IX. FREQUENCY MODULATION

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RESEARCH OBJECTIVES

The capture performance of a wideband FM system still holds our main interest in this research work. While we continue to be interested in developing new methods of suppressing FM disturbances for enhancing the capture of the stronger of two cochannel signals, the emphasis has shifted to the problem of capturing the weaker signal. We are probing the possibilities of several new methods that we classify as direct and indirect methods. In the direct approach, we attempt to capitalize on the distinguishability of the stronger signal in order to derive information that enables us to attenuate this signal continuously and instantaneously. The indirect approach involves spectrum-reshuffling techniques that are aimed at synthesizing a signal whose average frequency equals the instantaneous frequency of the weaker signal.

Our research is also concerned with: (a) analysis of FM transients and low-distortion transmission of FM signals, (b) analysis of low-distortion FM-to-AM conversion, (c) improvement of the locking range and decrease of the locking threshold of oscillating limiters, (d) AGC systems and their effect upon the interference and noise performance of FM receivers, (e) techniques of analysis and design of certain basic FM circuits, and (f) development of techniques and criteria for the objective evaluation of the capture performance of the FM receiver with the use of standard laboratory equipment. Of related interest to our work are the transistorization of certain basic circuits and the switching properties of semiconductor diodes for application in the analysis of transistor relaxation oscillators.

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A. CONDITION FOR QUASI-STATIONARY RESPONSE

The following discussion concerns a deduction of an upper bound on the error incurred in analyzing the response of a linear system to an FM excitation on a quasi-stationary (or instantaneous-frequency) basis.

Consider a linear (realizable and stable) system which is characterized by its unit-impulse response $h(t)$. Let the constant-amplitude, frequency-modulated excitation be described by

$$i(t) = e^{j[\omega_c t+\theta(t)]}$$

where $\theta(t)$ is some arbitrary function of time. If the forced-response voltage, after the initial transients have died out, is denoted by

$$e(t) = E(t)\, e^{j[\omega_c t+\theta(t)]}$$

then it follows that

$$E(t) = \int_0^\infty g(t, \tau)\, h(\tau)\, e^{-j\omega_i \tau}\, d\tau$$

where $\omega_i(t)$, the instantaneous frequency of excitation, $= \omega_c + d\theta/dt$, and

$$g(t, \tau) = e^{j[\theta(t-\tau) - \theta(t) + \tau\theta'(t)]}$$

If we assume that $\theta(t)$ and $\theta'(t)$ are continuous for all t, and that $\theta''(t)$ exists for $0 < t < \infty$, then, by Taylor's formula, we have $\theta(t-\tau) = \theta(t) - \tau\theta'(t) + R_{2\theta}$, where $R_{2\theta} = \frac{1}{2}\theta''(t - \eta\tau)(-\tau)^2$, $0 < \eta < 1$. It is evident that $|\theta''(t - \eta\tau)| \leq |\theta''(t)|_{max}$. Therefore $g(t, \tau) = \exp(j\, R_{2\theta}) = 1 + R_{1g}$, where $R_{1g} = j\, R_{2\theta} \exp(j\mu R_{2\theta})$, $0 < \mu < 1$, and $|R_{1g}| \leq |R_{2\theta}|$.

Thus we can write

$$E(t) = \int_0^\infty [1 + R_{1g}]\, h(\tau)\, e^{-j\omega_i \tau}\, d\tau$$

$$= Z(j\omega_i) + R_{1E}(t)$$

where $Z(j\omega_i) \equiv \int_0^\infty h(\tau) \exp(-j\omega_i\tau)\, d\tau$, and $R_{1E}(t) = \int_0^\infty R_{1g} h(\tau) \exp(-j\omega_i\tau)\, d\tau$. Since

$$|R_{1E}(t)| \leq \frac{1}{2}|\theta''(t)|_{max} \frac{d^2 Z(j\omega_i)}{d(j\omega_i)^2}$$

it is evident that $E(t)$ is closely approximated by the quasi-stationary term, $Z(j\omega_i)$, provided that $\theta(t)$ satisfies the indicated restrictions; $Z(s)$ is analytic in the right-half of the s-plane and on the $j\omega$-axis; and

$$\epsilon \equiv \frac{1}{2}\, |\theta''(t)|_{max} \cdot \left|\frac{Z''(j\omega_i)}{Z(j\omega_i)}\right| \ll 1$$

If the maximum possible value of ϵ is negligible compared with unity, this condition will be satisfied at all times. This maximum possible value occurs whenever $|(d/dt)\omega_i(t)|$ is maximum, while $\omega_i(t)$ corresponds to the value at which $|Z''(j\omega_i)/Z(j\omega_i)|$ is maximum. We therefore require that

$$\frac{1}{2}\, |\theta''(t)|_{max} \cdot \left|\frac{Z''(j\omega_i)}{Z(j\omega_i)}\right|_{max} = \epsilon_m \ll 1$$

if the frequency of the excitation can be expected to sweep the frequencies at which

$|Z''(j\omega_i)/Z(j\omega_i)|$ has its maxima. The quantity ϵ_m is then an upper bound on the relative error incurred in assuming that the forced response of the system is given by $e(t) = Z(j\omega_i) \exp\left(j \int_0^t \omega_i(t)\, dt\right)$, where $Z(j\omega_i)$ is the system function as a function of the instantaneous frequency, ω_i, of the excitation.

It is interesting to note that the sluggishness of the filter is completely characterized by $|Z''(j\omega)/Z(j\omega)|$. Thus we have chosen to call this ratio the "sluggishness ratio" of the filter. Sluggishness-ratio plots, as well as a detailed discussion of low-distortion transmission of FM signals through linear systems, based on the condition which has just been derived, will be presented in Technical Report 332 (to be published).

An alternative expression for $E(t)$ is given by

$$E(t) = \int_0^{\infty} g(t, \tau)\, h(\tau)\, e^{-j\omega_c \tau}\, d\tau$$

where $g(t, \tau) = \exp[-j\theta(t)] \exp[j\theta(t-\tau)]$. Here, by using Taylor's formula, we have been able to show that the error incurred in approximating $E(t)$ by the first n terms of the Carson and Fry expansion is dominated by the magnitude of the first rejected term in this expansion.

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