

Electroweak observables in a general 5D background

Antonio Delgado^a and Adam Falkowski^{ab}

^a*CERN Theory Division,
CH-1211 Geneva 23, Switzerland*

^b*Institute of Theoretical Physics, Warsaw University,
Hoża 69, 00-681 Warsaw, Poland*

E-mail: antonio.delgado@cern.ch, adam.falkowski@cern.ch

ABSTRACT: Warped extra dimensions provide a new playground to study electroweak breaking and the nature of the higgs field. In this paper we reanalyze the electroweak observables in theories with one extra dimension and a completely general warp factor. We demonstrate that, regardless of the 5D background, $SU(2)_R$ is needed in order to avoid excessive contributions to the T parameter. For higgsless theories cancellations between different contributions to the S parameter are needed.

KEYWORDS: Beyond Standard Model, Technicolor and Composite Models, Higgs Physics.

Contents

1.	Introduction	1
2.	Tools	2
2.1	Kaluza-Klein expansion in general warped backgrounds	3
2.2	Effective standard model and dimension-six operators	5
3.	No custodial	7
4.	Custodial	10
5.	Higgsless	14
6.	Conclusions	16
A.	Integrals	17

1. Introduction

The *precise* way the electroweak symmetry is broken is the last aspect left to be discovered to have a complete description of particle physics based on the group $SU(3)_c \times SU(2)_L \times U(1)_Y$. It is therefore one of the key aspects to be studied at the Large Hadron Collider (LHC). In the standard model (SM) one single fundamental scalar field, the higgs, is responsible for the breaking but its nature and the reason for that scalar to break the electroweak symmetry is not explained. This is the main reason to go beyond the SM. The two standard approaches are either to explain why a fundamental scalar is not unstable under radiative corrections (SUSY) or to construct models for dynamical electroweak breaking (technicolor or a pseudo-goldstone higgs).

Just before LEP-II the former approach was favoured due to the problems of models with dynamical breaking with the electroweak observables [1]. But after LEP-II and the *no*-discovery of the higgs or any supersymmetric particle there has been a revival of interest in models with a composite higgs or even without a higgs. The progress in model-building has been possible thanks to the formulation of strongly coupled theories as models with a *warped* extra dimension [2].

Theories formulated in 5D with the geometry of a slice of AdS_5 are in close relation [3] with 4D strongly coupled, approximately conformal theories which eventually confine and have also a fundamental sector that couples weakly to these bound states. The bound states map to the KK modes in the 5D theory whereas the fundamental fields correspond

to the zero modes of the 5D theory. One of the main advantages of this formulation is that in 5D one can carry out calculations and have observable quantities under control.

In this spirit, the electroweak sector of the SM can be formulated as a 5D warped model with two boundaries, one being the UV and the second one the IR brane. Gauge bosons and chiral fermions are free to move in the bulk of the extra dimension. The breaking of the electroweak symmetry occurs via a localized vev on the IR brane, which means that the higgs does not propagate into the bulk [4]. The hierarchy problem is solved because the higgs is interpreted as a composite object beyond the IR scale and this scale is set to be around a TeV. One can even send its vev to infinity and have a theory without a higgs (light scalar resonance) in the spectrum [5]. A third possibility is to suppose that the higgs is a 5D field but embedded in a gauge multiplet, so that its mass is protected [6]. This gauge symmetry is broken via boundary conditions and the higgs field arises as the lowest mode of the fifth component of the gauge field. This is mapped in the 4D picture to a theory where the higgs is a pseudo goldstone boson corresponding to a spontaneously broken global symmetry of the strongly coupled sector.

All these theories have been analyzed and different electroweak parameters have been calculated [4, 7–10]. In this paper we are going to revisit the first two scenarios, those where the breaking is localized on the IR brane, but for a general warped metric and not only the one of AdS_5 . On the 4D side, this corresponds to departure from an approximate conformal symmetry of the strongly coupled sector. We calculate tree-level contributions to the electroweak parameters both in the case with a higgs localized on the IR brane and in the higgsless theory (we leave the study of a pseudo-goldstone higgs for future publications). We present general formulas for the spectrum of gauge bosons and their contributions to S , T and $Zb\bar{b}$ couplings. The main conclusion is that, also in general backgrounds, $\text{SU}(2)_R$ is needed to cancel large contributions to T . The S parameter is under control for the case with a higgs, while in the higgsless case cancellations are necessary to happen independent of the background metric.

The paper is organized as follows, in section 2 different general formulas for the spectrum and the matching conditions between the 5D and the 4D theories are given, in section 3 the SM is studied leading to a large contribution to the T parameter, $\text{SU}(2)_R$ is introduced in section 4, higgsless models are studied in section 5 and finally our conclusions are presented in section 6. Some technical details are given in the appendix.

2. Tools

In this section we present the formalism we employ in order to derive electroweak precision constraints on 5D gauge theories in warped backgrounds. We choose to work in the KK picture.¹ We diagonalize the 5D action in the KK basis in the presence of electroweak breaking on the IR brane. This approach is conceptually clear. Moreover, derivation of the tree-level effective action for the SM fields is simplified, as the zero mode fields do not

¹Another approach, so-called holographic [13], consists in integrating out the bulk degrees of freedom and writing down an effective action for the UV brane degrees of freedom. Physical results, of course, do not depend on the approach.

mix the with the heavy modes. Thus, the gauge boson masses and the vertex corrections can be read off directly from the KK diagonalized 5D action. This information allows, in particular, to determine the oblique S and T parameters, which typically encode the most stringent bounds on the model. Determination of four-fermion operators in the effective theory still requires computing diagrams with the heavy KK mode exchange, but in most situations those do not impose additional constraints.

Below we introduce the techniques that allow to perform the KK diagonalization and the integrating-out procedure for an arbitrary warped background. We also review the basic facts concerning the parametrization of physics beyond the SM by dimension six operators.

2.1 Kaluza-Klein expansion in general warped backgrounds

We study 5D gauge theories with the fifth dimension being an interval, $x_5 \in [0, L]$. The gravitational background is described by the line element

$$ds^2 = a^2(x_5)\eta_{\mu\nu}dx^\mu dx^\nu - dx_5^2. \tag{2.1}$$

with the warp factor $a(x_5)$. We fix $a(0) = 1$. The choice $a(x_5) = 1$ corresponds to 5D flat spacetime, while $a(x_5) = e^{-kx_5}$ corresponds to AdS₅. For most of the subsequent discussion we do not specify the warp factor. We only assume that it is a monotonic and non-increasing function, so that it makes sense to define a UV brane at $x_5 = 0$ and an IR brane at $x_5 = L$, where the value of the warp factor is $a(L) \equiv a_L \leq 1$.

Consider the quadratic action for a 5D gauge field propagating in a warped background

$$S_5 = \int d^4x \int_0^L dx_5 \left\{ -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2}a^2(x_5)(\partial_5 A_\mu)^2 + \frac{1}{2}L\tilde{m}_L^2 A_\mu^2 \delta(L) \right\} \tag{2.2}$$

The boundary mass term represents the effect of the boundary higgs field vev. We expand the 5D gauge field in the KK basis

$$A_\mu(x, x_5) = \sum_n f_n(x_5)A_{\mu,n}(x) \tag{2.3}$$

and choose the profiles such that the quadratic action, *in the presence of the boundary mass term*, can be rewritten as a 4D action diagonal in n :

$$S_5 = \int d^4x \sum_n \left\{ -\frac{1}{4}(\partial_\mu A_{\nu,n} - \partial_\nu A_{\mu,n})^2 + \frac{1}{2}m_n^2(A_{\mu,n})^2 \right\}. \tag{2.4}$$

In this way, any mixing between a possible massless mode and the heavy KK modes induced by the higgs vev has already been taken into account (to all orders in \tilde{m}_L^2). The diagonalization is achieved if the profiles solve the bulk equation of motion:

$$\left(\partial_5^2 + 2\frac{a'}{a}\partial_5 + \frac{m_n^2}{a^2} \right) f_n(x_5) = 0 \tag{2.5}$$

and satisfy appropriate boundary conditions. On the UV brane, in absence of any localized mass or kinetic terms, these are the Neumann or Dirichlet boundary conditions,

$$\partial_5 f_n(0) = 0 \quad \text{or} \quad f_n(0) = 0 \tag{2.6}$$

On the IR brane we should impose

$$\partial_5 f_n(L) = -La_L^{-2} \tilde{m}_L^2 f_n(L) \tag{2.7}$$

The Dirichlet boundary conditions can be simulated in the limit $\tilde{m}_L^2 \rightarrow \infty$ (we call it the *higgsless limit*). The profiles should also satisfy the normalization condition $\int_0^L |f(y)|^2 = 1$.

