

IX. NOISE IN ELECTRON DEVICES*

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A. LANGEVIN EQUATION FOR MICROWAVE FERRITE WITH THERMAL NOISE

The Langevin equation for a ferrite at thermodynamic equilibrium is presented. The nature of the statistical "driving term" in this equation is such that it seems reasonable to retain it unchanged, even in the presence of the pumping excitation which would be used in a parametric ferrite amplifier.

The equation of a ferrite with loss (1), with the use of the Landau-Lifshitz representation of the loss, is

$$\frac{d\bar{M}}{dt} = -|\gamma|(\bar{M} \times \bar{H}) - \frac{a\gamma}{|\bar{M}|} \bar{M} \times (\bar{M} \times \bar{H}) \quad (1)$$

Now suppose that the ferrite is exposed to a time-independent magnetic field for producing the saturation magnetization \bar{M}_0 . The thermal agitation in the ferrite causes a deviation of the magnetization vector \bar{M} from alignment with \bar{M}_0 . This fluctuation can be taken into account by supplementing Eq. 1 with a random torque \bar{T} (in units of coul/m)

$$\frac{d\bar{M}}{dt} = -|\gamma|(\bar{M} \times \bar{H}) - \frac{a\gamma}{|\bar{M}|} \bar{M} \times (\bar{M} \times \bar{H}) + \bar{T} \quad (2)$$

Denote by $\bar{T}(\bar{r}, \omega)$ the complex Fourier amplitude of frequency ω at the point \bar{r} . The torque at the point \bar{r}' is denoted by $\bar{T}(\bar{r}', \omega)$. Then, we can show that the correct equilibrium fluctuations of the magnetization vector will be produced if the mean-square fluctuations of the three Cartesian torque components are

$$\begin{aligned} \overline{T_x(\bar{r}, \omega) T_x^*(\bar{r}', \omega)} &= \overline{T_y(\bar{r}, \omega) T_y^*(\bar{r}', \omega)} = \overline{T_z(\bar{r}, \omega) T_z^*(\bar{r}', \omega)} \\ &= 4|a\gamma M|kT\Delta f \delta(\bar{r} - \bar{r}') \end{aligned} \quad (3)$$

and mutually perpendicular torque components are uncorrelated. Here, $\delta(\bar{r} - \bar{r}')$ is a delta function, indicating that the torques in adjacent volume elements are uncorrelated.

It is noteworthy that the spectrum of the torque distribution indicated in Eq. 3 is uniform. Accordingly, the fluctuations of the torque are extremely rapid compared with any frequency for which Eq. 1 applies.

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The proof of Eq. 3 is accomplished by reduction of Eq. 2 to small-signal form. For the small-signal complex vector \bar{m} of the magnetization we obtain the relation

$$j\omega\mu_0\bar{m} = j\omega\mu_0\bar{\chi} \cdot \bar{h} + \bar{J}_M \quad (4)$$

where \bar{h} is the small-signal complex magnetic field vector, \bar{J}_M is the magnetic noise current density with a Fourier amplitude that is linearly related to $\bar{T}(\bar{r}, \omega)$. Then, we make use of the fact that, at equilibrium, the magnetic noise current density of a medium characterized by an equation in the form of Eq. 4 must satisfy the relation (see Quarterly Progress Report, July 15, 1957, p. 113, cf. Eq. 6, which was developed for an isotropic medium)

$$\overline{J_M(\bar{r}, \omega) \bar{J}_M(\bar{r}', \omega)^*} = 2j\omega\mu_0(\bar{\chi} - \bar{\chi}^\dagger) kT\Delta f \delta(\bar{r} - \bar{r}') \quad (5)$$

in which the dagger indicates the complex-conjugate transpose of the tensor $\bar{\chi}$. By comparing the explicit expression for $\bar{J}_M(\bar{r}, \omega)$, as obtained from Eq. 2, with Eq. 4 we find that the mean-square fluctuations of the torque components are perpendicular to the saturation magnetization \bar{M}_0 . The component of \bar{T} parallel to \bar{M}_0 does not affect the small-signal equilibrium fluctuations of \bar{M} , and therefore cannot be obtained from such considerations. However, by symmetry, it is reasonable to postulate a component of the torque in the direction of \bar{M}_0 that is equal to the component perpendicular to \bar{M}_0 . The resulting self-fluctuations and cross fluctuations of $\bar{T}(\bar{r}, \omega)$, in Eq. 3, are more easily generalized to parametrically excited ferrites.

We can develop a theory of noise in parametric ferrite amplifiers with the assumption that the equilibrium distribution of the torque, in Eq. 3, is unaffected by the application of the pump. This is a reasonable assumption in view of the statistical independence of the equilibrium torque at successive time intervals as indicated by its uniform frequency spectrum, and its statistical independence in two adjacent volume elements as indicated by the delta function in Eq. 3.

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References

1. N. Bloembergen, Magnetic resonance in ferrites, Proc. IRE 44, 1259-1269 (1956).