XIV. NOISE IN ELECTRON DEVICES*

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RESEARCH OBJECTIVES

Under this heading we shall combine the noise work which heretofore has been reported under the titles "Microwave Electronics" and "Semiconductor Noise." The major reasons for the change stem from the unifying directions in which the research has led us. The unifying concepts that we have thus far encountered originate from the general terminal description of noisy linear networks, and the discovery of significant invariant and minimal expressions combining noise and gain in such systems. The properties of the eigenvalues of the characteristic noise matrix $N = -\frac{1}{2} EE^{\dagger} (Z + Z^{\dagger})^{-1}$, and their significance as regards available power, noise performance (noise measure), and the canonic form of noisy linear networks (see Section XIV-B) suggest a connection with irreversible thermodynamics. (See Quarterly Progress Reports, Jan. 15, 1956, p. 124; April 15, 1956, p. 90; July 15, 1956, p.61.) However, the systems normally described by active linear noisy network models may actually be far from thermal equilibrium, and the present state of irreversible thermodynamics presumably does not include such cases. Still, the similarity of the matrix formalism is inescapable, and in the future we hope to establish the fact that our work is either a part of, or a logical extension of, the area of nonequilibrium thermodynamics. Collaterally, in view of the apparently nonequilibrium nature of the "gain" process, it is possible that intimate relations between gain and noise will be forthcoming. This possibility is also suggested by the noise measure concept itself, and by its specific interpretation in the microwave tube case. The molecular amplifier class of devices, and possibly the transistor under low bias conditions, may be particularly profitable examples which will later help to clarify the foregoing ideas.

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A. AN EXTENSION OF THE NOISE FIGURE DEFINITION

In the course of extending our previous general studies (1, 2) of noise performance of linear amplifiers, we were led to generalize the definitions of "available gain" G and "noise figure" F beyond their original meanings (3). The need arises from situations involving negative resistance, and stems from difficulties in such cases with the usual notion of available power. An extension of this concept is required.

Normally, the available power $P_{av|_s}$ of a source is defined as

$P_{av|_s} \equiv$ the greatest power which can be drawn from the source by arbitrary variation of its terminal current (or voltage). (1)

If the Thévenin representation of the source has rms open-circuit voltage E_s and

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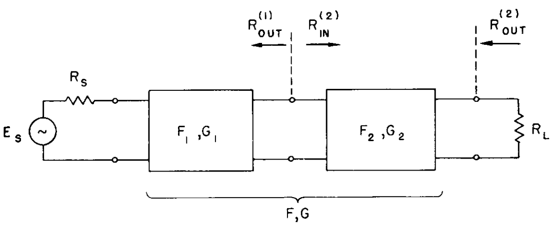

Fig. XIV-1. Two amplifiers in cascade. $R_s, R_L > 0$.

internal impedance Z_s, with $R_s = \mathrm{Re}(Z_s) > 0$, definition 1 leads to

$$P_{av}\big|_s = \frac{|E_s|^2}{4R_s} > 0 \text{ for } R_s > 0 \tag{2}$$

which is also a stationary value (extremum) of the power output regarded as a function of the complex terminal current. Moreover, the available power (Eq. 2) can actually be delivered to the (passive) load Z_s^*.

When $R_s < 0$, however, definition 1 leads to

$$P_{av}\big|_s = \infty \text{ for } R_s < 0 \tag{3}$$

since this is indeed the greatest power obtainable from such a source, and is achievable by loading it with the (passive) impedance, $-Z_s$. Observe that result 3 is neither a stationary value nor extremum of the power output as a function of terminal current.

The singular value and failure of the extremal property in Eq. 3 make definition 1 unsatisfactory in the negative resistance case. The following problem of noise figure for a cascade of two amplifiers (Fig. XIV-1) focuses attention clearly upon some of the details that support this statement.

In the neighborhood Δf of some frequency f, the first stage in Fig. XIV-1 has a noise figure F_1 and an available gain G_1. The second stage has a noise figure F_2 and an available gain G_2. The noise figure of the cascade is, then, presumably given by the well-known cascading formula

$$F = F_1 + \frac{F_2 - 1}{G_1} \tag{4}$$

Suppose, however, that the first amplifier has a negative output resistance $R_{out}^{(1)}$, whereas the second amplifier has positive input and output resistances $R_{in}^{(2)}$ and $R_{out}^{(2)}$. The closer the value of $R_{in}^{(2)}$ to $|R_{out}^{(1)}|$, the higher the transducer gain of the first stage. (The transducer gain is defined as the ratio of the actual power delivered at the output to the available power from the source.) Indeed, the condition $R_{in}^{(2)} = |R_{out}^{(1)}|$ would lead to infinite gain and instability, and, therefore, we assume $R_{in}^{(2)} \neq |R_{out}^{(1)}|$ in Fig. XIV-1. Thus the available gain G of the over-all amplifier is perfectly well defined in terms of the usual definition 1 of available power.

$$G = \frac{P_{av}|_{out}}{P_{av}|_{s}} \tag{5}$$

Similarly the noise figure F of the over-all amplifier is equally well defined by the general relation

$$F = 1 + \frac{N_{av}|_{out}}{GkT\Delta f} \tag{6}$$

where $N_{av}|_{out}$ is the noise power available at the output terminals in the frequency band Δf, when no noise is introduced by the source. It becomes clear, by visualizing the noise voltages (not shown in Fig. XIV-1) which characterize the noise of each amplifier, and by noting the condition $R_{in}^{(2)} \neq |R_{out}^{(1)}|$, that $N_{av}|_{out}$ is finite. So also is $GkT\Delta f$, which is the power available at the output caused only by the noise power $kT\Delta f$ available from the source resistance at temperature T.

