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Estimation of the Required Amount of Superconductors for High-field Accelerator Dipole Magnets

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Summary

The coil size and the corresponding amount of superconducting material that is used during the design process of a magnet cross-section have direct impacts on the overall magnet cost. It is therefore of interest to estimate the minimum amount of conductors needed to reach the defined field strength before a detailed design process starts. Equally, it is useful to evaluate the efficiency of a given design by calculating the amount of superconducting cables that are used to reach the envisaged main field by simple rule.

To this purpose, the minimum amount of conductors for the construction of a dipole of given main field strength and aperture size is estimated taking the actual critical current density of the used strands into account. Characteristic curves applicable for the NED Nb3Sn strand specification are given and some of the recently studied different dipole configurations are compared. Based on these results, it is shown how the required amount of conductors changes due to the iron yoke contribution and the loss of current-transporting surface by means of insulation and cabling.

1 Introduction

Different designs of superconducting coil cross-sections for dipole magnets vary mainly in the way how the superconducting cables are distributed around the aperture [1]. Since the cost of a superconducting magnet depends heavily on the amount of superconducting cable used for the coil winding, this number has to be examined [2] and taken into account if different designs are compared.

The magnetic field which can be obtained with a certain magnet configuration depends on the powering current. Due to the dependence of the maximum current density on the applied magnetic induction, a gain in main field strength can only[1] be achieved by increasing the cross-sectional area of the coil once the maximum current density is reached. In addition, the required amount of superconductors depends on the cable design since the overall current *density* in the cable cross-section scales with the cross-sectional area which is even further reduced by the required cable insulation. Finally, the maximum current density of the strands would be reduced by broken filaments [3], depending on the cabling process and the cable keystoning.

In this paper, we calculate the minimum amount of superconductors required for the construction of a dipole of given aperture size and peak field strength on conductor[2] by means of an ideal dipole model - two intersecting circles. The result is depicted by applying the specifications of the NED cable [4] and [5].

2 Analytical Model

The superconducting magnet coil is modeled by means of the ideal geometry of two intersecting circles shown in fig. 1, where R denotes the radii of the two circles, $2c$ the distance of the centers of the circles (that is the coil thickness at the midplane) and J the modulus of the oppositely directed and homogeneous current densities. The field inside the aperture is given by

$$B = \mu_0 c J, \tag{1}$$

and depends only on the distance of the circle centers $2c$ and the modulus of the current density J, whereas the radius of the circles R results from the chosen aperture size.

The maximum current density a superconductor is able to carry depends on temperature and applied magnetic induction. It is denoted by the critical current density $J_\mathrm{c}(B,T)$ usually given by a non-linear function as *e.g.* in [6] for Nb-Ti or in [7] for Nb_3Sn. Applying this function to (1) yields an implicit expression of the obtainable main field depending on the peak field on the conductor. For the considered ideal design, the homogeneous aperture field is maximum and the estimation can be carried out for the peak field only. For more realistic coil cross-sections the peak field on conductor typically exceeds the aperture field by some percent.

[1] The maximum allowed current density in a superconductor also depends on other quantities like applied stress and temperature, not considered here.

[2] Often, aperture size and field strength in the aperture are used as design criterion

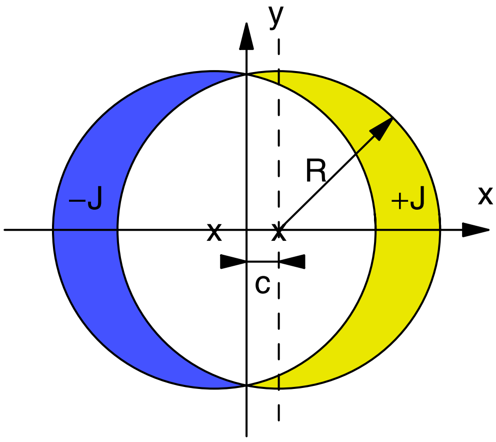


Figure 1: Ideal dipole geometry of two intersecting circles.

