II. MICROWAVE GASEOUS DISCHARGES[*]

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A. HIGHLY IONIZED PLASMAS

Thus far, the following optimum conditions for the production of a highly ionized plasma in a microwave cavity (Quarterly Progress Report, July 15, 1957, p. 7) have been determined. The cavity should be tightly coupled to the line; the best results are obtained with the cavity overcoupled by approximately 10 decibels. The dc magnetic field should be set at a value that is a few per cent smaller than the value required for cyclotron resonance. The optimum pressure is between 1 and 5 microns. The adjustment of the magnetic field is rather critical. When the magnetic field is at its optimum value, and the magnetron frequency is equal to the resonant frequency of the empty cavity, and the incident power is limited to approximately 1 watt, then the plasma appears to occupy (at least as far as its light output indicates) a region in the form of a narrow cylinder (diameter, approximately 2-3 mm) along the axis of the quartz tube (diameter, 10 mm). The plasma boundary can be made to appear rather sharp by careful adjustment of the magnetic field.

Optimum conditions are reached when the plasma is initially in the form of this "constricted" column. When the incident power is increased, the radius of the plasma column increases until the plasma fills the quartz tube. A further increase in incident power, and tuning of the magnetron frequency towards the new resonant frequency of the cavity, increase the plasma density. The highest plasma density obtained thus far (at an expenditure of 30 watts) was approximately 6×10^{11} cm^{-3}, which corresponds to a degree of ionization of approximately 2 per cent.

As the plasma density increases, so does the degree of dissociation of molecular hydrogen. This is evidenced by comparing the relative intensities of Balmer lines and molecular bands.

The mechanism by which the plasma "constricts" is not yet clear. The effect of dissociation on the radial distribution of electron density and electron temperature in the plasma was studied by using helium and argon which, of course, do not dissociate. Although a sharp constricted plasma could not be obtained with these gases, a constricted plasma with diffuse boundaries which remained away from the quartz walls was produced. It is believed, therefore, that the "constriction" is not the result of an electron-temperature gradient that could be produced if the gas were completely

dissociated in the inner core, molecular hydrogen were forming by recombination on the radial walls, and upon streaming back towards the axis of the tube, it were dissociated again with the result that the electron cloud in the immediate neighborhood of the quartz wall is cooled.

Experiments are in progress to determine the role which the radial walls of the quartz tube play in the production of the constricted plasma. When the tube radius is reduced one-half (0.5 cm), the constricted plasma is no longer in the form of a uniform cylinder but is in the form of a cylinder whose diameter decreases as the distance from the central plane increases. The plasma appears like an elongated football, suggesting, perhaps, that most of the plasma electrons are produced near the center of the tube, and, as a result of the decreased radius of the quartz tube, some of the electrons are lost in the radial direction as they move along the B-field lines toward the end walls.

More experiments are planned in order to determine the mechanism of the "constricted" plasma. We believe that the phenomenon is important, since it appears that only when the plasma is in the "constricted" state can high densities be produced.

S. J. Buchsbaum

B. PLASMA DIAMAGNETISM

It was shown in a previous report (1) that if we consider only the diffusion and mobility currents in a quasi-static plasma, the relationship between the plasma density $n(r)$, the electron and ion temperatures T_- and T_+, and the magnetic field $B(r)$ is given by

$$-\mu_0 nk(T_- + T_+)\mu_+\mu_- = \ell n \, B/B_0 + \frac{1}{2}\left(B^2 - B_0^2\right)\mu_+\mu_- \tag{1}$$

in which μ_+ and μ_- are the ion and electron mobilities in the absence of magnetic fields. Writing $B/B_0 = 1 - \delta B/B_0$, we can plot $\delta B/B_0$ as a function of $-\mu_0 nk(T_+ + T_-)\mu_+\mu_-$ with $\mu_+\mu_- B_0^2$ as a parameter. This is shown in Fig. II-1. For $nk(T_+ + T_-) < (1/5)B_0^2/2\mu_0$, that is, with the plasma pressure less than the magnetic pressure, a very good approximation to Eq. 1 can be written as

$$\delta B = \frac{\mu_+\mu_- B_0^2}{1 + \mu_+\mu_- B_0^2} \frac{\mu_0 nk(T_+ + T_-)}{B_0} \tag{2}$$