Now, if we try to apply the cascading formula 4, we find that we are in trouble. Indeed, the available gain G_1 of the first stage is infinite by definitions 5, 3, and 2; and thus, according to a cursory inspection of formula 4, the second stage does not seem to contribute to the over-all noise figure. That this conclusion is incorrect physically follows from a direct consideration of the contributions of the noise generators of the individual amplifiers, in the manner which would be used to obtain the over-all system noise figure (Eq. 6, and so on). A more careful examination of Eq. 4, in this case, reveals the following additional difficulties connected with the fact that $R_{out}^{(1)} < 0$:

(a) Noise figure F_1 is indeterminate when calculated from Eq. 6, because $N_{av}|_{out} = \infty$ according to definitions 1 and 3, and $G_1 = \infty$.

(b) Noise figure $F_2 = \infty$ by Eq. 6. This occurs because $G_2 = 0$, on the basis of definitions 3, 2, and 5. Thus the term $(F_2 - 1)/G_1$ in Eq. 4 is actually indeterminate also, which makes F in Eq. 4 entirely indeterminate.

(c) The use of $kT\Delta f$ in Eq. 6 for computing F_2, in this case, requires some comment, because $R_{out}^{(1)} < 0$ does not represent a resistance at thermal equilibrium temperature T, and the "available thermal noise power" $kT\Delta f$ has no clear physical meaning under the circumstances.

We shall now propose a new concept called the "exchangeable power," of a source, in terms of which an "exchangeable power gain" and a new noise figure can be defined. These new definitions remove all of the foregoing difficulties, and always reduce to the familiar ones whenever the latter apply.

The exchangeable power P_e of a source is defined as

$P_e|_s \equiv$ the stationary value (extremum) of the power output from the source, obtained by arbitrary variation of the terminal current (or voltage).

In terms of the Thévenin representation of the source (E_s, Z_s),

$$P_e|_s = \frac{|E_s|^2}{4R_s} \text{ for } R_s \neq 0 \tag{7}$$

Observe that $P_e|_s$ reduces to the conventional available power when $R_s > 0$. When $R_s < 0$, the exchangeable power is negative. As can be confirmed easily, the exchangeable power is, in this case, the maximum power that can be pushed into the "source," achievable by connecting the (nonpassive) impedance Z_s^* to the source terminals. The negative sign of the exchangeable power then conveniently underscores the fact that here we are speaking about a power extremum corresponding to a flow of power into, rather than from, the source.

The introduction of exchangeable power suggests the definition of a "ratio of exchangeable powers," the exchangeable power gain G_e.

$$G_e = \frac{P_e|_{out}}{P_e|_s} \tag{8}$$

The exchangeable power gain is the ratio of the exchangeable power at the output terminals of a network to the exchangeable power of the source connected to the input. It reduces to the conventional available power gain if both the output resistance and the source resistance are positive. If either one of these resistances is negative, $G_e < 0$. If both source and output resistance are negative, $G_e > 0$.

We can now extend the definition of the noise figure on the basis of exchangeable power. Let

$$F_e = 1 + \frac{N_e|_{out}}{G_e\, kT\Delta f} \tag{9}$$

where $N_e|_{out}$ is the exchangeable noise power at the output terminals with no noise from the source, and G_e is the exchangeable power gain of the system. The magnitude of the exchangeable noise power of the source is arbitrarily set equal to the standard $kT\Delta f$, simply for normalization purposes. It should be noted that $(F_e - 1) < 0$ only when the source resistance is negative: $(F_e - 1) > 0$ in all other cases.

We can now confirm that the cascading formula 4 has been extended to include all cases, provided F and G are reinterpreted as F_e and G_e. We have

$$N_e\big|_{out} = N_e^{(1)}\big|_{out} G_{e2} + N_e^{(2)}\big|_{out} \tag{10}$$

$$G_e = G_{e1}G_{e2} \tag{11}$$

and thus

$$F_e = 1 + \frac{N_e\big|_{out}}{G_e\, kT\Delta f} = 1 + \frac{N_e^{(1)}\big|_{out} G_{e2} + N_e^{(2)}\big|_{out}}{G_{e1}\, G_{e2}\, kT\Delta f} \tag{12}$$

or

$$F_e = F_{e1} + \frac{F_{e2} - 1}{G_{e1}}$$

The result (Eq. 12) differs from Eq. 4 only when the negative output resistance occurs somewhere in the cascade. Aside from the use of the generalization in such situations, we find it necessary for a careful treatment of the general noise theory of linear amplifiers.

