4.2.3	Entropy as Figure of Merit	105
4.2.4	Basic Operations	106
4.3	Ensemble Quantum Computation	108
4.4	Cooling with Filtering	112
4.5	Algorithmic Ground State Cooling	114
4.5.1	The Protocol	114
4.5.2	Theoretical Description	115
4.6	Conclusion	118
	References	119
5	Quantum Information Processing in Optical Lattices and Magnetic Microtraps	121
	<i>Philipp Treutlein, Tilo Steinmetz, Yves Colombe, Benjamin Lev, Peter Hommelhoff, Jakob Reichel, Markus Greiner, Olaf Mandel, Artur Widera, Tim Rom, Immanuel Bloch, and Theodor W. Hänsch</i>	
5.1	Introduction	121
5.2	Optical Lattices	122
5.2.1	Preparation of a Qubit Register	122
5.2.2	A Quantum Conveyer Belt for Neutral Atoms	123

5.2.3	Controlled Collisions	124
5.3	Magnetic Microtraps	127
5.3.1	Qubit States on the Atom Chip	128
5.3.2	State-dependent Microwave Potentials	132
5.3.3	Qubit Readout in Microtraps	135
5.3.3.1	Stable fiber Fabry-Perot Cavities	137
5.3.3.2	FFP Cavity Fabrication and Performance	137
5.3.4	On-chip Atom Detection with a FFP Cavity	138
5.3.5	Single Atom Preparation	141
5.4	Conclusion	142
	References	142
6	Two-dimensional Bose-Einstein Condensates in a CO₂-laser Optical Lattice	145
	<i>Giovanni Cennini, Carsten Geckeler, Gunnar Ritt, Tobias Salger, and Martin Weitz</i>	
6.1	Introduction	145
6.2	Experimental Setup and Procedure	146
6.3	Experimental Results	148
6.4	Conclusions	151
	References	153
7	Creating and Probing Long-range Order in Atomic Clouds	155
	<i>C. von Cube, S. Slama, M. Kohler, C. Zimmermann, and Ph.W. Courteille</i>	
7.1	Introduction	155
7.2	Collective Coupling	157
7.2.1	Experimental Setup	158
7.2.1.1	Ring Cavity	159
7.2.1.2	Dipole Trap for ⁸⁵ Rb	160
7.2.1.3	Optical Molasses	162
7.2.2	Signatures of Collective Atomic Recoil Lasing	163
7.2.2.1	Beat Note of Field Modes	163
7.2.2.2	Spectra of Recoil-induced Resonances	165
7.2.2.3	Atomic Transport	166
7.3	Creating Long-range Order	168
7.3.1	Analytic Treatment for Perfect Bunching	168
7.3.1.1	Radiation Pressure	170
7.3.1.2	Phase-locking by Imperfect Mirrors	171
7.3.2	Simulations of Atomic Trajectories with Friction and Diffusion	172
7.3.2.1	Lasing Threshold	173
7.3.2.2	Self-synchronization	174

7.4	Probing Long-range Order	176
7.4.1	Bragg Scattering	176
7.4.2	Heterodyned Bragg Spectra	178
7.4.3	Measuring the Bragg Scattering Phase	179
7.5	Conclusion	180
	References	181
8	Detecting Neutral Atoms on an Atom Chip	185
	<i>Marco Wilzbach, Albrecht Haase, Michael Schwarz, Dennis Heine, Kai Wicker, Xiyuan Liu, Karl-Heinz Brenner, Sönke Groth, Thomas Fernholz, Björn Hessmo, and Jörg Schmiedmayer</i>	
8.1	Introduction	185
8.2	Detecting Single Atoms	186
8.2.1	Measuring the Scattered Light: Fluorescence Detection	186
8.2.2	Measuring the Driving Field	187
8.2.2.1	Absorption on Resonance	187
8.2.2.2	Refraction	189
8.2.3	Cavities	189
8.2.3.1	Absorption on Resonance	189
8.2.3.2	Refraction	190
8.2.3.3	Many Atoms in a Cavity	190
8.2.4	Concentric Cavity	191
8.2.5	Miniaturization	191
8.3	Properties of Fiber Cavities	192
8.3.1	Loss Mechanisms for a Cavity	193
8.3.2	Losses due to the Gap Length	194
8.3.3	Losses due to Transversal Misalignment	195
8.3.4	Losses due to Angular Misalignment	196
8.3.5	Fresnel Reflections	197
8.4	Other Fiber Optical Components for the Atom Chip	199
8.4.1	Fluorescence and Absorption Detectors	199
8.4.2	A Single Mode Tapered Lensed Fiber Dipole Trap	199
8.5	Integration of Fibers on the Atom Chip	201
8.5.1	Building Fiber Cavities	201
8.5.2	The SU-8 Resist	203
8.5.3	Test of the SU-8 Structure	204
8.6	Pilot Test for Atom Detection with Small Waists	205
8.6.1	Dropping Atoms through a Concentric Cavity	205
8.6.2	Detecting Magnetically Guided Atoms	207
8.7	Conclusion	208
	References	209

