VIII. MICROWAVE ELECTRONICS*

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A. PERTURBATION FORMULA FOR FERRITE-FILLED CAVITY

Conventional perturbation formulae for cavities containing ferrites (1) usually start with the unperturbed cavity free of ferrite. The perturbing effect of the ferrite on the cavity modes is taken into account solely through the use of effective permeabilities (which include the demagnetization factors). Such an approach is adequate for all systems in which the stored energy is changed only by a small amount from the value computed from the empty cavity modes in the absence of the ferrite. In cases in which the ferrite stores substantial amounts of energy, for example, in a magnetostatic resonance of the ferrite, the conventional perturbation approach fails.

In an attempt to derive Suhl's expression (2) for the semistatic and static operation of the parametric ferrite amplifier by perturbation theory the author developed the perturbation formula reported here.

Denote by $\overline{E}_o$, $\overline{h}_o$, and $\overline{m}_o$ the electric field, small-signal magnetic field and magnetization of the mode o of the unperturbed lossless system, enclosed by a perfect electric conductor, that resonates at the frequency Ω_o. These fields satisfy the equations

$$\nabla \times \overline{E}_o = -j\Omega_o\mu_o(\overline{h}_o + \overline{m}_o) \tag{1}$$

$$\nabla \times \overline{h}_o = j\Omega_o\epsilon\overline{E}_o \tag{2}$$

The electric and magnetic fields satisfy the boundary conditions on the conductor

$$\overline{n} \times \overline{E}_o = 0; \quad \overline{n} \cdot \overline{h} = 0 \tag{3}$$

where $\overline{n}$ is the normal on the conductor. The magnetization $\overline{m}$ fulfills the small-signal ferrite equation

$$j\Omega_o\overline{m}_o = -|\gamma|(\overline{M}_o \times \overline{h}_o + \overline{m}_o \times \overline{H}_o) \tag{4}$$

The dielectric constant ϵ will be assumed to be a scalar. Consider now the same system perturbed by an electric current distribution $\overline{J}$ and a magnetic current distribution $\overline{J}_M$. The fields now satisfy the equations

$$\nabla \times \overline{E} = -j\omega\mu_o(\overline{h} + \overline{m}) - \overline{J}_M \tag{5}$$

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$$\nabla \times \bar{h} = j\omega\epsilon\bar{E} + \bar{J} \tag{6}$$

and the magnetization $\bar{m}$ still fulfills the relation

$$j\omega\bar{m} = -\ |\gamma|(M_o \times \bar{h} + \bar{m} \times \bar{H}_o) \tag{7}$$

The boundary conditions on $\bar{E}$ and $\bar{h}$ on the perfectly conducting enclosure remain unchanged.

We now dot-multiply Eq. 5 by $\bar{h}_o^*$, Eq. 6 by $-\bar{E}_o^*$, the complex conjugate of Eq. 1 by $\bar{h}$, and that of Eq. 2 by $-\bar{E}$. Adding the resulting equations, integrating over the volume τ of the perfectly conducting enclosure, using Gauss' theorem and the boundary conditions on $\bar{E}_o$, $\bar{h}_o$, $\bar{E}$, and $\bar{h}$, we obtain

$$\begin{aligned} 0 = &-j(\omega - \Omega_o)\int (\mu_o\bar{h} \cdot \bar{h}_o^* + \epsilon\bar{E} \cdot \bar{E}_o^*)\, d\tau \\ &+ j\Omega_o\mu_o\int \bar{h} \cdot \bar{m}_o^*\, d\tau - j\omega\mu_o\int \bar{m} \cdot \bar{h}_o^*\, d\tau \\ &-\int \bar{J}_M \cdot \bar{h}_o^*\, d\tau - \int \bar{J} \cdot \bar{E}_o^*\, d\tau \end{aligned} \tag{8}$$

Now, we transform the second and third terms in Eq. 8. Dot-multiplying Eq. 7 by $\bar{h}_o^*$, the complex conjugate of Eq. 4 by $\bar{h}$, and adding the two equations, we obtain

$$j\omega\bar{m} \cdot \bar{h}_o^* - j\Omega_o\bar{m}_o^* \cdot \bar{h} = -|\gamma|\ [\bar{m} \times \bar{H}_o \cdot \bar{h}_o^* + \bar{m}_o^* \times \bar{H}_o \cdot \bar{h}] \tag{9}$$

