XX. NETWORK SYNTHESIS

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RESEARCH OBJECTIVES

Of three major problems in progress a year ago, two have been concluded and one is continuing. The development of synthesis methods for prescribed voltage transfer functions (Sc. D. Thesis of P. M. Lewis II) and the development of a generalized analysis procedure for networks containing multiterminal elements instead of the conventional branches with two terminals (Sc. D. Thesis of J. Shekel) have been brought to a conclusion. The former is in process of publication as Technical Report 314. The latter is being prepared for publication. Investigation of the third problem — a study of the possibilities of all-pass networks for transient response correction (Sc. D. Thesis of John Pinson) — is in progress. (An interim summary appears below.)

For the coming year we shall be concerned with various specific investigations collaterally useful in time- and frequency-domain synthesis; and we plan additionally to study the possibilities of an approach — thus far unexplored — to the synthesis of active and/or nonbilateral networks. As pointed out by the writer in an informal paper presented at the A. I. E. E. Round Table Conference on Network Analysis and Synthesis, Jan. 20, 1940 (with the title "An Interpretation of Network Relations When Active Unilateral Elements Are Included With the Usual Passive Ones") it is possible to represent linear active and/or nonbilateral networks as ordinary passive bilateral ones in which either the independent or the dependent variables are subjected to an appropriate real nonsingular transformation. It is felt that the possibilities of this approach have lain dormant too long, and we shall devote our efforts this year (in part, at least) to remedying this situation.

E. A. Guillemin

A. TRANSIENT CORRECTION WITH ALL-PASS NETWORKS

In this study we are investigating the extent to which the transient response of a given network can be changed by adding phase correction. It can happen that a network designed to give a specified, desired amplitude characteristic is found to give an undesirable transient response. This response can be altered by means of phase correction; the correction is accomplished with an all-pass network, which has a transfer characteristic with constant amplitude and variable phase. The work that is progressing aims at determining just how much can be done by means of phase correction to change an undesirable transient into a more desirable one.

If $H(\omega)$ is the transfer characteristic of an all-pass network, then

$$H(\omega) = \epsilon^{-j\theta(\omega)} \tag{1}$$

Of course, considerations of physical realizability restrict the form of $\theta(\omega)$, so that

$$h(t) = F^{-1}[H(\omega)] = 0 \quad \text{for} \quad t < 0 \tag{2}$$

However, it can be shown that if a large enough delay, τ, is allowed, any phase function $\theta(\omega) + \omega\tau$ can be approximated as closely as desired by the phase of an all-pass network. In the problems that are being considered here, the introduction of a delay is not

undesirable. Then no restrictions are placed on the form of $\theta(\omega)$; $H(\omega) = \epsilon^{-j\theta(\omega)}$ need no longer represent a realizable network and Eq. 2 need not be satisfied, because it is known that the transfer function $\epsilon^{-j\theta(\omega)-j\omega\tau}$ can be approximated as closely as desired by a realizable all-pass network.

Let the undesirable transient that is to be corrected be denoted by i(t). If possible, phase correction should change i(t) into r(t), where r(t) is the desired transient. The network which changes i(t) to r(t) is

$$G(\omega) = \frac{R(\omega)}{I(\omega)} \triangleq |G(\omega)| \, \epsilon^{-j\phi(\omega)} \tag{3}$$

where $R(\omega)$ and $I(\omega)$ are the Fourier transforms of r(t) and i(t). Clearly, unless $|G(\omega)| = 1$, a phase correction network, $H(\omega)$, cannot change i(t) to r(t). The best that can be done with phase correction is to change i(t) into $r^*(t)$, where $r^*(t)$, the corrected transient, is an approximation of r(t). Then some sort of error criterion must be imposed in order to define the $r^*(t)$ which best approximates r(t). A convenient criterion is integral square error

$$\overline{\mathscr{E}^2} = \int_{-\infty}^{\infty} \left[r(t) - r^*(t) \right] dt \tag{4}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[R(\omega) - I(\omega) \, \epsilon^{-j\theta(\omega)} \right] \times \left[R(-\omega) - I(-\omega) \, \epsilon^{+j\theta(\omega)} \right] d\omega \tag{5}$$