The usual procedure is to solve the equations of motion eq. (2.5) for some particular background. In this paper we show how to obtain results valid for an arbitrary warp factor. To proceed, we denote the two independent solutions of eq. (2.5) by $C(x_5, m_n)$ and $S(x_5, m_n)$. We choose them such that they satisfy the initial conditions $C(0, m_n) = 1$, $C'(0, m_n) = 0$, $S(0, m_n) = 0$, $S'(0, m_n) = m_n$. These functions can be viewed as a warped generalization of the cosines and sines (in the flat background $C = \cos(x_5 m_n)$, $S = \sin(x_5 m_n)$). Using them, we can write down the profiles in a compact form. For example, a profile with the Neumann boundary conditions in the UV is written as $f_n(x_5) = \alpha_n C(x_5, m_n)$, where α_n is fixed by the normalization condition. The spectrum of the KK modes is determined by the IR boundary condition that, in this language, is written as $C'(L, m_n) = -La_L^{-2} \tilde{m}_L^2 C(L, m_n)$.

Our basic tool will be the expansion of the profiles corresponding to light fields in powers of m_n . Solving eq. (2.5) perturbatively in m_n we can expand the two solutions as

$$\begin{aligned} C(x_5, m_n) &= 1 - m_n^2 \int_0^{x_5} dy y a^{-2}(y) + \mathcal{O}(m_n^4) \\ S(x_5, m_n) &= m_n \int_0^{x_5} dy a^{-2}(y) + \mathcal{O}(m_n^3) \end{aligned} \tag{2.8}$$

We will employ this expansion for the profiles of the W and Z boson, whose masses are of the order of the electroweak scale. This makes sense when the electroweak scale is hierarchically smaller than the KK scale defined² as

$$M_{\text{KK}} = \frac{\pi}{\int_0^L dy a^{-1}(y)} \tag{2.9}$$

In any realistic set-up, a mass gap between the SM gauge fields and heavy resonances must be large enough to justify this expansion. Technically speaking, there are two ways to introduce the mass gap. One is to introduce it by hand by choosing $\tilde{m}_L/M_{\text{KK}} \ll 1$. In such a case the electroweak scale is of order \tilde{m}_L . In our setup the ratio $\tilde{m}_L/M_{\text{KK}}$ can be made arbitrarily small, however as soon as $M_{\text{KK}} \gg 1 \text{ TeV}$ we face the hierarchy problem. The mass gap may also exist in the higgsless limit when $\tilde{m}_L \rightarrow \infty$. In that case the electroweak (lightest resonance) scale and the KK (heavy resonance) scale are related by

$$m_W^2 \sim \frac{1}{\int_0^L ya^{-2}(y)} \sim \frac{2}{\pi^2} \frac{M_{\text{KK}}^2}{\mathcal{V}}. \tag{2.10}$$

If the warp factor decreases sharply toward the IR brane the denominator scales linearly with the size of the extra dimension L . To stress this, we introduced the *volume factor* \mathcal{V}

²This scale is parametrically of the order of the mass of light spin 1 resonances. In 5D Minkowski the first KK photon mass is exactly equal to M_{KK} , while in AdS₅ it is approximately $3/4 M_{\text{KK}}$.

defined as

$$\mathcal{V} = La_L^{-1}M_{\text{KK}}/\pi \tag{2.11}$$

To obtain the scale separation between the electroweak and the KK scale we need \mathcal{V} large enough. For backgrounds that solve the hierarchy problem we indeed expect the volume factor to be large. The argument is purely heuristic. Typically, solving the hierarchy problem involves generating the huge ratio M_{Pl}/TeV from a moderate number, say, of order 4π . Moreover, the hierarchy should be generated dynamically by stabilization, thus our moderate number should be somehow correlated with L . Next, we have $M_{\text{KK}} \sim a_L k$ where $k \sim a'(L)/a_L$ is the scale that describes how quickly the warp factor changes close to the IR brane. Then kL is a dimensionless parameter which we may identify with the one that generates hierarchy. This is of course not a proof and one can certainly find counterexamples, if the warp factor is complicated enough. For simple warp factors, however, the argument works. For example, for the AdS_5 background the volume factor is $\mathcal{V} = \log a_L^{-1} \sim \log(M_{\text{Pl}}/M_{\text{KK}}) \sim 30$. Thus, the volume factor is large when the hierarchy is generated by a dynamics that is approximately conformal over a large range of scales. On the other hand, in the flat space (which does not address the hierarchy problem) $\mathcal{V} = 1$.

A similar background independent formalism can be worked out for fermions. We do not review it here since we limit our study to gauge boson contribution, but see [14]. See also refs. [11, 12] for another background independent approach via sum rules.

2.2 Effective standard model and dimension-six operators

We write the effective low energy theory as $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{D6}$ where \mathcal{L}_{SM} is the electroweak part of the SM lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{SM}} &= -\frac{1}{2}\text{Tr}\{L_{\mu\nu}L_{\mu\nu}\} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} + i\sum_j \bar{\psi}_j\gamma_\mu D_\mu\psi_j + |D_\mu H|^2 - V(H) + \text{Yukawa} \\ D_\mu\psi_j &= (\partial_\mu - ig_L L_\mu^a T^a - ig_Y Y_j B_\mu)\psi_j \\ D_\mu H &= \left(\partial_\mu - ig_L L_\mu^a T^a - ig_Y \frac{1}{2}B_\mu\right)H \end{aligned} \tag{2.12}$$

and \mathcal{L}_{D6} are the dimension-six operators

$$\begin{aligned} \mathcal{L}_{D6} &= \alpha_T |H^\dagger D_\mu H|^2 + \alpha_S (H^\dagger T^a H) W_{\mu\nu}^a B_{\mu\nu} \\ &\quad - \left\{ i\beta_j g_L^2 (\bar{\psi}_j\gamma_\mu t^a\psi_j)(D_\mu H^\dagger t^a H) + i\frac{\gamma_j}{2} Y_j g_Y^2 \bar{\psi}_j\gamma_\mu\psi_j (D_\mu H^\dagger H) + \text{h.c.} \right\} \\ &\quad + \text{fermion}^4 \end{aligned} \tag{2.13}$$

The four-fermion terms will be ignored in the following (in this paper we restrict our discussion to the situations where they do not introduce significant constraints). There are also other dimension-six operators that can be generated by 5D physics (for example, $(\bar{\psi}_j\gamma_\mu t^a\psi_k)(H^T t^a D_\mu H)$) but are ignored here because they do not get large contributions from KK gauge bosons.

