XX. CIRCUIT THEORY

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RESEARCH OBJECTIVES

The long-range objective of the research that will be reported under Circuit Theory is to extend our knowledge of the properties of electric networks (and related systems), active or passive, reciprocal or nonreciprocal, linear or nonlinear. Current short-range projects include work on parametric amplifiers, devices for nonlinear filtering, general network topology, development of artificial noise sources, and the testing of a predetection diversity combiner.

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A. NETWORK TOPOLOGY

The original problem proposed for solution was to find a synthesis procedure for a network (graph) that has a specified number of nodes and branches and a maximum number of trees. By investigating numerous specific examples, the following necessary conditions were established: As nearly as possible, all vectors of the cut-set space defined by such a graph should have an equal number of nonzero components. Likewise, all of the primary vectors of the loop-set space (i.e., vectors having +1, −1, or 0 as components) should have, as nearly as possible, an equal number of nonzero components.

Therefore, the solution of this problem would involve synthesizing a branch-node incidence matrix that satisfies these conditions. A search of the available literature on the synthesis of incidence matrices revealed few specific results but pointed out two general methods of attack:

1. Use of the concepts of algebraic topology, such as complexes, cycles, boundary, chains, and so forth.

2. Use of tensor notation both for designation of the topological entities (and their electrical counterparts) and for obtaining various expansions of determinants.

In order to utilize these techniques, a study of algebraic topology and tensor algebra was undertaken. In the course of these studies an interesting fact pointed out by Okada (1) was discovered: Linear network analysis can be interpreted geometrically as a transformation of coordinates in a Euclidean space. The parameter matrices Z or Y can be interpreted as matrices of metric coefficients of such a transformation. Therefore, by defining different metrics in a more general affine space, relations between currents and voltages in nonlinear circuits could be obtained. In order to investigate this aspect of the problem, a more detailed study of affine geometry will be necessary.

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References

1. S. Okada, Report R-601-57, PIB-529, Microwave Research Institute, Polytechnic Institute of Brooklyn, Sept. 5, 1957, p. 8.

B. PARAMETRIC AMPLIFICATION

Developments in the microwave field have led to a renewal of interest in parametric amplifiers (1). In these amplifiers a circuit element, such as a capacitor, is varied by an external source that is independent of the electric signal that is to be amplified. Power flowing from the external source into the load is controlled by the desired signal.

At present, no general study of the circuit properties of such an amplifier is available. A study has just been started with the objective of investigating and deriving the properties of a single time-variant reactive element in the environment of lumped, linear, finite, passive, and bilateral elements. It is hoped that at the end of this investigation we shall be able to state limitations on gain and bandwidth, and to determine the need for undesired frequency components in the output of the circuit shown in Fig. XX-1.

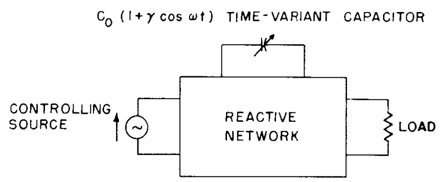

Fig. XX-1. Basic parametric amplifier.

When a capacitor that varies at a frequency ω_o, is placed in a circuit with an electric source at frequency ω_1, electric components at frequencies $\omega_1 + \omega_o$ and $|\omega_1 - \omega_o|$ are generated. These, in turn, react with the capacitor to give the additional frequency $|\omega_1 \pm 2\omega_o|$, and so on. Therefore, any investigation of such a device requires the study of a multiple-frequency system.

There are three approaches to this problem. First, there is a strictly time-domain approach. Second, there is the time-variant transform, $H(j\omega_1 t)$, suggested by Zadeh (2). Third, there is a strictly frequency-domain approach.

A time-domain attack is useful for simple systems, but if the reactive network is to have any generality, this approach will undoubtedly prove too complex mathematically. One of the other two methods, the use of a frequency-domain approach, may prove useful.

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References

1. H. A. Haus, The kinetic power theorem for parametric, longitudinal, electron-beam amplifiers, Nov. 11, 1957 (submitted for publication).