On the other hand, multiplying Eq. 7 by $\bar{m}_o^*$, the complex conjugate of Eq. 4 by $\bar{m}$, and adding the two equations, we obtain

$$j(\omega - \Omega_o)\bar{m} \cdot \bar{m}_o^* = -\ |\gamma|[\bar{M}_o \times \bar{h} \cdot \bar{m}_o^* + \bar{M}_o \times \bar{h}_o^* \cdot \bar{m}] \tag{10}$$

But $\bar{M}_o$ is parallel to $\bar{H}_o$ and

$$\bar{M}_o = \frac{M_o}{H_o}\bar{H}_o \tag{11}$$

Introducing this relation into Eq. 10 and comparing the results with Eq. 9, we find that

$$j\omega\bar{m} \cdot \bar{h}_o^* - j\Omega_o\bar{m}_o^* \cdot \bar{h} = \frac{H_o}{M_o}\, j(\omega - \Omega_o)\bar{m} \cdot \bar{m}_o^* \tag{12}$$

Finally, if we introduce this expression into the perturbation formula (Eq. 8) and solve for $\omega - \Omega_o$, we obtain

$$\frac{\omega - \Omega_o}{\Omega_o} = \frac{-j\left[\int \bar{J}_M \cdot \bar{h}_o^*\, d\tau + \int \bar{J} \cdot \bar{E}_o^*\, d\tau\right]}{\Omega_o\int\left(\epsilon\bar{E} \cdot \bar{E}_o^* + \mu_o\bar{h} \cdot \bar{h}_o^* + \frac{H_o}{M_o}\mu_o\bar{m} \cdot \bar{m}_o^*\right) d\tau} \tag{13}$$

If the perturbation is small and we are studying a resonance in the neighborhood of Ω_o, we can replace $\overline{E}$, $\overline{h}$, and $\overline{m}$ in the denominator of Eq. 13 by $\overline{E}_o$, $\overline{h}_o$, and $\overline{m}_o$ and obtain

$$\frac{\omega - \Omega_o}{\Omega_o} = \frac{-j\left[\int \overline{J}_M \cdot \overline{h}_o^* \, d\tau + \int \overline{J} \cdot \overline{E}_o^* \, d\tau\right]}{\Omega_o \int \left(\epsilon \overline{E}_o \cdot \overline{E}_o^* + \mu_o \overline{h}_o \cdot \overline{h}_o^* + \frac{H_o}{M_o} \mu_o \overline{m}_o \cdot \overline{m}_o^*\right) d\tau} \tag{14}$$

Equation 14 shows that in the analysis of magnetostatic operation, the stored electromagnetic energy must be supplemented by the term

$$\frac{1}{4} \int \frac{H_o}{M_o} \mu_o \overline{m}_o \cdot \overline{m}_o^* \, d\tau$$

This term can be recognized as the energy required to deflect the magnetization vector $\overline{M}$ from alignment with $\overline{M}_o$ and $\overline{H}_o$ by the angle $|\overline{m}|/M_o$.

If we start with Eq. 14, we can derive Suhl's formula for the semistatic and static operation of the parametric amplifier with little effort. In conclusion, it should be mentioned that a relation of the form of (Eq. 12) was first discovered by Mr. D. L. Bobroff, of Raytheon Manufacturing Company, who used it for proofs of orthogonality.

H. A. Haus

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B. SMALL-SIGNAL ENERGY THEOREM FOR BEAMS WITH ZERO CURL OF GENERALIZED MOMENTUM

The energy theorem of the theory of passive electromagnetic systems (1) relates the frequency derivative of the fields to energy storage. A corresponding theorem can be devised for the small-signal excitation in an electromagnetic system that contains an electron beam or an electron-space-charge cloud with zero curl of the generalized momentum. This theorem can be used, inter alia, to relate the group velocity of a small-signal wave excitation to the power and energy in that wave.