When the Higgs field acquires the vev we define the photon, W and Z as usual,

$$\begin{aligned}
 W_\mu^\pm &= \frac{1}{\sqrt{2}}(L_\mu^1 \mp iL_\mu^2) \\
 A_\mu &= \frac{1}{\sqrt{g_L^2 + g_Y^2}}(g_Y L_\mu^3 + g_L B_\mu) \\
 Z_\mu &= \frac{1}{\sqrt{g_L^2 + g_Y^2}}(g_L L_\mu^3 - g_Y B_\mu)
 \end{aligned}
 \tag{2.14}$$

but their masses and interactions are modified by the dimension six operators. The vertex correction β_j and γ_j modify the interactions of the SM fermions with the W and Z bosons:

$$\begin{aligned}
 \mathcal{L}_{\text{eff}} \rightarrow & \frac{gLg_Y}{\sqrt{g_L^2 + g_Y^2}}(t_i^3 + Y_i)\bar{\psi}_i\gamma_\mu\psi_i A_\mu + \frac{g_L}{\sqrt{2}}(1 + m_W^2\beta_j)\bar{\psi}_j\gamma_\mu t^\pm\psi_j W_\mu^\pm \\
 & + \frac{1}{\sqrt{g_L^2 + g_Y^2}}(g_L^2(1 + \beta_j m_Z^2)t_j^3 - g_Y^2(1 + \gamma_j m_Z^2)Y_j)\bar{\psi}_j\gamma_\mu\psi_j Z_\mu
 \end{aligned}
 \tag{2.15}$$

The Z boson mass is modified by α_T

$$m_W^2 = \frac{g_L^2 v^2}{4} \quad m_Z^2 = \frac{(g_L^2 + g_Y^2)v^2}{4} \left(1 + \frac{v^2}{2}\alpha_T\right)
 \tag{2.16}$$

Finally, α_S mixes the photon and the Z boson,

$$\mathcal{L}_{D6} \rightarrow -\frac{1}{4}\alpha_S v^2 L_{\mu\nu}^3 B_{\mu\nu}
 \tag{2.17}$$

We can adjust the coefficients of the dimension-six operators to match the effective lagrangian obtained by integrating out the KK modes. Note that this set of coefficient is redundant: the universal shift of the vertex corrections can be absorbed by redefinitions of the gauge couplings [4]. We can thus shift β_j and γ_j by $\Delta\beta$ and $\Delta\gamma$ without changing the physical content of the theory, provided that α_T and α_S are also shifted accordingly:

$$\begin{aligned}
 \beta_j &\rightarrow \beta_j + \Delta\beta \\
 \gamma_j &\rightarrow \gamma_j + \Delta\gamma \\
 \alpha_S &\rightarrow \alpha_S - \frac{gLg_Y}{2}(\Delta\beta + \Delta\gamma) \\
 \alpha_T &\rightarrow \alpha_T + g_Y^2\Delta\gamma
 \end{aligned}
 \tag{2.18}$$

One particular application of this result is when the vertex corrections are universal: $\beta_j = \beta$ and $\gamma_j = \gamma$. Then, choosing $\Delta\beta = -\beta$, $\Delta\gamma = -\gamma$ we can get rid of the vertex corrections, which reemerge as a shift of α_T and α_S . This is the *oblique case*, in which all the corrections from new physics can be parametrized by α_T and α_S . Those two are simply related to the familiar S and T parameters:

$$S = \frac{8\pi v^2}{gLg_Y}\alpha_S \quad T = -\frac{2\pi v^2}{e^2}\alpha_T
 \tag{2.19}$$

3. No custodial

We first consider a 5D model without a custodial symmetry. The bulk gauge symmetry is that of the SM, $SU(3)_c \times SU(2)_L \times U(1)_Y$. The electroweak group is broken to $U(1)_{em}$ by a higgs doublet H localized on the IR brane. The 5D action for the electroweak gauge bosons and the higgs reads

$$S = \int d^4x \int_0^L dx_5 \sqrt{g} \left\{ -\frac{1}{2} \text{Tr}\{L_{MN}L_{MN}\} - \frac{1}{4} B_{MN}B_{MN} \right\} + \int d^4x dx_5 \sqrt{g_4} \delta(L) \{ |D_\mu H|^2 - V(H) \}, \quad (3.1)$$

where $\partial_\mu H = (\partial_\mu - i\sqrt{L}g_L B_\mu - i\sqrt{L}g_Y Y_H B_\mu)H$ and $\sqrt{L}g_L, \sqrt{L}g_Y$ are the dimensionful gauge couplings of $SU(2)_L \times U(1)_Y$. The Higgs field acquires a vev $\langle H \rangle = (0, \tilde{v}a_L^{-1}/\sqrt{2})^T$. This results in the IR brane mass terms

$$\mathcal{L} = \delta(L) \left(\frac{1}{2} L \frac{g_L^2 \tilde{v}^2}{4} |W_\mu^+|^2 + \frac{1}{2} L \frac{(g_L^2 + g_Y^2) \tilde{v}^2}{4} |Z_\mu|^2 \right). \quad (3.2)$$

On the UV brane we impose the Neumann boundary conditions on the profiles

$$\partial_5 f_n^\gamma(0) = \partial_5 f_n^W(0) = \partial_5 f_n^Z(0) = 0 \quad (3.3)$$

so that no gauge symmetry gets broken there. Using the formalism introduced in section 2, the gauge boson profiles satisfying the UV boundary condition can be written as

$$f_n^W(x_5) = c_{W,n} C(x_5, m_{W,n}) \quad f_n^Z(x_5) = c_{Z,n} C(x_5, m_{Z,n}) \quad f_n^\gamma(x_5) = c_{\gamma,n} C(x_5, m_{\gamma,n}) \quad (3.4)$$

On the IR brane, in the presence of the boundary mass terms, the boundary conditions read

$$\partial_5 f_n^\gamma(L) = 0 \quad \partial_5 f_n^W(L) = -\frac{Lg_L^2 \tilde{v}^2}{4a_L^2} f_n^W(L) \quad \partial_5 f_n^Z(L) = -\frac{L(g_L^2 + g_Y^2) \tilde{v}^2}{4a_L^2} f_n^Z(L) \quad (3.5)$$

Those imply the quantization condition for the photon, W and Z mass towers

$$\begin{aligned} C'(L, m_{\gamma,n}) &= 0 \\ C'(L, m_{W,n}) + \frac{Lg_L^2 \tilde{v}^2}{4a_L^2} C(L, m_{W,n}) &= 0 \\ C'(L, m_{Z,n}) + \frac{L(g_L^2 + g_Y^2) \tilde{v}^2}{4a_L^2} C(L, m_{Z,n}) &= 0 \end{aligned} \quad (3.6)$$

Let us concentrate on the zero modes (we will omit the $n = 0$ index). The photon profile is a constant

$$f^\gamma(x_5) = \frac{1}{\sqrt{L}} \quad m_\gamma = 0 \quad (3.7)$$

The W and Z profiles are non-trivial due to the boundary mass terms and their shapes depend on the background geometry. However, we dispose of a small parameter — the

ratio of the gauge boson masses to the KK scale — in which we can expand the deviation of the profile from a constant. We find

$$\begin{aligned} f^W(x_5) &= \frac{1}{\sqrt{L}} \left(1 + m_W^2 [I_1(L) - I_2(x_5)] + \mathcal{O}(m_W^4/M_{KK}^4) \right) \\ f^Z(x_5) &= \frac{1}{\sqrt{L}} \left(1 + m_Z^2 [I_1(L) - I_2(x_5)] + \mathcal{O}(m_Z^4/M_{KK}^4) \right) \end{aligned} \quad (3.8)$$

where the integrals I_n depend on the warp factor and are defined in A. We can insert this expansion into the boundary conditions (3.4). As long as \tilde{v} much smaller than the compactification scale, the gauge boson masses can be perturbatively expanded in \tilde{v}^2 . Defining the electroweak scale v by

$$v^2 = \tilde{v}^2 \left(1 + \frac{g_L^2 \tilde{v}^2}{4} [I_1(L) - I_2(L)] \right) \quad (3.9)$$

we can write the gauge boson masses as

$$m_W^2 = \frac{g_L^2 v^2}{4} \quad m_Z^2 = \frac{(g_L^2 + g_Y^2) v^2}{4} \left(1 + \frac{g_Y^2 v^2}{4} [I_1(L) - I_2(L)] \right). \quad (3.10)$$

We move to discussing the fermionic sector of the model. The fermions can be simply realized by assigning a 5D bulk field to each SM fermion. For example, one quark generation is contained in the 5D fields