2. L. Zadeh, Frequency analysis of variable networks, Proc. IRE 38, 291 (1950).

C. A SIMPLE EXAMPLE OF NONLINEAR FILTERING

If a linear filter is used to remove a signal from noise when the two have nearly identical power spectra, not much gain in signal-to-noise ratio is achieved. Under certain circumstances, nonlinear devices can be made to single out properties of the noise or signal to achieve a better separation. An example of this fact was pointed out by the following experiment.

A noise waveform was generated by passing a square wave with independent, random axis crossings through an RC differentiating circuit with a time constant of approximately 50 μsec. The resulting exponential-pulse noise was then added to a music signal that was lowpass-filtered with a high-frequency cutoff of 3.2 kc.

Ideally, the noise could be entirely eliminated in three steps, as follows:

1. By passing the signal plus noise through a linear filter that converts the exponential-noise pulses into impulses.

2. By clipping the resulting signal to remove the impulses.

3. By passing the signal minus the impulses through an inverse linear filter to regain the original signal minus the noise.

A device for approximating these three steps was designed, and preliminary tests have been made. The filter required in step 1 could only be approximated and therefore did not produce true impulses, but rather short spikes of large amplitude. As a result of this limitation, the clipping process removed a portion of the impulse energy, and therefore the noise was partially removed. The filter recovered plainly intelligible speech and music from a noise level that was sufficiently high to make it barely perceptible to the ear.

Quantitative results and a more detailed discussion of the theory will be given later.

T. G. Stockham, Jr.

D. THERMAL INTEGRATING MULTIPLIER

For correlation, convolution, average-power determinations, and so on, it is necessary to find the average value of the product of two functions. If we utilize thermal heating to perform these operations, the integration and multiplication can be carried out simultaneously in a relatively simple manner, whereas the separate operations might be more difficult. Professor S. J. Mason suggested that a doubly excited bridge might be used to determine the average product of two variables. This report will describe the practical embodiment of this idea.

The basic circuit is shown in Fig. XX-2. The aim is to develop an output proportional to the average product of v_1 and v_2. The four arms of the bridge are fuses whose resistances are given very accurately by

$$r_f = r_o + pI_o^{-2} \tag{1}$$

where I_o and r_o are constants, and $p = \widehat{i_f^2}\, r_f$ is the power dissipated in the fuse.

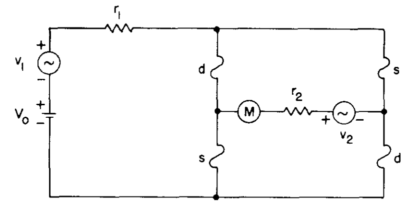

Fig. XX-2. Basic thermal integrating multiplier.
$r_1 = \left.\begin{matrix} a \text{ for ac} \\ b \text{ for dc} \end{matrix}\right\}$
$r_2 = \left.\begin{matrix} a \text{ for ac} \\ m \text{ for dc} \end{matrix}\right\}$
M = average-reading meter
$\hat{v}_1 = \hat{v}_2 = 0$

Expression 1 is accurate within approximately ±0.2 per cent for $r_f/r_o < 1.2$. For a 10-ma fuse, the value of r_o lies between 200 and 250 ohms, and I_o is approximately 11 ma. (Note that if we solve for r_f in Eq. 1 we have $r_f = \dfrac{r_o}{1 - \widehat{i_f^2}/I_o^2}$, and thus $r_f = \infty$ [i.e., the fuse blows] for $\widehat{i_f^2} = I_o^2$. Actually, I_o is slightly larger than the rated fuse current because Eq. 1 is not accurate when the heating becomes excessive.) The thermal time constant of the fuse is a few tenths of a second.

In Fig. XX-2 the fuses labeled "s" (with resistance s) have currents that are proportional to the sum of v_1 and v_2; the fuses labeled "d" (with resistance d) have currents that are proportional to the difference of v_1 and v_2. Since the heating of the fuse depends upon the square of the fuse current, the bridge will be unbalanced, with s − d proportional to the average value of the product of v_1 and v_2. To determine the unbalance of the bridge, a dc voltage is applied across one diagonal of the bridge, and the dc voltage that appears across the other diagonal of the bridge is measured by an average-reading dc meter. If we insist that v_1 and v_2 have no dc components, the meter reading will be proportional to the average value of the v_1v_2 product $(\widehat{v_1v_2})$. The limitation that v_1 and v_2 have no average values is not very serious, since the average values of a given waveform can be subtracted and multiplied separately by using relatively simple techniques.