The fundamental equations (2) of a single velocity space-charge cloud with zero curl of the generalized momentum and a time variation $e^{j\omega t}$ are

(a) the force equation,

$$\nabla(\overline{v}_o \cdot \overline{u}) = -j\omega\overline{u} + \frac{e}{m}\overline{E} \tag{1}$$

(b) the continuity equation,

$$\bar{v}_o \nabla \cdot \bar{J} = -j\omega(\bar{J} - \rho_o \bar{u}) \tag{2}$$

(c) and the two Maxwell equations,

$$\nabla \times \bar{E} = -j\omega\mu_o \bar{H} \tag{3}$$

$$\nabla \times \bar{H} = j\omega\epsilon_o \bar{E} + \bar{J} \tag{4}$$

in which no subscripts are used to indicate small-signal ac terms, and the subscript o is used to indicate the time-average components. The velocity $\bar{u}$ is the Eulerian velocity. $\bar{J}$ includes the ac surface current $\rho_o \bar{r}_1 \cdot \bar{n}$ on the beam surface (where $\bar{n}$ is the normal to the beam surface, and $\bar{r}_1$ is the small-signal displacement of an electron in the surface). In order to derive the energy theorem, we take the derivatives of Eqs. 1, 2, 3, and 4 with respect to ω. Then, we dot-multiply the resulting equations by $\frac{m}{e}\bar{J}^*$, $\frac{m}{e}\bar{u}^*$, and $-\bar{E}^*$, respectively, and add them. To this result we add the complex conjugates of Eqs. 1, 2, 3, and 4, after having dot-multiplied them by $\frac{m}{e}\frac{\partial J}{\partial \omega}$, $\frac{m}{e}\frac{\partial \bar{u}}{\partial \omega}$, $\frac{\partial \bar{H}}{\partial \omega}$, and $\frac{\partial \bar{E}}{\partial \omega}$, respectively. We obtain

$$-\frac{1}{4}\nabla \cdot \left(\frac{\partial \bar{E}}{\partial \omega} \times \bar{H}^* + \bar{E}^* \times \frac{\partial \bar{H}}{\partial \omega} + \frac{m}{e}\bar{v}_o \cdot \frac{\partial \bar{u}}{\partial \omega}\bar{J}^* + \frac{m}{e}\bar{v}_o \cdot \bar{u}^* \frac{\partial \bar{J}}{\partial \omega}\right)$$
$$= \frac{1}{4} j\left(\mu_o \bar{H} \cdot \bar{H}^* + \epsilon_o \bar{E} \cdot \bar{E}^* + \frac{m}{e}\bar{u} \cdot \bar{J}^* + \frac{m}{e}\bar{u}^* \cdot \bar{J} - \frac{m}{e}\rho_o \bar{u} \cdot \bar{u}^*\right) \tag{5}$$

This is the energy theorem. On the left-hand side, terms similar to power-flow terms appear, except that in each term one of the small-signal factors is differentiated with respect to ω. On the right-hand side, the sum of the electromagnetic energy and the small-signal kinetic energy (3), multiplied by j, appears.

As one application of the energy theorem (Eq. 5) we may consider a Brillouin space-charge cloud inside a magnetron cavity under small-signal excitation (zero anode current). The cavity opens into a uniform waveguide within which a steady state is maintained at the frequency ω. The walls of the magnetron cavity, the waveguide, and the cathode are all assumed to be lossless. The waveguide field at the reference plane within the waveguide may be characterized as

$$\bar{E} = V\bar{e} \text{ and } \bar{H} = I\bar{h} \tag{6}$$

where $\bar{e}$ and $\bar{h}$ are the frequency-independent field patterns of the dominant waveguide mode. The admittance Y is defined by

$$I = YV \tag{7}$$

Now suppose that the frequency of excitation is changed by $d\omega$ and that I is kept constant. Integrating Eq. 5 over the magnetron cavity up to the waveguide reference plane, we obtain

$$\frac{\partial Y}{\partial \omega} |V|^2 = -j \int \left[\mu_o \bar{H} \cdot \bar{H}^* + \epsilon_o \bar{E} \cdot \bar{E}^* + \frac{m}{e} (\bar{u} \cdot \bar{J}^* + \bar{u}^* \cdot \bar{J} - \rho_o \bar{u} \cdot \bar{u}^*)\right] dv \qquad (8)$$

All other terms have dropped out, since the boundary conditions have to be satisfied at the frequencies ω and $\omega + d\omega$. Over the entire surface of integration, $\bar{n} \cdot \bar{J}$ and $\partial\bar{J}/\partial\omega \cdot \bar{n}$, where $\bar{n}$ is the normal to the surface, are zero. Equation 8 shows that $\partial Y/\partial\omega$ is pure imaginary. Its sign is determined by the sign of the total small-signal energy, electromagnetic and kinetic. Thus $\partial Y/\partial\omega$ is negative imaginary when the energy is negative.