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \quad u^c \quad d^c \quad \mathbf{2}_{1/6} \quad \mathbf{1}_{2/3} \quad \mathbf{1}_{-1/3} \quad (3.11)$$

with the action

$$\begin{aligned} S &= \int d^4x \int_0^L dx_5 \sqrt{g} \{ \bar{q} (i\Gamma_N D_N + M_q) q + \bar{u}^c (i\Gamma_N D_N - M_u) u^c \\ &\quad + \bar{d}^c (i\Gamma_N D_N - M_d) d^c \} - \int d^4x dx_5 \sqrt{g_4} \delta(L) \left(\tilde{y}_u \bar{q}_L H^\dagger u_R^c + \tilde{y}_d \bar{q}_L H d_R^c + \text{h.c.} \right) \end{aligned} \quad (3.12)$$

For the light generations we can ignore the mixing between the zero modes and the heavy KK modes. The zero mode profiles are then

$$f_L^q \approx \frac{a^{-2}(x_5) e^{-M_q x_5}}{\int_0^L a^{-1}(y) e^{-2M_q y}} \quad f_R^{u^c} \approx \frac{a^{-2}(x_5) e^{-M_u x_5}}{\int_0^L a^{-1}(y) e^{-2M_u y}} \quad f_R^{d^c} \approx \frac{a^{-2}(x_5) e^{-M_d x_5}}{\int_0^L a^{-1}(y) e^{-2M_d y}} \quad (3.13)$$

and the quark masses are related to the boundary Yukawa couplings by

$$\begin{aligned} m_u^2 &\approx \frac{a_L^{-2} e^{-(M_q + M_u)L} (y_u \tilde{v})^2}{\int_0^L a^{-1}(y) e^{-2M_q y} \int_0^L a^{-1}(y) e^{-2M_u y}} \\ m_d^2 &\approx \frac{a_L^{-2} e^{-(M_q + M_d)L} (y_d \tilde{v})^2}{\int_0^L a^{-1}(y) e^{-2M_q y} \int_0^L a^{-1}(y) e^{-2M_d y}} \end{aligned} \quad (3.14)$$

We can now write down the interactions of the light fermions with the light gauge bosons

1. Electromagnetic currents:

$$\mathcal{L}_{em} = e(t_j^3 + Y_j)\bar{\psi}_j\gamma_\mu\psi_j A_\mu \quad e = \frac{g_L g_Y}{\sqrt{g_L^2 + g_Y^2}} \quad (3.15)$$

2. Charged currents:

$$\mathcal{L}_{cc} = \frac{g_L}{\sqrt{2}}[1 + m_W^2(I_2(L) - J_2(L, M_j))]\bar{\psi}_j\gamma_\mu t_j^\pm\psi_j W_\mu^\pm + \text{h.c.} \quad (3.16)$$

3. Neutral currents:

$$\mathcal{L}_{ncL} = \frac{g_L^2 t_j^3 - g_Y^2 Y_j}{\sqrt{g_L^2 + g_Y^2}}[1 + m_Z^2(I_2(L) - J_2(L, M_j))]\bar{\psi}_j\gamma_\mu\psi_j Z_\mu \quad (3.17)$$

These gauge interactions and the corrections to the Z boson mass can be reproduced by the SM lagrangian supplemented by the dimensions six operators defined in eq. (2.13). We find the coefficients

$$\begin{aligned} \alpha_T &= -\frac{1}{2}g_Y^2 (I_1(L) + I_2(L)) \\ \alpha_S &= g_L g_Y I_1(L) \\ \beta_j &= -J_2(L, M_j) \\ \gamma_j &= -J_2(L, M_j) \end{aligned} \quad (3.18)$$

The vertex corrections are in general non-universal, so that the corrections are not oblique. However, when the fermions are localized near the UV brane, the vertex corrections are negligibly small. This is the typical assumption for the first two generations of the SM fermions. In such case the corrections can be treated as oblique and adequately parametrized by the familiar S and T:

$$S = 8\pi v^2 I_1(L) \quad T = \frac{4\pi m_Z^2}{g_L^2} (I_1(L) + I_2(L)) \quad (3.19)$$

Because of the volume enhancement, the integral I_2 dominates for backgrounds with the large volume factor. Thus we get an approximate expression for the T parameter:

$$T \approx 4\pi m_Z^2 g_L^{-2} \int_0^{x_5} y a^{-2}(y) \sim \frac{2\pi^3 m_Z^2}{g_L^2 M_{KK}^2} \mathcal{V} \quad (3.20)$$

Here \mathcal{V} is the volume factor introduced in eq. (2.11). For backgrounds that solve the hierarchy problem we expect the volume factor to be large, which would strongly enhance the contribution to T. For example, in the Randall-Sundrum model $\mathcal{V} \sim \log(M_{Pl}/M_{KK}) \sim 30$ leading to a very strong constraint on the KK scale [7]. Our results show that the problem persists in any 5D warped model without a custodial symmetry, in which the solution to the hierarchy problem is associated with a moderately large volume factor.

4. Custodial

The well-known cure for an excessive T parameter is the custodial $SU(2)_R$ symmetry. In the context of 5D theories the custodial symmetry is promoted to a gauge symmetry. The hypercharge group is extended to $SU(2)_R \times U(1)_X$ that is broken to $U(1)_Y$ on the UV brane [4]. Thus, the bulk gauge symmetry is $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$. The IR brane higgs field Φ is in the $(2, \bar{2})_0$ representation with respect to $SU(2)_L \times SU(2)_R \times U(1)_X$. Below we apply our background independent techniques to this class of models.

The 5D action for the (extended) electroweak sector reads

$$S = \int d^4x \int_0^L dx_5 \sqrt{g} \left\{ -\frac{1}{2} \text{Tr}\{L_{MN}L_{MN}\} - \frac{1}{2} \text{Tr}\{R_{MN}R_{MN}\} - \frac{1}{4} X_{MN}X_{MN} \right\} \\ + \int d^4x dx_5 \sqrt{g_4} \delta(L) \left(\frac{1}{4} \text{Tr}|D_\mu \Phi|^2 - V(\Phi) \right) \tag{4.1}$$

The higgs field acquires the vev $\langle \Phi \rangle = \frac{\tilde{v}}{\alpha_L} I_{2 \times 2}$. This results in the mass terms

$$\mathcal{L}_{\text{mass}} = \frac{1}{8} L \tilde{v}^2 \delta(L) (g_L L_\mu^a - g_R R_\mu^a)^2 \tag{4.2}$$

that spontaneously break $SU(2)_L \times SU(2)_R$ to $SU(2)_V$ on the IR brane.

The UV boundary conditions that break $SU(2)_R \times U(1)_X$ down to $U(1)_Y$ impose the following conditions on the KK profiles:

$$\begin{aligned} \partial_5 f_{L,n}^a(0) &= 0 & a &= 1, 2, 3 \\ f_{R,n}^i(0) &= 0 & i &= 1, 2 \\ s_x \partial_5 f_{R,n}^3(0) + c_x \partial_5 f_{X,n}(0) &= 0 & s_x &= \frac{g_X}{\sqrt{g_X^2 + g_R^2}} \\ -c_x f_{R,n}^3(0) + s_x f_{X,n}(0) &= 0 & c_x &= \frac{g_R}{\sqrt{g_X^2 + g_R^2}} \end{aligned} \tag{4.3}$$

The profiles that solve the bulk equations of motion and satisfy the UV boundary conditions in our notation are written as

$$\begin{aligned} f_{L,n}^a(x_5) &= \alpha_{L,n}^a C(x_5, m_n) \\ f_{R,n}^i(x_5) &= \alpha_{R,n}^i S(x_5, m_n) \\ f_{R,n}^3(x_5) &= \alpha_{N,n} s_x C(x_5, m_n) - \alpha_{D,n} c_x S(x_5, m_n) \\ f_{X,n}(x_5) &= \alpha_{N,n} c_x C(x_5, m_n) + \alpha_{D,n} s_x S(x_5, m_n) \end{aligned} \tag{4.4}$$

The linear combination $B_\mu = s_x R_\mu^3 + c_x X_\mu$ survives on the UV brane and its zero mode is identified with the hypercharge gauge boson. B_μ couples to matter with the coupling $g_Y = g_X g_R / \sqrt{g_X^2 + g_R^2}$ and the hypercharge depends on the $SU(2)_R \times U(1)_X$ quantum numbers via $Y = t_R^3 + X$.