For the given setup of two intersecting circles the required "coil thickness", $2c$, to obtain a certain peak field is already given. The influence of the iron yoke or of the cable design on the magnet performance can be considered by:

- A constant scaling factor, k_{I}, to the peak field respectively main field which represents the field enhancement due to the iron:

$$B_{\mathrm{peak}} = \mu_0 k_{\mathrm{I}} c J(B_{\mathrm{peak}}). \tag{2}$$

- The specification of a superconducting strand often states the current density in the superconductor, $J_{\mathrm{c,sc}}$, or the non copper area, $J_{\mathrm{c,non\text{-}Cu}}$. The coil is built of superconducting strands which *e.g.* also consists of a copper matrix and thus the effective overall current density in the cross-sectional area of the strand $J_{\mathrm{c,strand}}$ is lower. It can be calculated by the superconductor to non-superconductor ratio, $\frac{\mathrm{sc}}{\mathrm{non\text{-}sc}}$, or the copper to non-copper ratio, $\frac{\mathrm{Cu}}{\mathrm{non\text{-}Cu}}$, respectively. *E.g.:*

$$J_{\mathrm{c,strand}}(B) = \frac{1}{1 + \frac{\mathrm{Cu}}{\mathrm{non\text{-}Cu}}} J_{\mathrm{c,non\text{-}Cu}}(B) \tag{3}$$

- By considering the reduction of the maximum overall current density in superconducting cables due to the greater cross-sectional area of the cable compared to the same number of single strands, the cable insulation, and the degradation due to cabling, the estimation can be carried out for realistic cable configurations. This way, the effective maximum overall current density which depends on the peak field in the conductor is given to

$$J_{\mathrm{c,eff}}(B_{\mathrm{peak}}) = k J_{\mathrm{c,strand}}(B_{\mathrm{peak}}) \tag{4}$$

with

$$k = k_{\mathrm{cab}} k_{\mathrm{ins}} k_{\mathrm{deg}}, \quad k < 1 \tag{5}$$

where the factor k_{cab} is the ratio of the area covered by strands to the total area of the cable and k_{ins} is given by the ratio non-insulated to insulated areas in the cable. The degradation due to cabling k_{deg} is taken from the cable specifications.

Current grading resulting from the use of key-stoned cables[3] or different cables in a multi-layer design as *e.g* in the LHC main bending magnets, is not taken into account due to the problem in the analytical calculation of the resulting field gradient over the cross-section.

By including the above modifications into (1) and rearranging the terms, the required circle off-centering $2c$ of the ideal intersecting-circles geometry carrying the same effective current density as a realistic cable is given by:

$$c = \frac{B_{\text{peak}}}{\mu_0 \lambda J_{\text{c,strand}}(B_{\text{peak}})}. \tag{6}$$

The influence of the iron yoke and the reduction of the current density in the cables is expressed by the factor $\lambda = k_{\text{I}} k$. *N.B.* the result neither depends on the size of the aperture nor on the radii of the two circles.

All parameters of the ideal intersecting circles configuration are fully determined and the required amount of superconductors can be calculated by means of the total area. The aperture radius r_A is given by the greatest circle which can be inscribed in the center current free region. This way, for a given aperture radius r_A and displacement c, the radius of the intersecting circles R results to

$$R = c + r_{\text{A}}. \tag{7}$$

For the calculation of the total cross-sectional area A_{con} covered by the current density J, *i.e.*, superconducting strand material, the geometrical relations are shown in fig. 2. One quarter of the total area indicated by $<1>$ is given by the difference of the two sectors of a circle indicated by $<2>$ and $<3>$ and twice the area of the triangle indicated by $<4>$.

$$A_{\text{con}} = 4\left(A_{<2>} - A_{<3>} + 2A_{<4>}\right). \tag{8}$$

The area of the sectors can be easily calculated by their fraction of a full circle and the triangle is given by means of PYTHAGOREAN's law:

$$\begin{aligned} A_{<2>} &= \frac{\pi - \alpha}{2} R^2, \\ A_{<3>} &= \frac{\alpha}{2} R^2, \\ A_{<4>} &= \frac{1}{2} c \sqrt{R^2 - c^2}. \end{aligned}$$

[3]Assuming a homogeneous magnetic field in the cross-section of a keystoned cable, the maximum current density is defined by the thin end which gives an overall current density, that is smaller than the maximum current density.

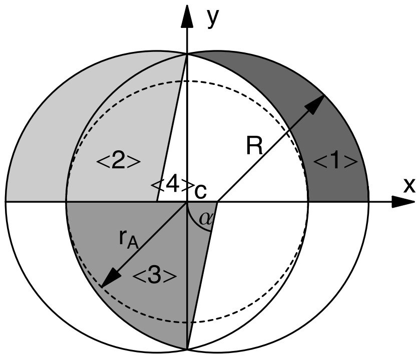


Figure 2: Geometry of two intersecting circles. Using the two shaded areas and the two triangles the area covered by the current density J can be calculated.