Steenbeck (2) has measured the change in the magnetic field as a result of the plasma diamagnetism by superimposing a sinusoidal electron-density modulation upon the steady-state density distribution and measuring the sinusoidal component of the magnetic-field strength. Figure II-2 shows the curve of Eq. 2 fitted to his data. This fit yields $nkT = 1.3 \times 10^{-1}$ erg-cm^{-3}, $\mu_+\mu_- = 4 \times 10^{-4}$ gauss^{-2} at a pressure of 3 microns.
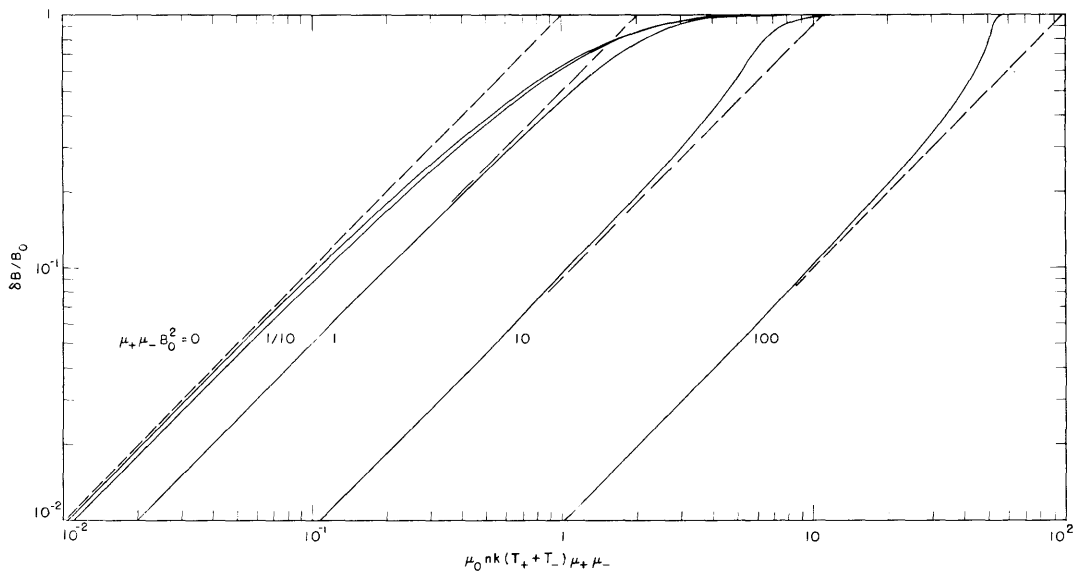
Fig. II-1. Relative change in field strength as a function of ion pressure.
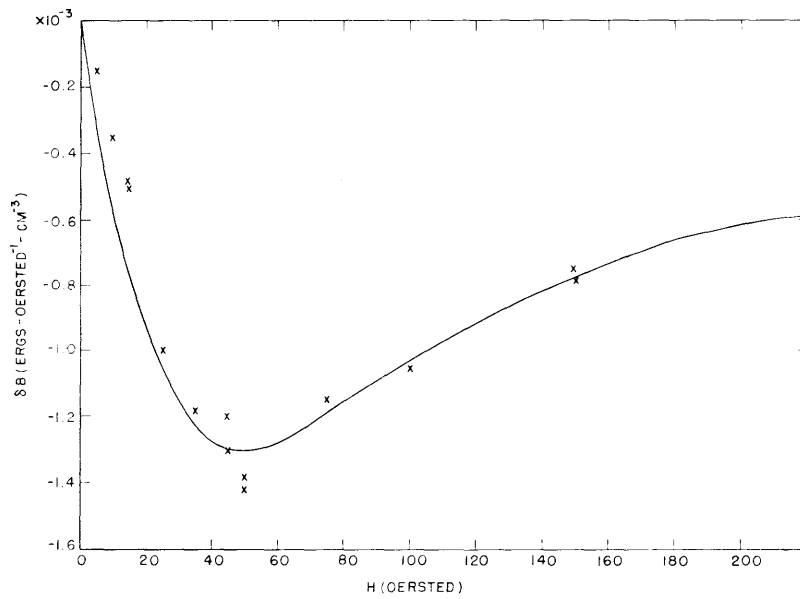Dashed lines refer to Eq. 2.

Fig. II-2. Change in magnetic-field strength as a function of applied field strength.