Now we turn to another application of the energy theorem. In a uniform lossless system containing an electron beam, consider a wave with the dependence $e^{-j\beta z}$, with β assumed real. We factor the common z-dependence from all field quantities, and indicate the remaining vector functions of x and y by a circumflex mark. If we apply the energy theorem (Eq. 5) to the fields of the wave, we have

$$\nabla \cdot \left(\frac{\partial \hat{E}}{\partial \omega} \times \hat{H}^* + \hat{E}^* \times \frac{\partial \hat{H}}{\partial \omega} + \frac{m}{e} \bar{v}_o \cdot \frac{\partial \hat{u}}{\partial \omega} \hat{J}^* + \frac{m}{e} \bar{v}_o \cdot \hat{u}^* \cdot \frac{\partial \hat{J}}{\partial \omega}\right) + \nabla \cdot \left\{\left[\hat{E} \times \hat{H}^* + \hat{E}^* \times \hat{H} + \frac{m}{e}\left(\bar{v}_o \cdot \hat{u}\hat{J}^* + \bar{v}_o \cdot \hat{u}^*\hat{J}\right)\right] j \frac{\partial \beta}{\partial \omega} z\right\}$$
$$= -j\left(\mu_o \hat{H} \cdot \hat{H}^* + \epsilon_o \hat{E} \cdot \hat{E}^* + \frac{m}{e} \hat{u} \cdot \hat{J}^* + \frac{m}{e} \hat{u}^* \cdot \hat{J} - \frac{m}{e} \rho_o \hat{u} \cdot \hat{u}^*\right) \qquad (9)$$

If we integrate Eq. 9 over a cylinder bounded by the perfectly conducting boundary of the system (or by a cylinder at infinity), and by two reference planes, separated by a distance L, the first term on the left-hand side integrates out to zero. We obtain

$$\frac{d\omega}{d\beta} = \frac{P}{w} \qquad (10)$$

where P is the small-signal power, electromagnetic and kinetic (2).

$$P = \frac{1}{2} \mathrm{Re} \int_{\text{cross section}} \left(\hat{E} \times \hat{H}^* + \frac{m}{e} \bar{v}_o \cdot \hat{u}\hat{J}^*\right) da \qquad (11)$$

and w is the small-signal stored energy per unit length

$$w = \frac{1}{4} \int \left[\mu_o \hat{H} \cdot \hat{H}^* + \epsilon_o \hat{E} \cdot \hat{E}^* + \frac{m}{e} (\hat{u} \cdot \hat{J}^* + \hat{u}^* \cdot \hat{J}) - \frac{m}{e} \rho_o \hat{\mu} \cdot \hat{\mu}^*\right] da \qquad (12)$$

Thus $\partial\omega/\partial\beta$ has formally the significance of energy velocity. Note, however, that w can now assume negative values.

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C. KLYSTRON GAP THEORY*

The small-signal longitudinal interaction of an electron stream with the electric fields of a gap can be represented, for convenience, by a three-port network, as shown

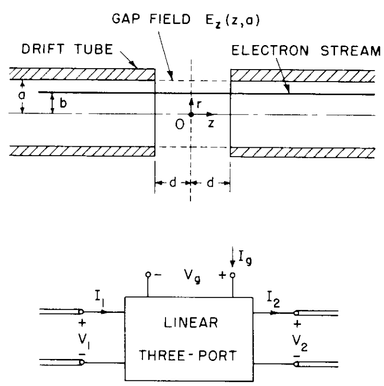

Fig. VIII-1. Electron-stream interaction with electric fields of a gap and small-signal equivalent circuit.

in Fig. VIII-1. The theoretical development of this problem was reported in the Quarterly Progress Report of July 15, 1958, page 49. Here we shall give the results of the evaluation of the gap matrix elements,

$$\begin{bmatrix} V_2 \\ I_2 \\ I_g \end{bmatrix} = \begin{bmatrix} y_{11} & 0 & y_{13} \\ Y_{21} & y_{22} & Y_{23} \\ Y_{31} & y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \\ V_g \end{bmatrix} \tag{1}$$

Consider a system consisting of a gap and a gap electric field with symmetry about the z = 0 plane, as shown in Fig. VIII-1. The matrix elements of Eq. 1 can be expressed in terms of the Fourier integrals or of the normalized field (1).

$$\beta = \int_{-\infty}^{\infty} \underline{\underline{E}}(\theta, \rho)\, e^{j\theta}\, d\theta \tag{2}$$