When the variational problem involved in finding the $\theta(\omega)$ which minimizes $\overline{\mathscr{E}^2}$ is solved, it is found that in order to minimize the integral square difference between the desired transient and the corrected transient, it must be that

$$\theta(\omega) = \phi(\omega) \tag{6}$$

The most frequent practical situation in which phase correction is of interest is being investigated in detail. Because of amplitude and phase distortions, an amplifier cannot reproduce a transient input exactly. A step-function input is usually used to judge the transient performance. Then phase correction can be added to improve this step response. Just what constitutes an improvement in the step response depends on the form of the uncorrected response. If the system is sluggish, the phase correction should give shorter rise time; if the uncorrected response has overshoot, the phase correction should decrease this overshoot.

It is interesting to note the phase that results when the minimum integral square-error criterion is applied to this problem. If $\mathscr{E}^2$ is to be minimized, the phase correction should be such that the phase of the corrected system is linearized. And this

linearization is ordinarily what is attempted in current engineering practice. It is not generally known, however, that this requirement can be derived on the basis of minimizing integral square error.

While the minimum integral square-error criterion can be considered to give a broadly conceived "best over-all result," there is no reason to believe that it leads to the best correction in terms of decreasing rise time or overshoot. In fact, it does not. A more general error criterion needs to be evolved. Consider the weighted integral square error

$$\overline{\mathscr{E}^2_a} = \int_{-\infty}^{\infty} a(t) \left[r(t) - r^*(t) \right]^2 dt \tag{7}$$

Obviously, great flexibility is given by the fact that errors at some times may be weighted more heavily than errors at other times. Extremals of the integral of Eq. 7 are those functions, $\theta(\omega)$, which satisfy the equation

$$\theta(\omega) = \arg \left\{ \int_{-\infty}^{\infty} A(\omega-y) \left[R(y) - I(y) \, \epsilon^{-j\theta(y)} \right] dy \right\} + \pi n(\omega) \tag{8}$$

where $A(\omega)$ is the Fourier transform of $a(t)$, and $n(\omega)$ is a real, antisymmetric function of ω whose value is always an integer except at discontinuities. In general, solutions of Eq. 8 cannot be found. But for the particular case of correcting the step response of a sluggish system to decrease the rise time, Eq. 8 reduces to an equation that can be solved. The corresponding corrected response has the shortest rise time that can be obtained by phase correction of the given system. Solution of Eq. 8 for the phase correction which reduces overshoot to a minimum is not as simple, but a possible method of solution is in sight.

J. C. Pinson

B. GEOMETRIC INTERPRETATION OF GEWERTZ'S STABILITY CONDITION USING THE ISOMETRIC CIRCLES

In terms of the notations of Fig. XX-1, Gewertz's stability condition for a bilateral, two terminal-pair network is

$$\left. \begin{array}{l} \text{Re } Z_{11} = R_{11} \geqslant 0 \\[2mm] \text{Re } Z_{22} = R_{22} \geqslant 0 \\[2mm] \text{Re } Z_{11} \cdot \text{Re } Z_{22} = R_{11} R_{22} \geqslant R_{12}^2 = \left(\text{Re } Z_{12} \right)^2 \end{array} \right\} \tag{1}$$

From Fig. XX-1, we also have

$$Z_{11} = \frac{a}{c}, \quad Z_{12} = \frac{1}{c}, \quad Z_{22} = \frac{d}{c} \tag{2}$$

so that

$$\left. \begin{array}{l} \text{Re}(a/c) \geqslant 0 \\[1.5ex] \text{Re}(d/c) \geqslant 0 \\[1.5ex] \text{Re}(a/c) \cdot \text{Re}(d/c) \geqslant [\text{Re}(1/c)]^2 \end{array} \right\} \tag{3}$$

In the Quarterly Progress Report of April 15, 1956, page 123, a graphical method for transforming impedances through bilateral, two terminal-pair networks was presented. The method utilizes two circles, the isometric circles, with their centers at $O_d = -d/c$ and $O_i = a/c$; both circles having the same radius, $R_c = 1/|c|$.