In the presence of the higgs vev, the IR boundary conditions for the profiles read

$$\begin{aligned}
 \partial_5 f_{X,n}(L) &= 0 \\
 g_R \partial_5 f_{L,n}^a(L) + g_L \partial_5 f_{R,n}^a(L) &= 0 \\
 g_L \partial_5 f_{L,n}^a(L) - g_R \partial_5 f_{R,n}^a(L) &= -\frac{1}{4}(g_L^2 + g_R^2) a_L^{-2} L \tilde{v}^2 (g_L f_{L,n}^a(L) - g_R f_{R,n}^a(L)) \quad (4.5)
 \end{aligned}$$

We can now solve these equations to find the mass eigenstates. The spectrum contains a tower of charge ± 1 massive vector bosons whose masses are given by the solutions of the quantization condition:

$$S'(L, m_n^W) C'(L, m_n^W) + \frac{a_L^{-2} L \tilde{v}^2}{4} (g_L^2 S'(L, m_n^W) C(L, m_n^W) + g_R^2 S(L, m_n^W) C'(L, m_n^W)) = 0 \quad (4.6)$$

and a tower of electrically neutral massive vector boson with masses

$$\begin{aligned}
 S'(L, m_n^Z) C'(L, m_n^Z) + \frac{a_L^{-2} L \tilde{v}^2}{4} \\
 (g_L^2 S'(L, m_n^Z) C(L, m_n^Z) + g_R^2 S(L, m_n^Z) C'(L, m_n^Z) + m_n^Z a_L^{-2} g_Y^2) = 0 \quad (4.7)
 \end{aligned}$$

There is also another tower of neutral vector bosons with masses given by

$$C'(L, m_n^\gamma) = 0 \quad (4.8)$$

which always includes the photon solution with $m_\gamma = 0$. The lightest solutions of eqs. (4.6) and (4.7) are identified with the W and Z boson masses.

We turn to discussing the profiles of the zero mode fields that correspond to the SM gauge bosons. In phenomenologically viable models these profiles can always be expanded in powers of the gauge boson masses over the KK scale. To first order³ m^2/M_{KK}^2 we find the SM gauge bosons are embedded into the 5D fields as follows

$$\begin{aligned}
 L_\mu^i(x, x_5) &\rightarrow \frac{1}{\sqrt{L}} (1 + m_W^2 [I_1(L) - I_2(x_5)]) W_\mu^i(x) \\
 R_\mu^i(x, x_5) &\rightarrow \frac{1}{\sqrt{L}} m_W^2 \frac{g_R}{g_L} I_3(x_5) W_\mu^i(x) \quad (4.9) \\
 L_\mu^3(x, x_5) &\rightarrow \frac{1}{\sqrt{L}} (\sin \theta_W A_\mu(x) \\
 &\quad + \cos \theta_W (1 + m_Z^2 [I_1(L) - I_2(x_5)]) Z_\mu(x)) \\
 R_\mu^3(x, x_5) &\rightarrow \frac{s_x}{\sqrt{L}} (\cos \theta_W A_\mu(x) \\
 &\quad - \sin \theta_W \left(1 + m_Z^2 \left[I_1(L) - I_2(x_5) + I_3(x_5) - \frac{1}{s_x^2} I_3(x_5) \right] \right) Z_\mu(x)) \\
 X_\mu(x, x_5) &\rightarrow \frac{c_x}{\sqrt{L}} (\cos \theta_W A_\mu(x) \\
 &\quad - \sin \theta_W (1 + m_Z^2 [I_1(L) - I_2(x_5) + I_3(x_5)]) Z_\mu(x)) \quad (4.10)
 \end{aligned}$$

³At the second order there exist m_W^4/M_{KK}^4 terms enhanced by the volume factor \mathcal{V} . Those terms can be neglected if $m_W^2/M_{\text{KK}}^2 < 1/\mathcal{V}$, which we assume in this section. On the other hand, in the higgsless case $m_W^2 \sim M_{\text{KK}}^2/\mathcal{V}$ and these terms have to be retained.

The integrals I_n are defined in A. Inserting this expansion into the 5D action we can read off the interactions between the SM gauge bosons and fermions. In general we should also perform the analogous KK expansion for the fermions. For the first two generations it is enough to convolute the gauge profiles with the massless fermionic profile

$$f_j(x_5) = \frac{e^{-M_j x_5} a^{-2}(x_5)}{\left(\int_0^L e^{-2M_j y} a^{-1}(y)\right)^{1/2}} \quad (4.11)$$

and the corrections are of order m_j^2/M_{KK}^2 . We find

1. Electromagnetic current:

$$\mathcal{L}_{em} = \frac{g_L g_Y}{\sqrt{g_L^2 + g_Y^2}} (t_{L,j}^3 + t_{R,j}^3 + X_j) \bar{\psi}_j \gamma_\mu \psi_j A_\mu \quad (4.12)$$

2. Charged current:

$$\mathcal{L}_{cc} = \frac{g_L}{\sqrt{2}} (1 + m_W^2 [I_1(L) - J_2(L, M_j)]) \bar{\psi}_j \gamma_\mu t_j^\pm \psi_j W_\mu^\pm \quad (4.13)$$

3. Neutral current:

$$\begin{aligned} \mathcal{L}_{ncL} = \frac{1}{\sqrt{g_L^2 + g_Y^2}} \{ & g_L^2 t_{L,j}^3 (1 + m_Z^2 [I_1(L) - J_2(L, M_j)]) \\ & - g_Y^2 Y_j (1 + m_Z^2 [I_1(L) - J_2(L, M_j) + J_3(L, M_j)]) \\ & + g_R^2 t_{R,j}^3 m_Z^2 J_3(L, M_j) \} \bar{\psi}_j \gamma_\mu \psi_j Z_\mu \end{aligned} \quad (4.14)$$

At the zeroth order these are just the SM gauge interactions. The corrections of order m^2/M_{KK}^2 depend not only on the bulk mass M_j but also on the embedding of the SM fermions into $SU(2)_R$ representations. There are several choices one can make. The simplest is to embed the SM left doublets into $SU(2)_R$ singlets and the SM left singlets into $SU(2)_R$ doublets [4]. For example, the lightest quark generation can be embedded as follows

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \quad u = \begin{pmatrix} u^c \\ \tilde{d}^c \end{pmatrix} \quad d = \begin{pmatrix} \tilde{u}^c \\ d^c \end{pmatrix} \quad (4.15)$$

$(\mathbf{2}, \mathbf{1})_{1/6} \quad (\mathbf{1}, \mathbf{2})_{1/6} \quad (\mathbf{1}, \mathbf{2})_{1/6}$

The singlet quarks should be put into two different bulk multiplet to give mass for both the u and d quarks. The tilded fermions can be removed from the low energy spectrum by $SU(2)_R$ breaking boundary conditions on the UV brane. Another possibility is to embed the SM left doublets into the bifundamental representation of $SU(2)_L \times SU(2)_R$ [15]. Then the SM left singlets should be placed into $SU(2)_R$ singlets or triplets. For example [17]

$$Q = \begin{bmatrix} u & \chi \\ d & \tilde{u} \end{bmatrix} \quad u^c \quad D = \begin{bmatrix} \frac{1}{\sqrt{2}} \tilde{u}^c & \chi^c \\ d^c & -\frac{1}{\sqrt{2}} \tilde{u}^c \end{bmatrix} \quad (4.16)$$

$(\mathbf{2}, \mathbf{\bar{2}})_{2/3} \quad (\mathbf{1}, \mathbf{1})_{2/3} \quad (\mathbf{1}, \mathbf{3})_{2/3}$

Here χ^c is an exotic charge 5/3 quark. Again, by an appropriate choice of boundary conditions we can ensure that only the SM quarks show up in the low-energy spectrum.