Converting to more conventional units, we obtain $(\mu_- p)$ $(\mu_+ p)$ = 3.6 × 10^7 cm^4-sec^{-2}-$volts^{-2}$-(mm Hg)^2. For mercury, Aisenberg (3) gives

$$\mu_- p = 1.49 \times 10^5 \left(\frac{e}{kT}\right)^{1/2}$$

$$\mu_+ p = 3.05 \times 10^3 (E/p)^{-1/2}$$

(3)

If we assume as appropriate values for Steenbeck's discharge kT/e = 1.5 volts and E/p = 100 volts/cm mm Hg, Eq. 3 yields $(\mu_- p)$ $(\mu_+ p)$ = 3.72 × 10^7.

For the varying part of Steenbeck's density, we can assume that n = 5 × 10^{10} cm^{-3}, within one order of magnitude. This yields nkT = 1.2 × 10^{-1} erg cm^{-3}. The very good agreement in both mobility and density is probably accidental, but it does indicate that Eq. 2 may be valid.

Steenbeck's measurements were made in the positive column of a mercury arc. Our present experiments are directed toward observing the diamagnetism in a micro-wave maintained discharge.

E. I. Gordon

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1. Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., July 15, 1957, p. 14, cf. Eq. 7.
2. M. Steenbeck, Über die magnetischen Eigenschaften des Plasmas von Gasentladungen, Wiss. Veroff. Siemens-Werken 15, 2, 1 (1936).
3. S. Aisenberg, Ph.D. Thesis, Department of Physics, M.I.T., June 1957.

C. MEASUREMENTS OF ELECTRON COLLISION PROBABILITIES IN GASES

Measurements of the electron collision probability for momentum transfer in hydrogen with the use of techniques described elsewhere (1) are in progress. At present, results are available only for the case in which the electrons are in thermal equilibrium with the gas at room temperature.

The means of determining electron collision probability consists, essentially, of measuring the conductivity in the afterglow of a microwave discharge. In the absence of inelastic collisions, the complex radiofrequency conductivity σ is given by

$$\sigma = -\frac{4\pi ne^2}{3m\omega} \int_0^\infty \frac{(\nu/\omega - j)}{1 + (\nu/\omega)^2} v^3 \frac{\partial f}{\partial v} \, dv$$

(1)

where v is the electron velocity, e and m are the electronic charge and mass, ω is the angular frequency of the rf field, f is the electron velocity distribution function, and
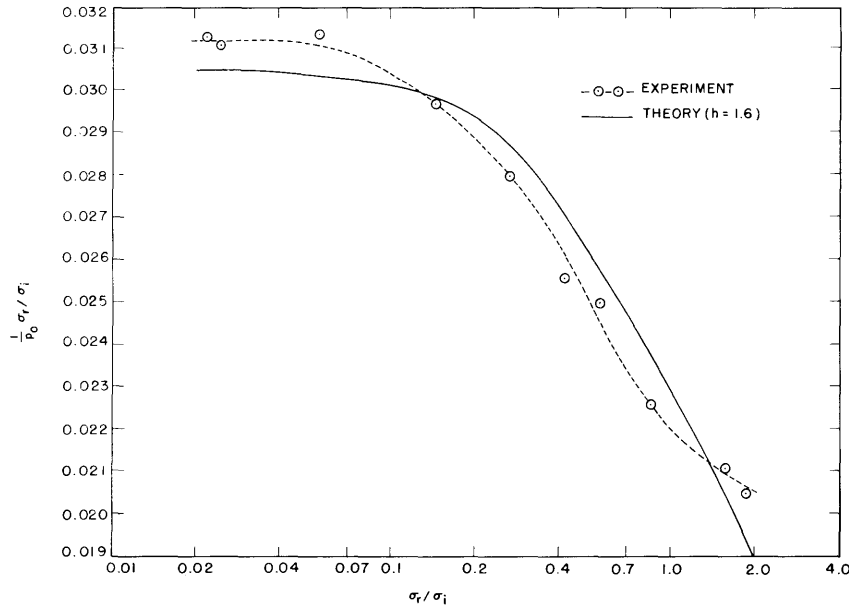
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Fig. II-3. Conductivity ratio in the afterglow of a hydrogen plasma
as a function of pressure.

n is the electron density. The collision probability P_m is related to the collision frequency ν through

$$\nu = P_m p_o v$$

where p_o is the gas pressure normalized to $0°$ C.