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and

$$\gamma = \int_{-\infty}^{\infty} - j\theta \, \underline{\underline{E}}(\theta, \rho) \, e^{j\theta} \, d\theta \tag{3}$$

both of which are real functions. The parameters β and γ can be evaluated if the electric field at the gap, $E_z(z, a)$, is known, or they can be measured in an actual gap system by a perturbation technique. The matrix elements and their designations are:

Drift coefficients (θ_d is the normalized distance of the gap field extension)

$$y_{11} = y_{22} = e^{-j2\theta_d} \tag{4}$$

Coupling coefficients

$$y_{13} = y_{32} = \beta \, e^{-j\theta_d} \tag{5}$$

Bunching admittance

$$Y_{21} = \frac{1}{2} G_o(j2\theta_d) \, e^{-j2\theta_d} \tag{6}$$

Induced-current admittance

$$Y_{23} = Y_{31} = \frac{1}{2} G_o(\gamma + j\theta_d\beta) \, e^{-j\theta_d} \tag{7}$$

Electronic-loading admittance

$$\left.\begin{aligned} Y_{33} &= \frac{1}{2} G_o(\gamma\beta + j2b_{e\ell}) \\ b_{e\ell} &= \operatorname{Re} \int_{-\infty}^{\infty} d\theta \, \underline{\underline{E}}(\theta, \rho) \, e^{j\theta} \int_{-\infty}^{\theta} d\xi(-\xi) \, \underline{\underline{E}}(\xi, \rho) \, e^{-j\xi} \end{aligned}\right\} \tag{8}$$

in which $G_o = I_o/V_o$ is the dc beam conductance, and all other quantities are as previously defined (1).

The equivalent circuit at the gap-circuit terminals follows directly from the kinetic power theorem and has been given previously (1). When the gap circuit is terminated in a passive admittance Y_c (Fig. VIII-2a), the gap terminals are loaded by the electronic loading admittance (Eqs. 8), and driven by current generators that are proportional to the excitations present in the incoming electron stream, as shown in Fig. VIII-2b. At the same time, the electron-beam kinetic voltage V_1 and beam current I_1 undergo a transformation that can be represented by a linear two-port, as in Fig. VIII-2c.

For convenience, let

$$Z \equiv (Y_c + Y_{33})^{-1} \tag{9}$$

$$G_{e\ell} \equiv \mathrm{Re}\,(Y_{33}) = \frac{1}{2} G_o \gamma \beta \tag{10}$$

and

$$\phi \equiv \frac{\beta}{\gamma} \theta_d \tag{11}$$

Making use of Eqs. 1 and 4 - 8, and of the definitions that have been given, we obtain the following description of the electron-beam two-port network of Fig. VIII-2c.

$$\begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} e^{-j2\theta_d} \begin{pmatrix} V_1 \\ I_1 \end{pmatrix} \tag{12}$$

with

$$A = D = 1 - (G_{e\ell} Z)(1 + j\phi) \tag{13}$$

$$B = -\frac{\beta^2}{G_{e\ell}} (G_{e\ell} Z) \tag{14}$$

$$C = \frac{G_{e\ell}}{\beta^2} \left[j2\phi - (G_{e\ell} Z)(1 + j\phi)^2 \right] \tag{15}$$

In the important case of a resonant circuit, we have

$$Z = \frac{1}{G} \frac{1}{(\lambda + 1)} \tag{16}$$

where $G = G_c + G_{e\ell}$ is the total gap conductance; and $\lambda = \dfrac{s^2 + \omega_o^2}{2\alpha s}$ is the normalized

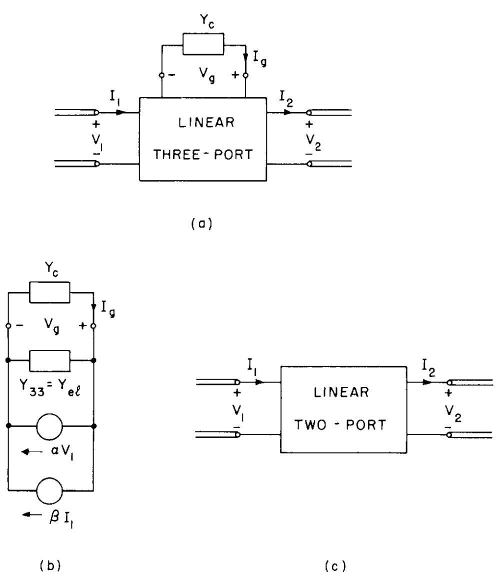

Fig. VIII-2. (a) Gap circuit terminated in a passive admittance.
(b) Equivalent circuit at the gap terminals.
(c) Equivalent circuit at the beam terminals.