By expressing the angle α in terms of the two quantities R and c the total area is finally given by:

$$A_{\text{con}}(c,R) = 4\left[\left(\frac{\pi}{2} - \arccos\left(\frac{c}{R}\right)\right)R^2 + c\sqrt{R^2 - c^2}\right], \tag{9}$$

With Eq. (7) the total area given in Eq. (9) can be written as

$$A_{\text{con}}(c,r_{\text{A}}) = 4\left[\left(\frac{\pi}{2} - \arccos\left(\frac{c}{c + r_{\text{A}}}\right)\right)(c + r_{\text{A}})^2 + c r_{\text{A}}\sqrt{2\frac{c}{r_{\text{A}}} + 1}\right]. \tag{10}$$

Note, that this equation merely depends on the aperture radius r_{A} and the off-centering c which is resulting from the requested strength of the peak field in Eq. (1).

3 Graphical Solution

Due to the implicit formulation of (1) the resulting peak field for a given cross-sectional area can only be determined graphically or numerically. The graphical solution is chosen since it gives a better impression of the steer-ability and interdependencies than a point-by-point numerical calculation.

The graphical solution of Eqs. (6) and (10) is depicted in fig. 3 by means of a set of characteristic curves where the lower diagram gives the value for the distance of the circles to the center c depending on the wanted main field, the critical current density of the used strand, the reduction of current density due to the used cable and the influence of the iron yoke. The upper diagram gives the required cross-sectional area depending on the aperture radius r_{A} and the distance c.

The required amount of superconductors can be estimated by means of elementary algebraic operations only:

(a) Draw a horizontal line for the wanted peak field.

(b) Calculate the factor λ, describing the cable and the influence of the iron yoke, and pick the corresponding line.