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2. H. A. Haus and R. B. Adler, Limitations on noise performance of linear amplifiers, paper presented at the Congrès International "Tubes Hyperfréquences," Paris, June 1956 (to be published).
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B. CANONICAL FORM OF A NOISY LINEAR n TERMINAL-PAIR NETWORK

In previous work (1, 2) a characteristic noise matrix N was associated at a particular frequency with a noisy linear network. If the network has the impedance matrix Z and the column matrix of open-circuit noise voltages E, the characteristic noise matrix has the form

$$N = \frac{1}{2}\overline{EE^{\dagger}}\,(Z + Z^{\dagger})^{-1} \tag{1}$$

where the dagger denotes conjugate transpose.

Consider a lossless imbedding (Fig. XIV-2) of the n terminal-pair network in a

2n terminal-pair, lossless network, so that a new n terminal-pair network with the column matrix E' and the impedance matrix Z' is formed. The transformations of $\overline{EE^{\dagger}}$ and $(Z + Z^{\dagger})$ obey the law

$$\overline{E'E'^{\dagger}} = Q\,\overline{EE^{\dagger}}\,Q^{\dagger} \tag{2}$$

$$(Z' + Z'^{\dagger}) = Q(Z + Z^{\dagger})Q^{\dagger} \tag{3}$$

where Q is a matrix of the n^{th} order. Q can, in fact, be chosen quite arbitrarily by making a proper choice of the lossless imbedding network (1, 2).

One consequence of transformations 2 and 3 is that the eigenvalues of N stay invariant under lossless imbedding (1, 2), because

$$N' = \frac{1}{2}\,\overline{E'E'^{\dagger}}\left(Z' + Z'^{\dagger}\right)^{-1} = \frac{1}{2}\,Q\,\overline{EE^{\dagger}}\left(Z + Z^{\dagger}\right)^{-1} Q^{-1} \tag{4}$$

represents a similarity transformation of N. Another consequence of transformations 2 and 3 will now be considered.

Choose Q, and accordingly the imbedding network, so that both $\overline{EE^{\dagger}}$ and $Z + Z^{\dagger}$ are diagonalized simultaneously. Since Q can be chosen arbitrarily, the simultaneous diagonalization is possible by virtue of the positive definite character of $\overline{EE^{\dagger}}$ and the Hermitian character of $Z + Z^{\dagger}$. One can, in addition, transform $\overline{EE^{\dagger}}$ into the identity matrix I.

Once such a diagonalization is accomplished, we have

$$\overline{E'E'^{\dagger}} = I$$

and

$$Z' = \text{diag}\,(R_1, R_2, \ldots, R_n) + Z_o$$

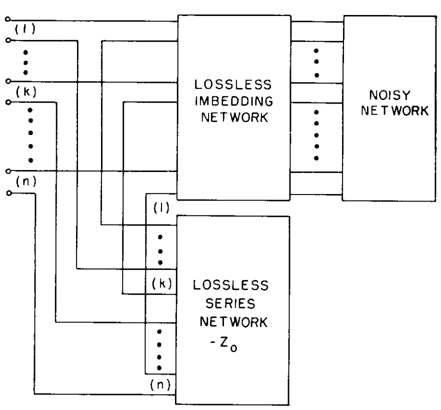

Fig. XIV-2. Lossless imbedding of a noisy network.

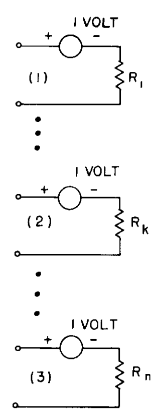

Fig. XIV-3. Canonical form of a noisy network. Voltage sources are uncorrelated and given in terms of rms values.

where Z_o fulfills the condition

$$Z_o + Z_o^{\dagger} = 0$$

This means that Z_o is the matrix of a lossless network.

A series connection of the network E', Z' with the lossless n terminal-pair network Z_o (Fig. XIV-2) leads to the network E'', Z'', where

$$\overline{E''E''^{\dagger}} = \overline{E'E'^{\dagger}} = I$$

and

$$Z'' = \text{diag}\,(R_1, R_2, \ldots, R_n)$$

Thus, the two operations shown in Fig. XIV-2 reduce every noisy n terminal-pair network to the canonical form indicated in Fig. XIV-3. The resistances can be either positive or negative, depending upon the original network. The noise generators are all of unit rms voltage and uncorrelated. The resistances R_1 to R_n can be found directly from the eigenvalues of the N matrix. Indeed, the transformed N' matrix has the eigenvalues $1/4R_1, 1/4R_2, \ldots, 1/4R_n$, which are also the eigenvalues of N.

The preceding decomposition of a noisy network into its canonical form also proves a converse theorem.

Theorem. Any linear noisy n terminal-pair network can be represented by a canonical network imbedded in an appropriate lossless network.

Indeed, we obtain the original network of Fig. XIV-2 by an imbedding of its canonical form (Fig. XIV-3) in the inverse of the lossless network employed in Fig. XIV-2. The foregoing development gives a new network interpretation of the eigenvalues of N.

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References

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