frequency parameter with ω_o the loaded resonant frequency, and α the loaded damping constant. Three special cases of varying circuit loading with respect to beam loading are:

a. Lossless circuit with $G_c = 0$

$$A = 1 - \frac{(1 + j\phi)}{(\lambda+1)} \tag{17}$$

$$B = -\frac{\beta^2}{G_{e\ell}} \frac{1}{(\lambda+1)} \tag{18}$$

$$C = \frac{G_{e\ell}}{\beta^2} \left[j2\phi - \frac{(1 + j\phi)^2}{(\lambda+1)} \right] \tag{19}$$

b. Lossy circuit with $G_c \gg G_{e\ell}$

$$A \doteq 1 - \frac{G_{e\ell}}{G_c} \frac{(1 + j\phi)}{(\lambda+1)} \tag{20}$$

$$B \doteq -\frac{\beta^2}{G_c} \frac{1}{(\lambda+1)} \tag{21}$$

$$C \doteq \frac{G_{e\ell}}{\beta^2} \left[j2\phi - \frac{G_{e\ell}}{G_c} \frac{(1 + j\phi)^2}{(\lambda+1)} \right] \tag{22}$$

c. Loss loading = beam loading with $G_c = G_{e\ell}$

$$A = 1 - \frac{1}{2} \frac{(1 + j\phi)}{(\lambda+1)} \tag{23}$$

$$B = -\frac{\beta^2}{2G_{e\ell}} \frac{1}{(\lambda+1)} \tag{24}$$

$$C = \frac{G_{e\ell}}{\beta^2} \left[j2\phi - \frac{1}{2} \frac{(1 + j\phi)^2}{(\lambda+1)} \right] \tag{25}$$

Most gaps that are of interest have $(\beta^2/G_{e\ell}) \gg 1$ and $\phi \sim 1$. For such gaps, in all of the cases that we have mentioned, $C \approx 0$ is a good approximation. For case 2, in which the circuit conductance is much larger than the electronic conductance, we may also assume that $A \approx 1$. Only for this case can we approximate the two-port network of Fig. VIII-2c by a series admittance that is equal to the circuit admittance, as shown in Fig. VIII-3 (for convenience, $\exp(-j2\theta_d)$ is omitted).

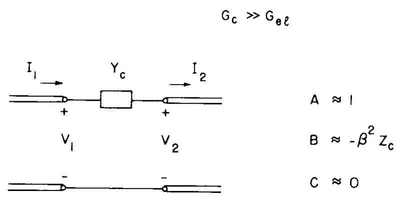

Fig. VIII-3. Approximate equivalent circuit for the electron beam when electronic loading is negligible compared with circuit loading $[\exp(-j2\theta_d)$ phase shift omitted$]$.

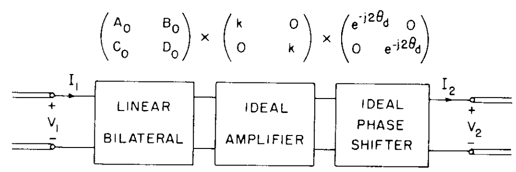

Fig. VIII-4. Equivalent circuit for electron beam interacting with passive circuit fields.

1. Reciprocity

From Eqs. 12 - 15, we have

$$AD - BC = 1 - 2G_{e\ell}Z \equiv k^2 \tag{26}$$

The linear two-port network of Fig. VIII-2c is reciprocal if $k^2 \exp(-j4\theta_d) = 1$; this will be so when $\theta_d = 0$ and $k = 1$. It is interesting to note that when $k = 1$, but $\theta_d \neq 0$, the two-port network of Fig. VIII-2c can be considered as reciprocal with respect to a reversal of the direction of flow of the electron stream. Two such cases occur when $Z = 0$ (which is a drift region) and when $G_{e\ell} = 0$. In the general case, Eq. 12 can be rewritten (2) as

$$\begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} A_o & B_o \\ C_o & D_o \end{pmatrix} k\, e^{-j2\theta_d} \begin{pmatrix} V_1 \\ I_1 \end{pmatrix} \tag{27}$$

where $A_o = A/k$, $B_o = B/k$, and so forth; and therefore we now have $A_oD_o - B_oC_o = 1$. The representation that is equivalent to Fig. VIII-2c is as shown in Fig. VIII-4. The location of the ideal amplifier and phase shifter is arbitrary.

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