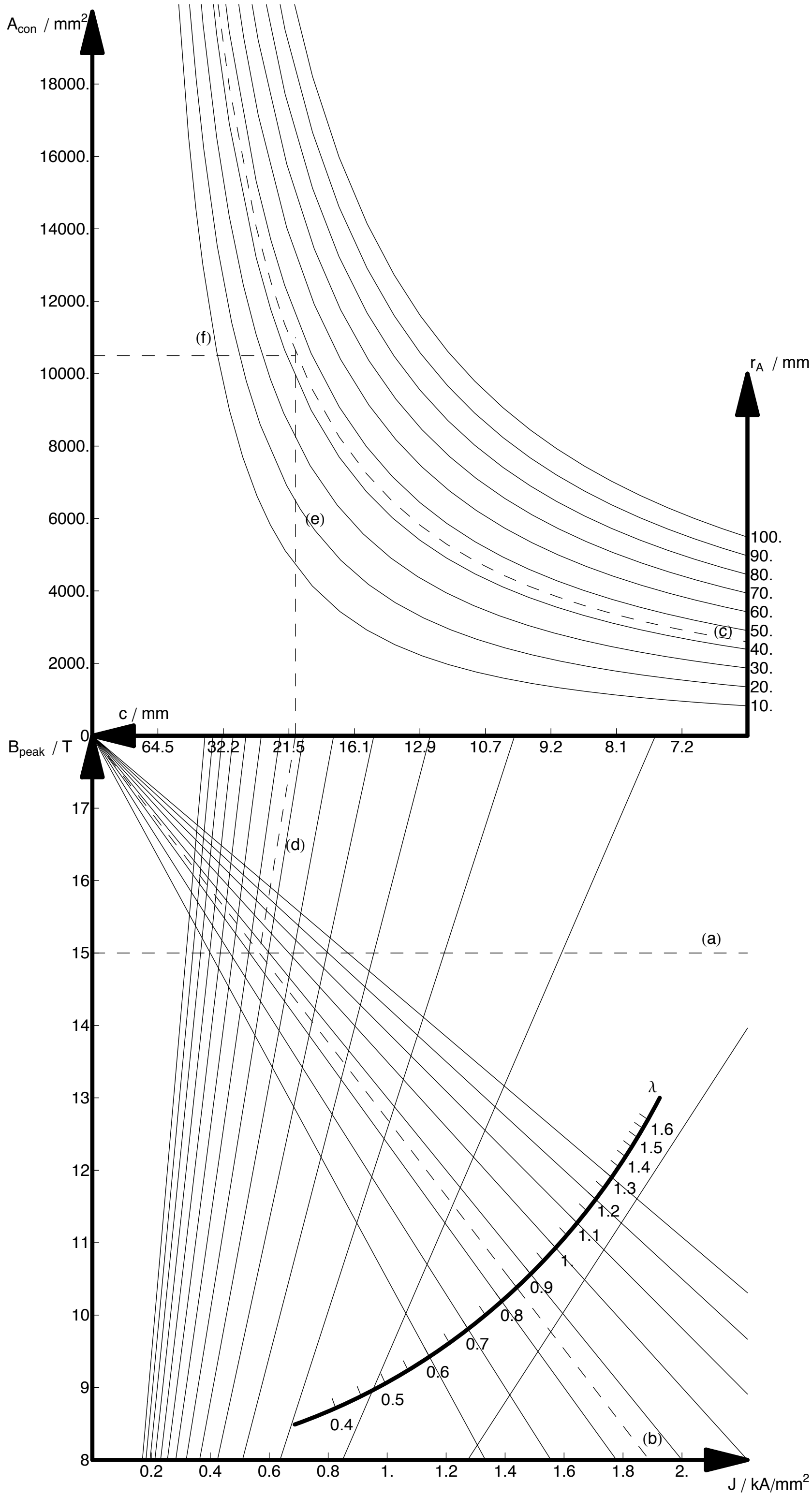


Figure 3: Set of characteristic curves for the estimation of the minimum cross-sectional area of high-field dipole geometries using the NED strand as example. The dashed lines illustrate the results given in section 3.

(c) Pick a line corresponding to the wanted aperture size.

(d) Determine the intersection of the two lines and from there follow the vertical lines up to the axis showing the values for c.

(e), (f) Read off the cross-sectional area!

Figure 3 shows an example for a wanted peak field of 15 T, $\lambda = 0.85$ and an aperture radius of 44 mm using the strand specifications of the NED cable [4] and a linear approximation of the critical current density [5]. The non-copper critical current density at high field and constant temperature is given by

$$J_{\text{non-Cu}}(B) = J_0 - \frac{\Delta J}{\Delta B}(B - B_0) \tag{11}$$

where $J_0 = 3\,\text{kA/mm}^2$ denotes the critical current density at a magnetic flux density $B_0 = 12\,\text{T}$ and $\Delta J/\Delta B = 0.5\,\text{kA/(mm}^2\text{T)}$ is the slope of the critical current density curve at B_0.

4 Sensitivity

As one can see from the graphical approach shown in fig. 3, for high peak fields B_{peak} the distance of the circles $2c$ is very sensitive to small variations of the parameters λ, B_{peak}. In addition, the resulting variations of c for high peak fields further amplify the change in the cross-sectional area.

For the NED 88 mm 15 T dipole the sensitivity of the different parameters is shown in tab. 1 assuming a parameter variation of $\pm 1\%$. The contribution of the iron yoke and the degradation have little influence on the results and changes of the aperture radius, *e.g.* due to thermal contraction are small as well.

Table 1: Variation of the estimated number of conductors for a change of the assumed values by 1%.

Parameter	Influence on the amount of superconductor
r_A	$\pm 0.7\%$
λ	$\pm 1.3\%$
B_{peak}	$\pm 8\%$

5 Results for the Next European Dipole

The new tool is illustrated by comparing the results of the current NED magnet design activities [8], all using the same strand specifications. To this purpose, we express the required amount of superconductor by the number of strands/conductors per quadrant.

5.1 Estimation of the Minimum Amount of Conductors

The specifications of the NED strand [4] are applied to the approach derived above and the number of strands are calculated for a dipole of 15 T peak field B_{peak}.

Based on the numerical results obtained for the $\cos\theta$-layer design with 44 mm aperture radius [9] the contribution of the iron yoke to the peak field at maximum excitation taken as 20% yielding $k_{\text{I}} = 1.2$. The required minimum number of strands for the three aperture diameters used within the NED program are given in tab. 2.

Table 2: Required minimum number of strands per quadrant for a 15 T dipole made of NED strands.

Aperture diameter r_A in mm	88	130	160
Number of strands	1418	1933	2299

For the $\cos\theta$-layer design an insulated, keystoned RUTHERFORD-type cable is used. With the specifications given in [4], the constants describing the reduction of the current density due to the cable geometry can be calculated and yield $k_{\text{cab}} = 0.83$ and $k_{\text{ins}} = 0.84$. For the maximum degradation due to cabling 10% reduction is taken into account by $k_{\text{deg}} = 0.9$. The resulting minimum number of conductors, insulated and non-insulated, are shown in tab. 3.