If the boundary Higgs vev is much smaller than the KK scale we can also solve eqs. (4.6) and (4.7) perturbatively in $\tilde{v}^2/M_{\text{KK}}^2$. Defining

$$v^2 = \tilde{v}^2 \left(1 + \frac{\tilde{v}^2}{4} [g_L^2 [I_1(L) - I_2(L)] - g_R^2 I_3(L)] \right) + \mathcal{O}(\tilde{v}^6) \quad (4.17)$$

the gauge boson masses are given by

$$m_W^2 = \frac{g_L^2 v^2}{4} \quad m_Z^2 = \frac{(g_L^2 + g_Y^2) v^2}{4} \left(1 + \frac{g_Y^2 v^2}{4} [I_1(L) + I_4(L)] \right) + \mathcal{O}(v^6) \quad (4.18)$$

We have now all the necessary information to read off the coefficient of the dimension six operators defined in eq. (2.13)

$$\begin{aligned} \alpha_T &= \frac{1}{2} g_Y^2 (-I_1(L) + I_4(L)) \\ \alpha_S &= g_L g_Y I_1(L) \\ \beta_j &= -J_2(L, M_j) \\ \gamma_j &= -J_2(L, M_j) + J_3(L, M_j) - \frac{g_R^2 t_{R,j}^3}{g_Y^2 Y_j} J_3(L, M_j) \end{aligned} \quad (4.19)$$

The vertex corrections are non-universal. They depend on both the bulk fermion masses and the embedding into $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_X$. Note that in this case the non-universality persists even when all M_j are equal. The vertex corrections can be however safely neglected for fermions localized close to the UV brane. In such case the corrections can be adequately parametrized by the S and T parameters:

$$S = 8\pi v^2 I_1(L) \quad T = 4\pi m_Z^2 g_L^{-2} (I_1(L) - I_4(L)) \quad (4.20)$$

The oblique parameters do not depend on the fermionic representation. The S parameter is given by exactly the same integral as in the case without the custodial symmetry. For the T parameter the custodial symmetry is at work: the combination of the integrals that enters there is suppressed, rather than enhanced, by the volume factor \mathcal{V} . Thus the tree-level T parameter ends up being tiny for the backgrounds with a large volume factor.

For the third generation fermions we cannot assume that their profiles are localized in the UV since some relevant coupling to the IR brane is needed to give mass to the top quark. Thus their interaction vertices with the SM gauge bosons can receive sizable corrections. Parameterizing the Z-boson vertex as

$$\mathcal{L}_{Zjj} = \frac{g_L^2 t_{L,j}^3 - g_Y^2 Y_j}{\sqrt{g_L^2 + g_Y^2}} (1 + \delta g_{Zjj}) \bar{\psi}_j \gamma_\mu \psi_j Z_\mu \quad (4.21)$$

we find the corrections from the gauge bosons exchange⁴

$$\delta g_{Zjj} = m_Z^2 (J_3(L, M_j) - J_2(L, M_j)) - m_Z^2 J_3(L, M_j) \frac{g_L^2 t_{L,j}^3 - g_R^2 t_{R,j}^3}{g_L^2 t_{L,j}^3 - g_Y^2 Y_j} \quad (4.22)$$

The second term, if not suppressed by the bulk mass exponential, is volume enhanced and typically dominates the vertex correction:

$$\delta g_{Zjj} \approx - \frac{g_L^2 t_{L,j}^3 - g_R^2 t_{R,j}^3}{g_L^2 t_{L,j}^3 - g_Y^2 Y_j} m_Z^2 L \frac{\int_0^L a^{-1}(y) e^{-2M_j y} \int_0^y a^{-2}(y')}{\int_0^L a^{-1}(y) e^{-2M_j y}} \quad (4.23)$$

If $e^{-2M_j y} < a^{-1}(y)$ close to the IR brane this expression scales linearly with L , so that $\delta g_{Zjj} \sim \mathcal{V} m_Z^2 / M_{\text{KK}}^2$, similarly as T in the absence of the custodial symmetry. This can be dangerous for the $Z b_L \bar{b}_L$ vertex that is well constrained by experiment, $\delta_{Z b_L \bar{b}_L} < 0.0025$. Indeed, in some cases, e.g. for the pseudo-goldstone higgs, this provides the tightest constraint on the parameter space [8]. One can however introduce a symmetry that allows to keep this vertex under control [15] (for another approach, see [16]). As can be seen from eq. (4.22), we need a LR parity symmetry that sets $g_L = g_R$. Then the second term vanishes if b_L has $t_L^3 = t_R^3$. This is possible if the third generation left doublet originates from the $(2, \bar{2})$ representation of $SU(2)_L \times SU(2)_R$. For example, the embedding in eq. (4.16) satisfies this requirement, although there we need additional multiplets in the $(3, 1)_{2/3}$ representation to keep the LR parity stable. The symmetry advocated in [15] also protects the vertex against order m_T^2 / M_{KK}^2 corrections from a mixing of the bottom quark with the fermionic KK modes. When the volume enhanced contribution is canceled there still remains an order m_Z^2 / M_{KK}^2 contribution from the first term in eq. (4.22). However, it always represents a weaker constraint on the KK scale than that from the S parameter.

5. Higgsless

The study we performed is slightly modified in the higgsless limit $\tilde{v} \rightarrow \infty$. Below we consider the custodially symmetric model of section 4 (that of section 3 is totally unrealistic in this limit, as it predicts $m_W = m_Z$). We cannot, of course, expand in powers of $\tilde{v} / M_{\text{KK}}$ anymore. Thus, eq. (4.17) and the following expression for the gauge boson masses are no longer valid. On the other hand, the ratio of the gauge boson masses to the KK scale remains a perfect expansion parameter in any realistic setup (including the AdS_5 background). Nevertheless, there is one qualitative change. Before, m_W / M_{KK} was a tunable parameter controlled by \tilde{v} . In the higgsless limit m_W is intimately tied to M_{KK} . The precise relation depends on the background geometry and it turns out to be of the form $m_W^2 \sim M_{\text{KK}}^2 / \mathcal{V}$.

⁴For the third generation there can also be corrections of order m_T^2 / M_{KK}^2 from a mixing with the fermionic KK modes. Those depend on a precise realization of the fermionic sector and are not discussed here.

Let us first discuss the SM gauge boson masses in a quantitative way. Taking the $\tilde{v} \rightarrow \infty$ limit of eq. (4.6) and eq. (4.7) we get the quantization conditions

$$\begin{aligned} (g_L^2 + g_R^2)S(L, m_W)C'(L, m_W) + m_W a_L^{-2} g_L^2 &= 0 \\ (g_L^2 + g_R^2)S(L, m_Z)C'(L, m_Z) + m_Z a_L^{-2} (g_L^2 + g_Y^2) &= 0 \end{aligned} \quad (5.1)$$

The lightest solution is identified with the SM gauge bosons. Expanding in $m_{W,Z}/M_{\text{KK}}$ we find

$$\begin{aligned} m_W^2 &= \frac{g_L^2 v^2}{4} \\ m_Z^2 &= \frac{(g_L^2 + g_Y^2)v^2}{4} \left[1 + g_Y^2 \frac{v^2}{4} (I_1(L) + I_5(L)) \right] + \mathcal{O}(v^6) \end{aligned} \quad (5.2)$$

where we defined

$$v^2 = \frac{4}{(g_L^2 + g_R^2)L \int_0^L a^{-2}(y)} \left[1 + g_L^2 \frac{v^2}{4} (I_1(L) + I_5(L)) \right] \quad (5.3)$$

As advocated, the gap between the electroweak and the resonance scales is controlled by the volume factor, $m_W^2 \approx M_{\text{KK}}^2/(\pi^2 \mathcal{V})$. In the following we assume that the 5D background is such that the gap is large enough to allow for expansion in m_W^2/M_{KK}^2 .