In the circuit of Fig. XX-2 the ac resistances of the sources v_1 and v_2 are equal to a; the dc source resistance is b; and the meter resistance is m. The fuses have the same resistance for both ac and dc, which is given by s or d. Solving for the meter current, we find that

$$\frac{I_m}{V_o} = \frac{A(\widehat{v_1v_2})}{1 + B\left(\widehat{v_1^2} + \widehat{v_2^2}\right) + C\left(V_o^2\right)} = k\,\widehat{v_1v_2} \tag{2}$$

where

$$A = \frac{b^2}{(b+s)(m+d)+(m+s)(b+d)}\left[\frac{s}{(a+s)^2}+\frac{d}{(a+d)^2}\right]\frac{1}{I_o^2}$$

$$B = \frac{sd-a^2}{4(a+s)^2(a+d)^2 I_o^2}$$

$$C = \frac{sd-m^2}{[(b+s)(m+d)+(m+s)(b+d)]^2 I_o^2}$$

If k were a constant independent of v_1 and v_2, we would have an ideal average-reading multiplier, but since s and d depend somewhat on v_1 and v_2, k will not be absolutely constant. By judicious choice of circuit parameters, however, k can be made to be practically constant; typically, k will vary by less than ±0.5 per cent from a constant value for reasonable values of v_1 and v_2. If we approximate Eq. 2 for a judicious choice of variables, we find

$$\frac{I_m}{V_o} = \left[4(b+a)(m+a)\,aI_o^2\right]^{-1}\widehat{v_1 v_2} \qquad (3)$$

with m, b > 10a; $s_o = d_o = a$; $s = (1.00 \pm .05)s_o$; and $d = (1.00 \pm .05)d_o$.

If we use a 10-ma fuse (I_o = 10 ma, $s_o = d_o$ = 250 ohms), with V_o/b = 2 ma and m = 2K, we have $I_m \approx 10\,\widehat{v_1 v_2}$ μamp if $\dfrac{\widehat{v_1^2}+\widehat{v_2^2}}{4a} < 16$ milliwatts (to prevent errors and fuse burnout). The error involved in neglecting B and C will be less than 0.5 per cent for this choice of circuit values. Thus we have a possibility of building an accurate average-reading multiplier with a practically unlimited frequency response and with only a few milliwatts of power required from each source. A practical circuit, shown in Fig. XX-3, has an over-all accuracy of approximately ±1 per cent.

Any small metallic conductor can be used to replace the fuses, but a fuse is

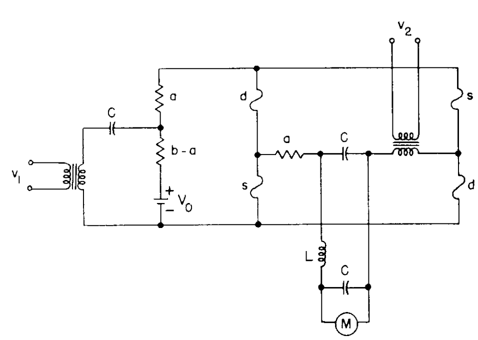

Fig. XX-3. A practical form of Fig. XX-2.

relatively cheap and requires only a few milliwatts of power to change its resistance 10 per cent. It is necessary to measure the characteristics of a number of fuses and select a matched set of fuses to achieve satisfactory operation. If we match I_o for the four fuses within approximately 0.2 per cent, good operation is possible. Furthermore, the amplitudes of v_1 and v_2 must be sufficiently small so that the fuse resistance does not change by more than approximately ±5 per cent from some average value. Note that the resistance of the fuse is a relatively constant parameter, and hence this circuit has advantages over a similar circuit that uses diodes which must be accurately matched.

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