Table 3: Required minimum number of conductors per quadrant for a 15 T dipole made of keystoned NED cables.

Aperture diameter r_A in mm	88	130	160
Number of conductors (not insulated)	43	57	68
Number of conductors (insulated)	45	60	70

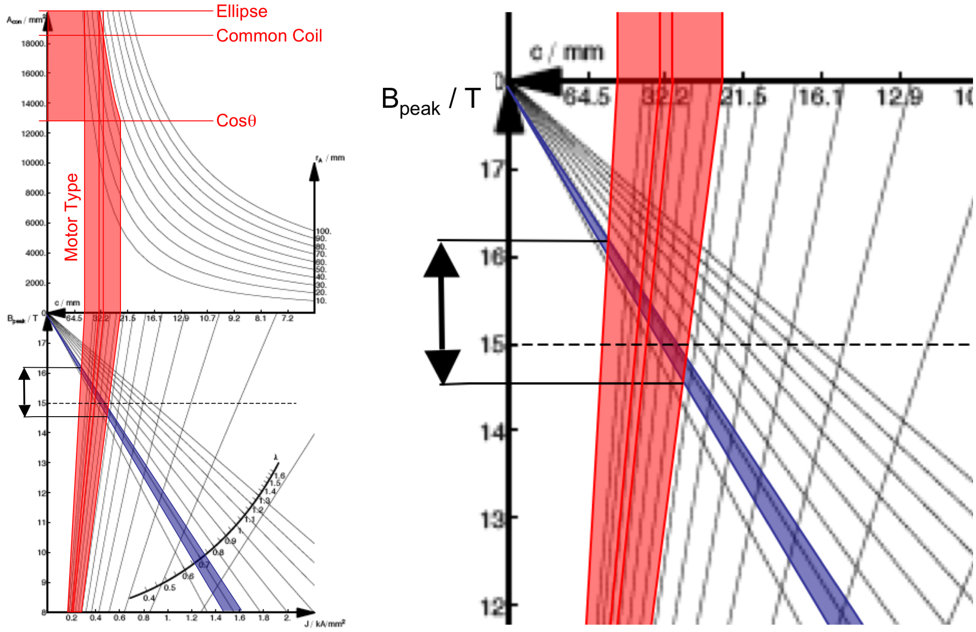
Comparing these results with the results of the preliminary design for NED published by LEROY and VINCENT-VIRY [10], a peak field approximately 1.7% higher is obtained. This deviation is likely resulting from the omittance of the cable keystoning and the respective current grading. Another comparison with the model of an ideal $\cos\theta$ configuration as used by CASPI [11] yields an about 2% greater number of conductors if the cross-sectional area is homogenized to an equivalent area of constant current density.

5.2 Comparison of Different Designs of the Magnet Design and Optimization Working Group

Within the NED collaboration very different magnet designs like, *e.g.* the motor type, the common coil, the ellipse design and, of course, the $\cos\theta$-design were studied. In order to compare and evaluate the efficiency of the different designs, the results for the 88 mm aperture are applied to the graphical solution. Figure 4 shows the final total cross-sectional area of each design [8] as well as the range of the λ values representing the different used cables and the differences in the contribution of the iron yoke [12].

It can be seen that by combining the best used conditions of the cable design and the iron yoke contribution a maximum field of approximately 16.2 T should be possible to reach. Nevertheless all designs, except for the ellipse design, show a peak field of 15 T.

As expected, the cosθ-design shows the highest efficiency of conductor use with respect to the obtainable peak field and the motor-type design the lowest. *N.B.*, although the ellipse design uses more superconductors as the common coil design, it is still of higher efficiency since it shows a peak field of nearly 15.5 T.



(a) Use of the graphical solution for the NED values.

(b) Details and the range of theoretical maximum field (14.5T to 16.2T).

Figure 4: Comparison of the different design approaches used in NED. Based on the used cross-sectional area and the corresponding value of λ, the theoretically maximum field strength is calculated.