9	High Resolution Rydberg Spectroscopy of Ultracold Rubidium Atoms	211
	<i>Axel Grabowski, Rolf Heidemann, Robert Löw, Jürgen Stuhler, and Tilman Pfau</i>	
9.1	Introduction	211
9.2	Experimental Setup and Cold Atom Preparation	212
9.2.1	Vacuum System and Magneto Optical Trap (MOT)	212
9.2.2	Rydberg Laser System and Rydberg Excitation	215
9.2.3	Detection of the Rydberg Atoms	216
9.2.4	Excitation Sequence	217
9.3	Spectroscopy of Rydberg States, $ m_j $ Splitting of the Rydberg States	219
9.4	Spatial and State Selective Addressing of Rydberg States	220
9.4.1	Spatial Selective Rydberg Excitation	220
9.4.2	Hyperfine Selective Rydberg Excitation	221
9.5	Autler-Townes Splitting	222
9.6	Conclusion and Outlook	224
	References	224
10	Prospects of Ultracold Rydberg Gases for Quantum Information Processing	227
	<i>Markus Reetz-Lamour, Thomas Amthor, Johannes Deiglmayr, Sebastian Westermann, Kilian Singer, André Luiz de Oliveira, Luis Gustavo Marcassa, and Matthias Weidemüller</i>	
10.1	Introduction	227
10.2	Excitation of Rydberg Atoms from an Ultracold Gas	229
10.3	Van-der-Waals Interaction	230
10.3.1	Blockade of Excitation	231
10.3.2	Ionization	232
10.4	States with Permanent Electric Dipole Moments	234
10.5	Förster Resonances	236
10.6	Conclusion	239
	References	241
11	Quantum State Engineering with Spins	243
	<i>Andreas Heidebrecht, Jens Mende, and Michael Mehring</i>	
11.1	Introduction	243
11.1.1	Quantum States of Spins	244
11.2	Deutsch-Josza Algorithm	246
11.2.1	The Deutsch-Josza Algorithm	246
11.2.2	Implementation of the 3-qubit Deutsch-Josza Algorithm Using Liquid State NMR	247

11.2.2.1	2,3,4-Trifluoroaniline	247
11.2.2.2	Preparation of Pseudo-pure States	248
11.2.2.3	Results on the 3-qubit DJ-algorithm	249
11.3	Entanglement of an Electron and Nuclear Spin in $^{15}\text{N}@C_{60}$	251
11.4	Spin Quantum Computing in the Solid State: S-bus	253
11.4.1	The S-bus Concept	253
11.4.2	Single Crystal $\text{CaF}_2 : \text{Ce}^{3+}$ as an S-bus system	255
11.4.3	Experimental Details	256
11.4.4	3-qubit Pseudo-pure States	258
11.4.5	2-qubit Deutsch-Josza Algorithm	259
11.4.5.1	Controlling Nuclear Spin Decoherence in $\text{CaF}_2 : \text{Ce}$	260
11.5	Summary and Outlook	263
	References	263
12	Improving the Purity of One- and Two-qubit Gates	265
	<i>Sigmund Kohler and Peter Hänggi</i>	
12.1	Introduction	265
12.2	Quantum Gate with Bit-flip Noise	266
12.2.1	Bloch-Redfield Master Equation	267
12.2.2	Purity Decay	268
12.2.3	Numerical Solution	269
12.3	Coherence Stabilization for Single Qubits	270
12.3.1	Dynamical Decoupling by Harmonic Driving	271
12.3.2	Coherent Destruction of Tunneling	272
12.4	Coherence Stabilization for a CNOT Gate	275
12.4.1	Heisenberg vs. Ising Coupling	276
12.4.2	Coherence Stabilization by an AC Field	278
12.4.3	Numerical Solution	279
12.4.4	Implementation with Quantum Dots	282
12.5	Conclusions	282
A	Appendix	283
	References	284
13	How to Distill Entanglement from a Finite Amount of Qubits?	287
	<i>Stefan Probst-Schendzielorz, Thorsten Bschorr, and Matthias Freyberger</i>	
13.1	Introduction	287
13.2	Entanglement Distillation	288
13.2.1	The Protocol	289
13.3	CNOT Distillation for a Finite Set of Entangled Systems	293
13.3.1	Iterative Distillation	294
13.4	Example of the Iterative Distillation for Small Finite Sets	297