Measurements were carried out in hydrogen over a range of pressures p_o from approximately 0.5 to 100 mm Hg. The results are shown in Fig. II-3, in which the quantity $\sigma_r/p_o \sigma_i$ is plotted against σ_r/σ_i, the latter being an indirect measure of the pressure at which the measurement was made. Each experimental point shown represents an average of five or more readings taken at various times in the afterglow, covering a range from 100 μsec to 12 msec. Within this interval the conductivity ratio was quite independent of the time.

The collision probability P_m is determined by comparing the experimental results of Fig. II-3 with predictions from Eq. 1. This is achieved by postulating for ν a reasonable velocity dependence, preferably in the form of a power series, and then fitting the coefficients to the experimental curve. In view of the difficulties of evaluating the ensuing integrals at any pressures except low pressures $(\nu/\omega \ll 1)$, workers in the field have assumed the following simple power law:

$$\nu = a p_o v^h \tag{2}$$

If we make a substitution from Eq. 2 in Eq. 1, we obtain

$$-\frac{1}{\gamma}\frac{\sigma_r}{\sigma_i} = \int_0^\infty e^{-x}\frac{x^{1.5+0.5h}}{1+\gamma^2 x^h}\,dx \bigg/ \int_0^\infty e^{-x}\frac{x^{1.5}}{1+\gamma^2 x^h}\,dx \tag{3}$$

with $\gamma = (2kT/m)^{h/2}\,ap_o/\omega$, where k is Boltzmann's constant, and T is the absolute temperature of the gas. The best fit between experiment and Varnerin's (2) numerical solution of Eq. 3 is found for the case h = 1.6 (see Fig. II-3), which yields the following value for the collision probability

$$P_m = 28(v/\bar{v})^{0.6}\text{ cm}^{-1}\text{ mm}^{-1}\text{ Hg};\quad \bar{v} = 0.161\,(\text{volt})^{1/2}$$

where $\bar{v}$ is the most probable velocity at a temperature of 300° K. This value of P_m differs greatly from that of Phelps, Fundingsland, and Brown (3), who quote a value of 46, subject to the assumption that P_m is independent of velocity, but it is in substantial agreement with Varnerin's (2, 4) value, $P_m \sim 34\,(v/\bar{v})^{0.6}$.

Table II-1.

h	Error in $\sigma_r/\gamma\sigma_i$	Number of Terms of Series
$0 \le h \le 1.0$	$\le 2\%$ for $0 \le \gamma \le 1$	first term only
$0 \le h \le 1.0$	$\le 5\%$ for $1 < \gamma \le 1.5$	first two terms
$1 < h \le 1.6$	$\le 3\%$ for $0 \le \gamma \le 1$	first term only
$1.6 < h \le 2$	$\le 7\%$ for $0 \le \gamma \le 1$	first two terms

As we have mentioned, in those cases for which it is not desirable to satisfy the condition $v/\omega \ll 1$, the conductivity integral must, in general, be evaluated numerically. Moreover, the complexity increases manyfold if the distribution function departs from Maxwellian or if the plasma is acted upon by a magnetic field. Approximate solutions were sought and good results were obtained with the aid of a recently derived (5) asymptotic solution of the integral

$$I = \int_0^\infty e^{-px}x^{c-1}F(x)\,dx = p^{-c}\,\Gamma(c)\sum_{k=0}^\infty c\,(c+1)\ldots(c+k-1)\,p^{-2k}\,F_k\left(\frac{c+k}{p}\right)$$

where

$$F_O(x) = F(x) ; \qquad F_{k+1}(x) = \frac{d}{dx}\left\{ -\frac{F_k(x) - F_k\left(\frac{c+k}{p}\right)}{x - \frac{c+k}{p}} \right\}$$

and p must be "sufficiently" large and may be complex; Re $c > 0$, and $F(x)$ is an analytic function.

For example, in applying this expansion to the evaluation of Eq. 3, we set $p = 1$, $F(x) = (1 + \gamma^2 x^h)^{-1}$, and $c = 2.5 + 0.5h$ or $c = 2.5$, depending upon which of the two integrals is being evaluated. Table II-1 shows the degree of accuracy attainable with the asymptotic formula as compared with a numerical evaluation of the integrals.