5.3 Influence of the Iron Yoke on the Total Number of Conductors

The set of characteristic curves is now used to demonstrate the iron yoke contribution on a magnet design applying the rule-of-thumb. As suggested by Todesco and Devred, the iron yoke for the current NED design could be left away in order to cut the saturation induced field errors. This idea is resulting from the facts that firstly the removal of the iron reduces the main field without changing the current density in the conductors. Secondly, the iron yoke contribution to the total aperture field for the NED design can be compensated up to 3% by an increase of the excitation current in order to keep the peak field value. Their estimations can be fully verified for the NED 88 mm 15 T dipole by means of the shown set of characteristic curves.

From the diagram 3 it can be seen that the missing 3% in main field, nearly 0.5 T, can only be compensated for if the number of superconductors in the cross-section is increased. From the sensitivity analysis shown in table 3, one could already see that an increase in peak field of 3% would require an increase in superconductors of 24%. For the design of the 15 T, 88 mm aperture dipole for NED, due to the non-linearity of the critical current, nearly 28% more conductors are needed to keep the aperture field if the iron yoke is removed compared to the traditional design with iron yoke. This corresponds to a minimum number of 58 conductors per quadrant which due to space limitations would not match within the cross-section. Consequently, without iron yoke, the peak field cannot be reached.

6 Conclusion

A simple analytical model for the calculation of the required amount of superconductors for the design of a dipole magnet of given aperture size and peak field has been derived that allow to study differences in cable design and yoke contribution by scaling the effective current density or enhancing the peak field, respectively. The application of the graphical solution of the NED model illustrates the parameter space for the designer and gives a practical tool for fast estimations.

References

[1] M. N. Wilson, *Superconducting Magnets.* New York, USA: Monographs on Cryogenics, Oxford University Press, 1983.

[2] H. Brechna, *Superconducting Magnet Systems*, 1st ed. Berlin, Germany: Springer, 1973.

[3] A. Devred, "Practical Low-Temperature Superconductors For Electromagnets," CERN, Geneva, Switzerland, Tech. Rep. CERN-2004-006, 2004. [Online]. Available: http://doc.cern.ch/yellowrep/2004/2004-006/p1.pdf

[4] A. Devred, *et al.*, "Status of the Next European Dipole (NED) Activity of the Collaborated Accelerator Research in Europe (CARE) Project," in *Transactions on Applied Superconductivity*, vol. 15, no. 2, June 2005, pp. 1106–1112. [Online]. Available: http://doc.cern.ch/archive/electronic/cern/preprints/at/at-2005-002.pdf

[5] N. Schwerg, A. Devred, and C. Vollinger, "Estimation of the critical current density for the strand used for ned," Technical Note 2005-07, EDMS 638344, CERN, Geneva, Switzerland, Sept. 2005.

[6] L. Bottura, "A Practical Fit for the Critical Surface of NbTi," in *16th International Conference on Magnet Technology MT16*, Ponte Vedra Beach, USA, 1999. [Online]. Available: http://doc.cern.ch/archive/electronic/cern/preprints/lhc/lhc-project-report-358.pdf

[7] L. T. Summers, *et al.*, "A Model for the Prediction of Nb_3Sn Critical Current as a Function of Field, Temperature, Strain and Radiation Damage," in *IEEE Transactions On Magnetics*, vol. 27, no. 2, Mar. 1991, pp. 2041–2044. [Online]. Available: http://ieeexplore.ieee.org/iel1/20/3657/00133608.pdf?tp=&arnumber=133608&isnumber=3657

[8] F. Toral, *et al.*, "Comparison of 2-d magnetic designs of selected coil configurations for the next european dipole (ned)," IEEE Trans. Mag. - accepted for publication, August 2006.

[9] N. Schwerg, "Electromagnetic Design Study for a Large Bore 15T Superconducting Dipole Magnet," Diploma Thesis, Technische Universität Berlin, Berlin, Germany, Nov. 2005. [Online]. Available: http://cdsweb.cern.ch/search.py?recid=914515&ln=en

[10] D. Leroy and O. Vincent-Viry, "Preliminary Magnetic Designs for Large-Bore and High-Field Dipole Magnets," CERN, Geneva, Switzerland, Tech. Rep., Dec. 2004. [Online]. Available: http://doc.cern.ch/archive/electronic/cern/preprints/at/at-2004-022.pdf

[11] S. Caspi, P. Ferracin, and S. Gourlay, "Graded high field Nb_3Sn dipole magnets," in *IEEE Transactions on Applied Superconductivity*, vol. 16, no. 2. IEEE, June 2006, pp. 354–357.

[12] F. Toral, "Iron contribution of different NED designs," Private Communciation, June 2006.