13.5	Conclusions	299
A	Appendix	300
	References	301
14	Experimental Quantum Secret Sharing	303
	<i>Christian Schmid, Pavel Trojek, Sascha Gaertner, Mohamed Bourennane, Christian Kurtsiefer, Marek Zukowski, and Harald Weinfurter</i>	
14.1	Introduction	303
14.2	Theory	304
14.2.1	The GHZ-protocol	304
14.2.2	The $ \Psi_4^-\rangle$ -protocol	305
14.2.3	The Single Qubit Protocol	306
14.2.4	Security of the Protocols	307
14.3	Experiment	309
14.3.1	The $ \Psi_4^-\rangle$ -protocol	309
14.3.2	The Single-qubit Protocol	310
14.4	Conclusion	312
	References	314
15	Free Space Quantum Key Distribution: Towards a Real Life Application	315
	<i>Henning Weier, Tobias Schmitt-Manderbach, Nadja Regner, Christian Kurtsiefer, and Harald Weinfurter</i>	
15.1	Introduction	315
15.2	Setup	316
15.2.1	Transmitter Unit	316
15.2.2	Free Space Link	317
15.2.3	Receiver Unit	318
15.2.4	Synchronisation and Automatic Alignment Control	319
15.2.5	Sifting, Error Correction and Privacy Amplification	319
15.2.6	Experimental Results	320
15.3	Conclusion	322
	References	323
16	Continuous Variable Entanglement Between Frequency Modes	325
	<i>Oliver Glöckl, Ulrik L. Andersen, and Gerd Leuchs</i>	
16.1	Introduction	325
16.2	Sideband Separation	327
16.2.1	Theory	328
16.2.2	Pictorial Description	331
16.3	Experiment and Results	331

16.4	Conclusion and Discussion	335
	References	336
17	Factorization of Numbers with Physical Systems	339
	<i>Wolfgang Merkel, Ilya Sh. Averbukh, Bertrand Girard, Michael Mehring, Gerhard G. Paulus, and Wolfgang P. Schleich</i>	
17.1	Introduction	339
17.2	Chirping a Two-photon Transition	340
17.2.1	Chirped Laser Pulses	340
17.2.2	Excitation Probability Amplitude	341
17.2.3	Example for Factorization	342
17.3	Driving a One-photon Transition	343
17.3.1	Model	344
17.3.2	Floquet Ladder	345
17.3.3	Pulse Train	346
17.4	Factorization	347
17.4.1	Factorization with Floquet Ladder	348
17.4.2	Factorization with a Pulse Train	349
17.5	NMR-experiment	350
17.6	Conclusions	352
	References	353
18	Quantum Algorithms for Number Fields	355
	<i>Daniel Haase and Helmut Maier</i>	
18.1	Introduction	355
18.1.1	Outline of the Survey	355
18.1.2	Why Number Fields?	356
18.1.3	Some History of the Subject	356
18.2	Geometry of Numbers	357
18.2.1	Number Fields	357
18.2.2	Lattices	358
18.2.3	Integral Elements	359
18.2.4	The Class Number	360
18.2.5	The Regulator	361
18.2.6	Complexity Results	361
18.3	Reduction	362
18.3.1	Reduced Ideals	362
18.3.2	Infrastructure	363
18.3.3	Geometric Interpretation of \mathbf{G}	364
18.4	Results from Analytic Number Theory	366
18.4.1	Distribution of Prime Numbers	366
18.4.2	Class Number Formulas	367