The SM gauge boson profiles are those of eq. (4.9) and eq. (4.10) with $I_1(L) \rightarrow (I_4(L) + I_5(L))/2$.⁵ From this, we find the coefficients of the dimension-six operators

$$\begin{aligned} \alpha_T &= 0 \\ \alpha_S &= \frac{g_L g_Y}{2} (I_4(L) + I_5(L)) \\ \beta_j &= -J_2(L, M_j) \\ \gamma_j &= -J_2(L, M_j) + J_3(L, M_j) - \frac{g_R^2 t_{R,j}^3}{g_Y^2 Y_j} J_3(L, M_j) \end{aligned} \quad (5.4)$$

where in the oblique parameters we dropped all terms suppressed by the volume factor. The T parameter vanishes at this order thanks to the custodial symmetry. We can write the S parameter as⁶

$$S = 4\pi v^2 (I_4(L) + I_5(L)) \sim \frac{12\pi}{\mathcal{V}(g_L^2 + g_R^2)} \quad (5.5)$$

We would need a large volume factor to suppress the gauge contribution to S. Recall that in AdS₅ $\mathcal{V} \sim 30$, which is not enough. Note that a volume factor large enough to make $S < 0.3$ would also make the KK scale heavier than 1 TeV. It seems unlikely that the longitudinal WW scattering amplitude could be unitarized by such heavy resonances. Therefore the only way to bring S down to acceptable level is by a suitable choice of fermionic profiles. More precisely, one assumes that the fermion bulk masses corresponding to the light SM

⁵The normalization factor changes because there exist $\mathcal{V}m_W^4/M_{\text{KK}}^4$ terms which are of the same order as m_W^2/M_{KK}^2 terms in the higgsless case. For this reason the expression for S is different than in the case with a higgs, where these higher-order terms were neglected.

⁶This expression is consistent with the one derived for general metrics in [11].

generations are such that the integrals J_2 and J_3 are not suppressed. Moreover, the bulk masses for the different light SM fields should be almost equal, $M_j \approx M_{\text{ref}}$. By eq. (2.18), choosing $\Delta\beta = J_2$, $\Delta\gamma = J_2 - J_3$, the universal part of the vertex corrections can be traded for a shift in S :

$$\Delta S = -8\pi v^2 \left(J_2(L, M_{\text{ref}}) - \frac{1}{2} J_3(L, M_{\text{ref}}) \right) \quad (5.6)$$

Since the shift is negative, we can cancel the positive gauge contribution in eq. (5.5) (in AdS_5 , the cancellation occurs for $M \sim 0.47k$). Note however that there will still remain sizable non-universal vertex corrections, $\gamma_j = -(g_R^2 t_{R,j}^3 / g_Y^2 Y_j) J_3(L, M_j)$, which depend on the embedding of the SM fields into $\text{SU}(2)_R$. Thus constraints based just on the oblique parameters can be misleading and one would need a more refined fit as in [10].

6. Conclusions

In this paper we have studied different models for electroweak breaking based on an extra dimension with a completely general warp factor. Under very broad conditions one can obtain useful expressions for the spectrum of the model and one can then match to a 4D theory with only light particles plus higher dimensional operators. This operators give contributions to the electroweak observables S, T , three main conclusions can be drawn:

- If the model is only based on the SM gauge groups there are very large contributions to the T parameter unless the KK scale is very large.
- If the model has $\text{SU}(2)_R$ as a symmetry in the bulk and then T is under control. As long as there is a higgs in the theory then the model has no problems with EW observables for KK masses of a few TeV.
- If we go to the higgsless limit then the S parameter in general grows and some careful choice of fermionic parameters are needed to ensure that the model passes the EW observables test. It is important to note that in this kind of models there is no free parameter to tune in order to reduce S since the KK masses are closely related to the EW scale

All of the above conclusions are *independent* of the geometry of the extra dimension so our conclusions are general.

Once this models are in agreement with present day bounds one could study the different experimental signatures and resonances that can be produced at LHC. In general the KK modes for gauge bosons tend to be too heavy ~ 3 TeV. More promising signal comes from some light fermionic resonance that can appear when the extra symmetry to cancel $Zb\bar{b}$ couplings is implemented [17]. Another possibility is to study the signal of KK gluons that can be detected even when they are as heavy as 4-5 TeV [18]. We postpone a detailed study of these signatures until future publications.

Acknowledgments

AF is partially supported by the European Community Contract MRTN-CT-2004-503369 for the years 2004–2008 and by the MEiN grant 1 P03B 099 29 for the years 2005–2007. The authors want to express gratitude to Manuel Pérez-Victoria for some useful discussions.

A. Integrals

The profiles and the masses of the SM gauge bosons depend on certain integrals of the warp factor. Here is the complete list relevant to us:

$$\begin{aligned}
 I_1(x_5) &= L^{-1} \int_0^{x_5} \int_0^y y' a^{-2}(y') \\
 I_2(x_5) &= \int_0^{x_5} y a^{-2}(y) \\
 I_3(x_5) &= L \int_0^{x_5} a^{-2}(y) \\
 I_4(x_5) &= \int_0^{x_5} \int_0^y a^{-2}(y') \\
 I_5(x_5) &= \frac{\int_0^{x_5} a^{-2}(y) \int_0^y \int_0^{y'} a^{-2}(y'')}{\int_0^L a^{-2}(y)} \tag{A.1}
 \end{aligned}$$

Notice that $I_4(L) = I_3(L) - I_2(L)$. All these integrals have dimension $[\text{mass}]^{-2}$, therefore $I_n(L)$ is expected to be of order $1/M_{\text{KK}}^2$. However $I_2(L)$ and $I_3(L)$ can be parametrically enhanced when the volume factor defined in eq. (2.11) is large. The argument goes as follows. If the warp factor decreases sharply toward the IR brane, we get $\int_0^L a^{-2} \sim a_L^{-2}/k$ and also $M_{\text{KK}} \sim a_L k$. Here, $k \sim a'(L)/a_L$ is the scale that describes how quickly the warp factor changes close to the IR brane. Thus $I_3(L) \sim kL/M_{\text{KK}}^2 = \mathcal{V}/M_{\text{KK}}^2$. Similarly $I_2(L) \sim \mathcal{V}/M_{\text{KK}}^2$ as the integral is dominated by $y \sim L$. Therefore, those electroweak parameters that depend on $I_2(L)$ and $I_3(L)$ end up being larger than the naive estimate v^2/M_{KK}^2 , and the constraints on the resonance scale become more stringent. Using analogous estimates, the remaining integrals: $I_1(L)$, $I_4(L)$ and $I_5(L)$ are $\sim 1/M_{\text{KK}}^2$, with no volume enhancement. Note also that $I_4(L) - I_1(L) = \int \int (1 - y/L) a^{-2}$ is suppressed, because the integrand vanishes at $y = L$.

As an example consider the familiar AdS_5 background corresponding to $a(x_5) = \exp(-kx_5)$. For $a_L \ll 1$ the resonance scale is, $M_{\text{KK}} \approx \pi k a_L$ and the volume factor $\mathcal{V} = kL$. We find the integrals

$$I_1(L) \approx \frac{\pi^2}{4M_{\text{KK}}^2} \left[1 - \frac{1}{kL} \right] \quad I_4(L) \approx \frac{\pi^2}{4M_{\text{KK}}^2} \quad I_5(L) \approx \frac{\pi^2}{8M_{\text{KK}}^2} \tag{A.2}$$

$$I_2(L) \approx \frac{\pi^2}{4M_{\text{KK}}^2} [2kL - 1] \quad I_3(L) \approx \frac{\pi^2}{4M_{\text{KK}}^2} 2kL \tag{A.3}$$

We can see that, indeed, the integrals in the second line are enhanced by the volume factor, which is of order $\log(M_{\text{Pl}}/\text{TeV}) \sim 30$ in the Randall-Sundrum setup.