G. Bekefi

References

1. L. Gould and S. C. Brown, Phys. Rev. 95, 897 (1954).

2. L. J. Varnerin, Jr., Progress Report CR31-555-21, Sylvania Electric Products, Inc., Boston, Massachusetts, 1951.

3. A. V. Phelps, O. T. Fundingsland, and S. C. Brown, Phys. Rev. 84, 559 (1951).

4. L. J. Varnerin, Jr., Phys. Rev. 84, 563 (1951).

5. J. Franklin and B. Friedman, Research Report No. BR-9, Institute of Mathematical Sciences, New York University, 1954.

D. HIGH-CURRENT ION SOURCE

The design and initial testing of an ion source capable of producing a 20-ma beam of hydrogen ions at an energy of approximately 500 volts have been completed. A magnetic field was used to focus the beam, and to maintain the discharge at a pressure of approximately one micron. Differential pumping was employed to reduce the pressure in the accelerating section of the ion source.

The electrode configuration in the ion source is shown in Fig. II-4. The source was modeled after a device built by F. M. Penning and J. H. A. Moubis (1). The cathode is a pure tungsten ribbon, 0.001 inch thick, with an emitting surface approximately 0.25 inch square. It is heated to electron-emission temperatures by an ac heating unit.

The electrodes are constructed of nonmagnetic molybdenum, and can operate at high temperatures in a hydrogen atmosphere. The accelerating electrode draws electrons from the hot cathode but cannot collect them because the magnetic field prevents their reaching the accelerating electrode. The deaccelerating electrode, which is at the same potential as the cathode, stops the electrons and returns them in the direction toward
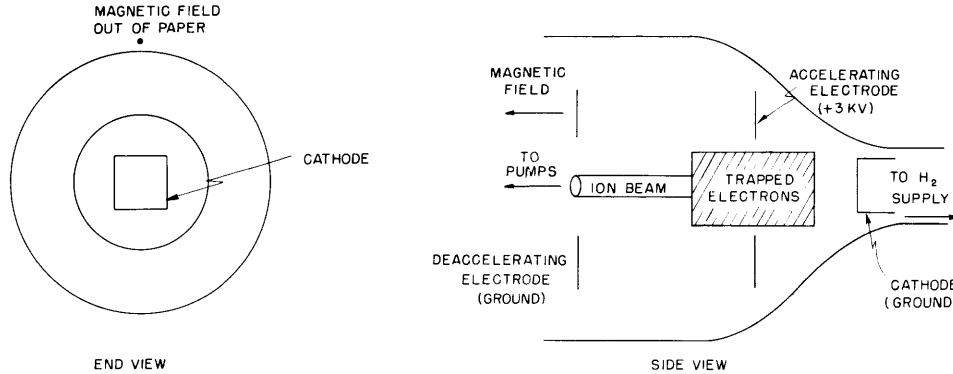
Fig. II-4. Electrode configuration in the ion source.

the cathode. The electrons perform many oscillations in this manner, until they are eventually lost to the accelerating electrode through collisions with hydrogen atoms. Thus a single electron can make many ionizing collisions in its lifetime; only a reasonable electron current need be emitted from the cathode to produce a large ion current.

For ions produced in the source, the functions of the electrodes are reversed, since the "trapping" action works only for negatively charged particles. The positively charged ions are accelerated toward the ground potential of the deaccelerating electrode and leave the source as a beam.

A steady magnetic field of approximately 800 gauss is produced by an external solenoid, and serves the dual purpose of trapping the electrons and preventing the spread of the ion beam which is caused by space charge.

R. G. Meyerand, Jr.

References

1. F. M. Penning and J. H. A. Moubis, Eine Neutronenröhre ohne Pumpvorrichtung, Physica **4**, 1190 (1937).

E. PULSE BREAKDOWN IN A MAGNETIC FIELD

By observing the rate of increase of electron density in a microwave discharge, it is possible to study quantitatively some of the processes that lead to the gain of electrons in the plasma, and also the interdependence of various parameters involved in these processes.

The rate of growth of electron density can be written generally as

$$\frac{\partial N}{\partial t} = (\nu_i - \nu_L)N \tag{1}$$

where v_i is the frequency of electron production, and v_L represents the frequency of electron loss. Equation 1 shows that a measurement of time for the build-up of electron density to a value that is large enough to be detected yields information about the net electron gain, whereas a similar measurement when the plasma is decaying $(v_i = 0)$ will give information about v_L. Determinations of frequency of ionization and coefficient of diffusion, by using a pulsed microwave discharge, in the absence of a magnetic field, have already been reported (1, 2). In the present report an analogous technique for studying the behavior of a microwave pulsed discharge in the presence of a magnetic field is described.