18.5	Examples of Minima Distributions	368
18.6	Computing the Regulator	370
18.6.1	Real Quadratic Case	370
18.6.2	Hallgren's Algorithm	371
18.6.3	Generalization of the Weak Periodicity Condition	372
18.7	Computation of Other Invariants	374
18.7.1	The Principal Ideal Problem	374
18.7.2	Computing the Class Number	375
	References	376
19	Implementation Complexity of Physical Processes as a Natural Extension of Computational Complexity	377
	<i>Dominik Janzing</i>	
19.1	Introduction	377
19.2	Similar Complexity Bounds for Different Tasks	379
19.3	Relating Control Problems to Hard Computational Problems	385
19.4	The Need for a Control-theoretic Foundation of Complexity	388
19.5	Hamiltonians that Compute Autonomously	393
	References	396
20	Implementation of Generalized Measurements with Minimal Disturbance on a Quantum Computer	399
	<i>Thomas Decker and Markus Grassl</i>	
20.1	Introduction	399
20.2	Minimal-disturbing Implementations of POVMs	401
20.2.1	Generalized Measurements of Quantum Systems	401
20.2.2	Positive-operator Valued Measures	402
20.2.3	Orthogonal Measurements	403
20.2.4	Disturbance of a Generalized Measurement	404
20.2.5	Minimal-disturbing Implementation of a POVM	405
20.3	Symmetric Matrices and their Structure	406
20.3.1	Representations of Finite Groups	407
20.3.2	Projective Representations	408
20.3.3	Symmetry of a Matrix and Schur's Lemma	410
20.3.4	Symmetric POVMs Define Matrices with Symmetry	411
20.4	Implementation of Symmetric POVMs	413
20.5	Cyclic and Heisenberg-Weyl Groups	416
20.5.1	Cyclic Groups	416
20.5.2	Heisenberg-Weyl Groups	419
20.6	Conclusions and Outlook	423
	References	424

21	Full Counting Statistics of Interacting Electrons	425
	<i>D. A. Bagrets, Y. Utsumi, D. S. Golubev, and Gerd Schön</i>	
21.1	Introduction	425
21.2	Concepts of FCS	428
21.3	Full Counting Statistics in Interacting Quantum Dots	435
21.3.1	FCS of a Set for Intermediate Strength Conductance	437
21.3.2	Non-Markovian Effects: Renormalization and Finite Lifetime Broadening of Charge States	440
21.3.3	Keldysh Action and CGF in Majorana Representation	442
21.4	FCS and Coulomb Interaction in Diffusive Conductors	443
21.4.1	Model and Effective Action	445
21.4.2	“Cold Electron” Regime	447
21.4.3	“Hot Electron” Regime	453
21.5	Summary	454
	References	455
22	Quantum Limit of the Carnot Engine	457
	<i>Friedemann Tonner and Günter Mahler</i>	
22.1	Introduction	457
22.2	Spin-oscillator Model	458
22.2.1	Basic Definitions	458
22.2.2	Thermodynamic Variables for G	461
22.3	Master Equation	462
22.3.1	Lindblad Superoperator	462
22.3.2	Time Slot Operators	463
22.4	Machine Cycles	465
22.4.1	Choice of Amplitudes $a_{\pm}^{(j)}$ and Control Functions $\theta^{(j)}(\tau)$	465
22.4.2	Heat and Work	467
22.4.3	Energy Balance	468
22.4.4	Fluctuations	468
22.5	Numerical Results	470
22.5.1	Heat Engine	470
22.5.2	Heat Pump	474
22.5.3	Longtime Limit	475
22.5.4	Quantum Limit and Classical Limit	475
22.6	Summary and Conclusions	477
	References	479
	Appendix: Colour Plates	481
	Index	491