Another, less known example is when the scale factor in 5D varies according to a power law: $a(x_5) = \left(1 - \frac{kx_5}{\gamma-1}\right)^\gamma$. The parameter $1/\gamma$ allows to describe a departure from conformal symmetry; in the conformal limit $\gamma \rightarrow \infty$ we are back to AdS₅. For $a_L \ll 1$ and $\gamma \gg 1$ the resonance scale is $M_{\text{KK}} \approx \pi k a_L^{1-1/\gamma}$ and the volume factor $kL a_L^{-1/\gamma}$. We obtain:

$$\begin{aligned}
 I_1(L) &\approx \frac{\pi^2}{2M_{\text{KK}}^2} \frac{\gamma-1}{2\gamma-3} \left[1 - \frac{2(\gamma-1)}{kL(2\gamma-1)} \right] \\
 I_4(L) &\approx \frac{\pi^2}{2M_{\text{KK}}^2} \frac{\gamma-1}{2\gamma-1} \\
 I_5(L) &\approx \frac{\pi^2}{2M_{\text{KK}}^2} \frac{\gamma-1}{4\gamma-3}
 \end{aligned} \tag{A.4}$$

$$\begin{aligned}
 I_2(L) &\approx \frac{\pi^2}{2M_{\text{KK}}^2} kL a_L^{-1/\gamma} \left[1 - \frac{\gamma-1}{kL(2\gamma-1)} \right] \\
 I_3(L) &\approx \frac{\pi^2}{2M_{\text{KK}}^2} kL a_L^{-1/\gamma} \frac{2(\gamma-1)}{2\gamma-1}
 \end{aligned} \tag{A.5}$$

We again observe the volume factor popping out in $I_2(L)$ and $I_3(L)$. As we move γ away from the conformal limit the effect of volume enhancement becomes more dramatic: $I_2(L)$ and $I_3(L)$ grow as a power of the large number $1/a_L$ (rather than a logarithm as in AdS₅). The remaining integrals, those that are not volume enhanced, are of order $1/M_{\text{KK}}^2$ and weakly depend on the shape of the warp factor. In consequence, we cannot significantly reduce the S parameter by varying γ (γ too close to 1 is not attractive as we need to fine-tune L to generate large hierarchy).

In the flat space, where there is no hierarchy of scales, all the integrals are of the same order,

$$6I_1(L) = 2I_2(L) = I_3(L) = 2I_4(L) = 6I_5(L) = \frac{\pi^2}{M_{\text{KK}}^2} \tag{A.6}$$

In order to describe the SM gauge interactions with fermion we define another class of integrals:

$$J_n(L, M) = \frac{\int_0^L a^{-1}(y) e^{-2My} I_n(y)}{\int_0^L a^{-1}(y) e^{-2My}} \tag{A.7}$$

that depends on the bulk fermion masses.

If the bulk mass is large, such that $a^{-1}(y) e^{-2My} \ll 1$ close to the IR brane, then those integrals are suppressed wrt to $1/M_{\text{KK}}^2$. For a small bulk mass, however, J_n is of the same order as I_n .

Consider AdS₅ once again and parametrize $M = ck$. For $c \gg 3/2$ we find $J_n \sim M_{\text{KK}}^{-2} a_L^2$, a Planck scale suppressed result. For $1/2 \ll c \ll 3/2$ $J_n \sim M_{\text{KK}}^{-2} a_L^{2c-1}$, still suppressed by an intermediate scale. But for $c \ll 1/2$ the suppression is gone and $J_n \sim M_{\text{KK}}^{-2}$. In particular

$$J_2(L, ck) \approx \frac{\pi^2}{4M_{\text{KK}}^2} \frac{(1-2c)(2kL(3-2c) - 2c + 5)}{(3-2c)^2} \quad J_3(L, ck) \approx \frac{\pi^2}{4M_{\text{KK}}^2} 2kL \frac{1-2c}{3-2c} \tag{A.8}$$

are volume enhanced, just like $I_2(L)$ and $I_3(L)$. But note that $J_3 - J_2$ is safe. For the crossover value of the bulk mass, $c = 1/2$, we obtain

$$J_2(L, k/2) \approx \frac{\pi^2}{4M_{\text{KK}}^2} \left(1 - \frac{1}{kL}\right) \quad J_3(L, k/2) \approx \frac{\pi^2}{4M_{\text{KK}}^2} \quad (\text{A.9})$$

so that there is no volume enhancement yet.

References

- [1] M.E. Peskin and T. Takeuchi, *Estimation of oblique electroweak corrections*, *Phys. Rev. D* **46** (1992) 381.
- [2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [[hep-ph/9905221](#)].
- [3] J.M. Maldacena, *The large- N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [[hep-th/9711200](#)];
 N. Arkani-Hamed, M. Porrati and L. Randall, *Holography and phenomenology*, *JHEP* **08** (2001) 017 [[hep-th/0012148](#)];
 R. Rattazzi and A. Zaffaroni, *Comments on the holographic picture of the Randall-Sundrum model*, *JHEP* **04** (2001) 021 [[hep-th/0012248](#)];
 M. Pérez-Victoria, *Randall-Sundrum models and the regularized AdS/CFT correspondence*, *JHEP* **05** (2001) 064 [[hep-th/0105048](#)];
 T. Gherghetta, *Warped models and holography*, [hep-ph/0601213](#).
- [4] K. Agashe, A. Delgado, M.J. May and R. Sundrum, *RS1, custodial isospin and precision tests*, *JHEP* **08** (2003) 050 [[hep-ph/0308036](#)].
- [5] C. Csáki, C. Grojean, L. Pilo and J. Terning, *Towards a realistic model of higgsless electroweak symmetry breaking*, *Phys. Rev. Lett.* **92** (2004) 101802 [[hep-ph/0308038](#)].
- [6] R. Contino, Y. Nomura and A. Pomarol, *Higgs as a holographic pseudo-Goldstone boson*, *Nucl. Phys. B* **671** (2003) 148 [[hep-ph/0306259](#)];
 K. Agashe, R. Contino and A. Pomarol, *The minimal composite Higgs model*, *Nucl. Phys. B* **719** (2005) 165 [[hep-ph/0412089](#)].
- [7] C. Csáki, J. Erlich and J. Terning, *The effective lagrangian in the Randall-Sundrum model and electroweak physics*, *Phys. Rev. D* **66** (2002) 064021 [[hep-ph/0203034](#)].
- [8] K. Agashe and R. Contino, *The minimal composite Higgs model and electroweak precision tests*, *Nucl. Phys. B* **742** (2006) 59 [[hep-ph/0510164](#)].
- [9] G. Cacciapaglia, C. Csáki, C. Grojean and J. Terning, *Oblique corrections from higgsless models in warped space*, *Phys. Rev. D* **70** (2004) 075014 [[hep-ph/0401160](#)]; *Curing the ills of Higgsless models: the S parameter and unitarity*, *Phys. Rev. D* **71** (2005) 035015 [[hep-ph/0409126](#)].
- [10] M. Carena, E. Ponton, J. Santiago and C.E.M. Wagner, *Electroweak constraints on warped models with custodial symmetry*, [hep-ph/0701055](#).
- [11] J. Hirn and V. Sanz, *The fifth dimension as an analogue computer for strong interactions at the LHC*, *JHEP* **03** (2007) 100 [[hep-ph/0612239](#)].
- [12] J. Hirn and V. Sanz, *(Not) summing over Kaluza-Kleins*, [hep-ph/0702005](#).

- [13] R. Barbieri, A. Pomarol and R. Rattazzi, *Weakly coupled higgsless theories and precision electroweak tests*, *Phys. Lett.* **B 591** (2004) 141 [[hep-ph/0310285](#)].
- [14] A. Falkowski, *About the holographic pseudo-Goldstone boson*, *Phys. Rev.* **D 75** (2007) 025017 [[hep-ph/0610336](#)].
- [15] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, *A custodial symmetry for Zb anti- b* , *Phys. Lett.* **B 641** (2006) 62 [[hep-ph/0605341](#)].
- [16] A. Djouadi, G. Moreau and F. Richard, *Resolving the $A(FB)(b)$ puzzle in an extra dimensional model with an extended gauge structure*, *Nucl. Phys.* **B 773** (2007) 43 [[hep-ph/0610173](#)].
- [17] M. Carena, E. Ponton, J. Santiago and C.E.M. Wagner, *Light Kaluza-Klein states in Randall-Sundrum models with custodial $SU(2)$* , *Nucl. Phys.* **B 759** (2006) 202 [[hep-ph/0607106](#)].
- [18] K. Agashe, A. Belyaev, T. Krupovnickas, G. Perez and J. Virzi, *LHC signals from warped extra dimensions*, [hep-ph/0612015](#);
B. Lillie, L. Randall and L.T. Wang, *The bulk RS KK -gluon at the LHC*, [hep-ph/0701166](#).