We get, from Eqs. 3,

$$\left. \begin{array}{l} \text{Re } O_d \leqslant 0 \\[1.5ex] \text{Re } O_i \geqslant 0 \\[1.5ex] |\text{Re } O_d| \ |\text{Re } O_i| \geqslant R_c^2 \end{array} \right\} \tag{4}$$

Thus the isometric circle of the direct transformation, C_d, must have its center in the left half-plane of the complex impedance plane, while the isometric circle of the inverse transformation, C_i, has its center in the right half-plane. The radii R_c of the circles must have a value given by the third of the conditions of Eq. 4.

It can easily be shown that the conditions of Eq. 4 are obtained by specifying that, after performing the different steps of the isometric circle method, all points in the right half-plane or on the imaginary axis remain in the right half-plane or on the
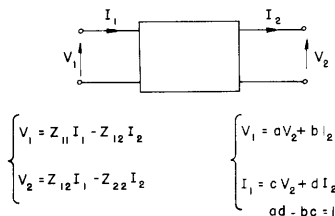

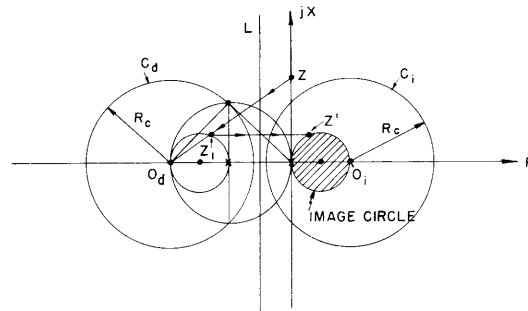
Fig. XX-1. Two terminal-pair network.

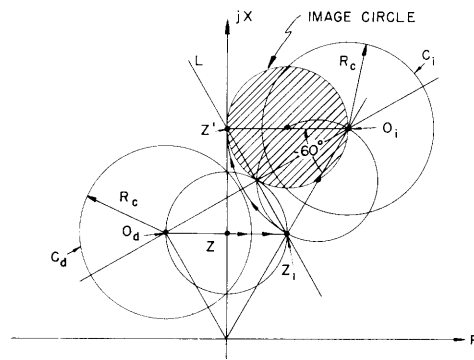Fig. XX-2. Limiting, nonloxodromic example, $|O_d| \cdot |O_i| = R_c^2$, $O_d < 0$, $O_i > 0$.

Fig. XX-3. Limiting, loxodromic example,

$$\left| \mathrm{Re}\ O_d \right| \cdot \left| \mathrm{Re}\ O_i \right| = R_c^2, \quad \mathrm{Re}\ O_d < 0, \quad \mathrm{Re}\ O_i > 0.$$

imaginary axis. Two illustrative, limiting examples are shown in Fig. XX-2 and Fig. XX-3. Figure XX-2 shows a nonloxodromic (hyperbolic) transformation $(a = \sqrt{2}, \ b = 0, \ c = 1, \ d = 1/\sqrt{2})$. Figure XX-3 shows a loxodromic transformation $(a = 2\sqrt{2}\ \exp(j60°), \ b = 3, \ c = 1, \ d = \sqrt{2}\ \exp(-j60°), \ -2\ \arg\ (a+d) = -60°)$. In both examples a point on the imaginary axis is transformed into another point on the same axis.

E. F. Bolinder