The experimental arrangement (1) is nearly the same as that used before, except that the plasma is now produced in a narrow cylindrical tube, which is placed along the axis of a cylindrical microwave cavity $(TE_{111}$ mode). The quartz tube is placed in a static magnetic field parallel to the axis of the tube. The principle of cyclotron resonance is utilized as an efficient means of energy transfer from the microwave field to the electron, and the working pressures are of the order of 10^{-3}-10^{-4} mm Hg.
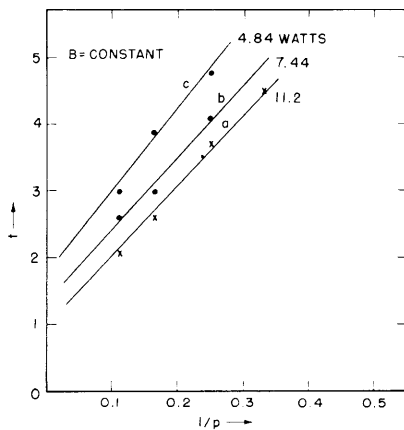

Fig. II-5. Breakdown time against 1/p with input power as a parameter, for a constant magnetic field.

Preliminary measurements indicated that with the application of a microwave pulse, the breakdown of the gas takes place erratically. Therefore, in order to reduce the statistical lag, a low-density plasma is maintained with a low-power cw signal, then the density of the plasma in the quartz tube is increased by a microwave pulse. The time for the build-up of the electron density to a value N_m, taken as the number of electrons that causes the electric field in the cavity to decrease 5 per cent (1), is measured with the aid of a Tektronix oscilloscope as a function of the power level in the microwave pulse and as a function of the static magnetic field close to the value that corresponds to cyclotron resonance. Time for breakdown is minimum when the cyclotron frequency ω_b equals the microwave frequency ω. It also decreases when the input microwave power is increased.

Figure II-5 shows that at a constant value of input power and of the magnetic field the time for breakdown varies inversely with pressure. That this should be so can be seen from Eq. 1, in which the total number of electrons is written as

$$N = N_0 \exp(\nu_i - \nu_L)t \tag{2}$$

or

$$t = \frac{1}{(\nu_i - \nu_L)} \ln \frac{N}{N_0} \tag{3}$$

To a first approximation we can assume that ν_L is small, and write

$$t \approx \frac{1}{h_i \nu_c} \ln \frac{N}{N_0} \approx \frac{1}{h_i \, 5.9 \times 10^9 p} \ln \frac{N}{N_0} \tag{4}$$

The slope of the lines in Fig. II-5 yields only qualitative information about the probability of ionization per collision, h_i, since both N and N_0 are unknown. But let us assume
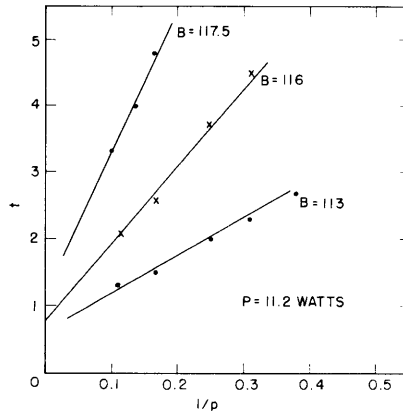

Fig. II-6. Breakdown time against 1/p with magnetic field (in relative units) as a parameter, for constant input power.

that the density builds up over, say, two orders of magnitude. Then, from curve a in Fig. II-5, we obtain $h_i \approx 0.7$, which is reasonable. In Fig. II-6 similar curves (i.e., t against 1/p with magnetic field as a parameter) for a constant input power are plotted. These two plots show that the changes in slope are small as the input power is increased, whereas the changes in slope are large as the magnetic field is varied. This behavior of the curves indicates that large changes in the electron energy take place as the magnetic field is varied over a small region close to its cyclotron-resonance value.

M. P. Madan

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1. M. P. Madan, E. I. Gordon, S. J. Buchsbaum, and S. C. Brown, Determination of the coefficient of diffusion and frequency of ionization in microwave discharges, Phys. Rev. 106, 839 (1957).

2. M. P. Madan, Microwave determinations of frequency of ionization and coefficient of free diffusion, Quarterly Progress Report, Research Laboratory of Electronics, Oct. 15, 1956, p. 15.