## XVII. CIRCUIT THEORY*

| Prof. S. J. Mason | Prof. R. D. Thornton | B. J. Leon |
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| Prof. H. J. Zimmermann | J. T. Andreika | T. G. Stockham, Jr. |
| Prof. J. Granlund | D. G. Kocher | E. H. Sussenguth |
| Prof. C. L. Searle | D. B. Leeson | S. Weinreb |

## A. NONLINEAR REACTANCE FREQUENCY MULTIPLIERS

We have completed a circuit analysis of an $\mathrm{N}^{\text {th }}$-order frequency multiplier using a nonlinear reactance in the circuit configurations previously reported (1). A paper giving the details of this analysis has been prepared and submitted for publication.

The analysis given in this paper spec-


Fig. XVII-1. Calculated efficiency versus harmonic order ifies impedence levels, conditions for optimum efficiency, and the value of optimum efficiency for a given lossy nonlinear reactance and for given lossy tuned circuits. Figure XVII-l shows the theoretical optimum efficiency that can be obtained by using a lossy abrupt-junction nonlinear capacitor as a frequency multiplier. $Q_{F}$ is the reciprocal of the sum of reciprocal Q's of the nonlinear capacitor and linear inductors evaluated at the fundamental frequency. $\mathrm{Q}_{\mathrm{H}}$ is the identical quantity evaluated at the $\mathrm{N}^{\text {th }}$ harmonic frequency. We have assumed that the nonlinear capacitor does not conduct during any part of the cycle.
Briefly, the analysis begins with a Taylor-series expansion of the nonlinear chargevoltage relation in the case of a nonlinear capacitor, or flux linkage-current relation in the case of a nonlinear inductor. Fourier-series expressions for the variables are substituted in this Taylor-series expansion and components at identical frequencies are equated. This procedure results in algebraic equations relating Fourier charge coefficients to voltage coefficients. With the addition of circuit constraints and small-signal approximations the analysis is complete.

The results of this analysis have been applied in detail to the case of the semiconductor nonlinear capacitor. Experimental measurements of input impedance and efficiency of frequency doublers containing abrupt-junction nonlinear capacitors have

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been made, and good agreement between experiment and theory has been obtained. It is evident that these devices make highly efficient frequency doublers. Efficiencies of $1 \mathrm{db}(1.5 \mathrm{mc}$ to 3.0 mc ) and $2.3 \mathrm{db}(100 \mathrm{mc}$ to 200 mc ) have been measured. A method of obtaining efficient high-order multiplication is to cascade doublers. By using this method an octupler ( 2 mc to 16 mc ) with an efficiency of 6.5 db has been constructed.
D. B. Leeson, S. Weinreb

## References

1. D. B. Leeson and S. Weinreb, Frequency multiplication with nonlinear capacitors, Quarterly Progress Report No. 53, April 15, 1959, p. 203.

## B. A NUMERICAL METHOD FOR COMPUTING POLE AND ZERO LOCATIONS

Methods for determining the pole-zero pattern associated with a given magnitudephase specification are discussed in almost all textbooks on network synthesis. These methods employ tedious analytical calculations or approximate graphical techniques. A method of using the principle of analytic continuation, in which an electronic computer performs all of the calculations and decisions, has been investigated.

The procedure applies the Cauchy-Riemann conditions of analyticity to the data, which are prescribed along a vertical line in the complex plane, and determines the values of magnitude and phase along a second line close to the first. The second line is then used to determine a third, the third a fourth, and so on until a pole or zero is reached. Poles and zeros are located by examining the relative magnitudes at the various mesh points. For each point, the ratio of the magnitude at that point to the preceding point is formed and, if at some point the ratio exceeds two and at the next point is less than one-half, we know that a pole lies between the two points. The influence of this pole is then "subtracted" from the orignal data and the computation begun anew. By repeating this process a sufficient number of times a system of poles and zeros is built up that approximates the original specification.

This technique has been programmed on the IBM 704 computer, at the M.I.T. Computation Center, with only moderate success. The poles and zeros of elementary specifications are located accurately, but in more realistic specifications a build-up of error in the neighborhood of prospective poles distorts the values so much so that the ratio criterion will not locate the pole. (The ratio criterion was derived under the assumption that the true values of the function were available.) Various smoothing and averaging techniques that were employed to counteract the error were not satisfactory.
E. H. Sussenguth


[^0]:    *This research was supported in part by Purchase Order DDL-B222 with Lincoln Laboratory, a center for research operated by M.I.T., which is supported by the U.S. Air Force under Air Force Contract AF19(604)-5200.

