VIII. PROCESSING AND TRANSMISSION OF INFORMATION*

Prof. P. Elias
Prof. R. M. Fano
Prof. D. A. Huffman
Prof. C. E. Shannon
Dr. M. V. Cerrillo
W. G. Breckenridge
E. Ferretti
K. Joannou
R. M. Lerner
J. B. O'Loughlin
L. S. Onyshkevych
A. J. Osborne
E. T. Schoen
F. F. Tung
S. H. Unger
M. L. Wetherell
J. M. Wozencraft
W. A. Youngblood

A. CHANNEL CAPACITY WITHOUT CODING

1. Introduction

Kelly (1) recently gave an illustration of a gambling situation in which channel capacity has a meaning independent of any requirement to code the signals that are to be transmitted over the channel. Shannon (2) gave a "cheating channel" example of a procedure which, given a noisy forward channel of capacity C_1, a noiseless feedback channel, and a noiseless forward channel of capacity C_2, would permit transmission at a rate $C_1 + C_2$ over the combination of the two forward channels with a simple, constructive coding procedure that would be easy to implement, as opposed to the complex codebook coding procedures that are required to transmit over a single noisy channel with low error probability, in the absence of feedback. (It has been shown (3) that for a channel without memory, the presence of the noiseless feedback channel does not affect the capacity of the forward channel.)

For channels with additive white gaussian noise, it is possible to strengthen Shannon's example. Two such forward channels, both noisy, of common bandwidth W and capacities C_1 and C_2, can be used to obtain in analog fashion, without substantial coding or delay, one channel of capacity $C_1 + C_2$, by making use of a noiseless feedback channel. This leads to the interesting result that, if a noiseless feedback channel is available, a single forward channel of large bandwidth, perturbed by additive white gaussian noise of power N_o watts/cycle, can transmit an analog signal of bandwidth W with transmitter power S which will be received with an effective signal-noise power ratio σ_o^2 that is given by

$$\sigma_o^2 = e^{\sigma_i^2} - 1$$

where

$$\sigma_i^2 = S/N_oW$$

is the signal-noise power ratio obtainable by simple transmission without coding or

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feedback, and

$$\sigma_o^2 = S/N_o'W$$

is the signal-noise power ratio of the received signal after a detection operation, still in bandwidth W and identical to the transmitted signal except for the added noise of spectral density N_o' watts/cycle and a delay of about 1/2W sec. Thus, a 10-db signal-noise ratio in band W is equivalent to a signal-noise ratio of $e^{10}-1 = 22,000$, or approximately 43.5 db, if the available forward channel is wideband and a noiseless feedback channel is available.

This behavior is not quite as exceptional as it sounds: pulse-code modulation achieves the same character of behavior without the noiseless feedback channel, but an extra 8 db or so is required (4), which would make it useless in a channel with an initial signal-noise ratio as low as 10 db.

2. Parallel Gaussian Channels

Consider two channels of unit bandwidth, with additive gaussian noise, and signal-noise power ratios σ_1^2 and σ_2^2, respectively. Their capacities will be

$$\begin{aligned} C_1 &= \log\left(1 + \sigma_1^2\right) \\ C_2 &= \log\left(1 + \sigma_2^2\right) \end{aligned} \qquad (1)$$

in which natural logarithms are used, and information is in natural units (nats). If the same signal is applied to both channels, it is known (5, 6) that the optimum way to combine the two signals at the receiver is to take a sum weighted by the square root of the signal-noise power ratio of each channel, and that this gives a received signal-noise power ratio of $\sigma_o^2 = \sigma_1^2 + \sigma_2^2$, and thus an effective total capacity for the two channels which are used in this way, of

$$C_o = \log\left(1 + \sigma_1^2 + \sigma_2^2\right) \qquad (2)$$

However, the sum of the capacities of the two channels, which is the total capacity available for forward transmission, is

$$\begin{aligned} C = C_1 + C_2 &= \log\left(1 + \sigma_1^2\right) + \log\left(1 + \sigma_2^2\right) = \log\left(1 + \sigma_1^2\right)\left(1 + \sigma_2^2\right) \\ &= \log\left(1 + \sigma_1^2 + \sigma_2^2 + \sigma_1^2\,\sigma_2^2\right) \end{aligned} \qquad (3)$$

Now, Eq. 3 implies that the attainable receiver signal-noise power ratio should be

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_1^2 \sigma_2^2 \tag{4}$$

which is not much greater than σ_o^2 if $\sigma_1^2 < 1$, $\sigma^2 < 1$. But if both channels have good signal-noise ratios, then σ_o^2 is approximately equal to $2\sigma^2$, and σ_1^2 and σ_2^2 should then multiply, not add.

To obtain a received signal-noise ratio σ^2, we send the noisy output of the first channel back to the transmitter over the noiseless feedback path, and send a signal over the second channel which is a linear combination of the original signal and the first-channel noise. This is added (with appropriate weighting) to the first received signal. The net result, using optimum linear combinations at both ends, is a received signal at the signal-noise ratio given by Eq. 4, which gives the channel capacity $C = C_1 + C_2$. The process is illustrated in Fig. VIII-1. The fact that this is a maximum in received signal-noise power ratio is a consequence of the channel capacity theorem: it can be proved, with patience, by substituting undetermined coefficients at the two adders and varying in order to maximize the received signal-noise ratio.

The statements in Section A.1 now follow, if a channel of wide bandwidth is turned into many channels, each of bandwidth W, by time- or frequency-multiplexing. The capacity of the sum channel, iterating the procedure given above, is the sum of the channel capacities, the signal power available being divided among the channels. In the limit, this gives

$$C = \lim_{K \to \infty} KW \log\left(1 + S/KN_oW\right) = S/N_o \qquad \text{nats/sample}$$

Then, σ_o^2, the effective received signal-noise power ratio, is given by

$$C = W \log\left(1 + \sigma_o^2\right) = S/N_o$$

$$\log\left(1 + \sigma_o^2\right) = S/N_oW = \sigma_i^2$$

$$\sigma_o^2 = e^{\sigma_i^2} - 1$$

If noise is added to the feedback channel, the optimum behavior attainable by the addition of two forward channels, making use of feedback, gives a forward capacity that is less than $C_1 + C_2$, and a σ_o^2 that is less than $\sigma_1^2 + \sigma_2^2 + \sigma_1^2\sigma_2^2$. In fact, if σ_3^2 is the signal-noise power ratio in the feedback channel,

$$\sigma_o^2 = \sigma_1^2 + \sigma_2^2 + \frac{\sigma_1^2 \sigma_2^2 \sigma_3^2}{\left(1 + \sigma_1^2\right)\left(1 + \sigma_2^2\right) + \sigma_3^2} \tag{5}$$

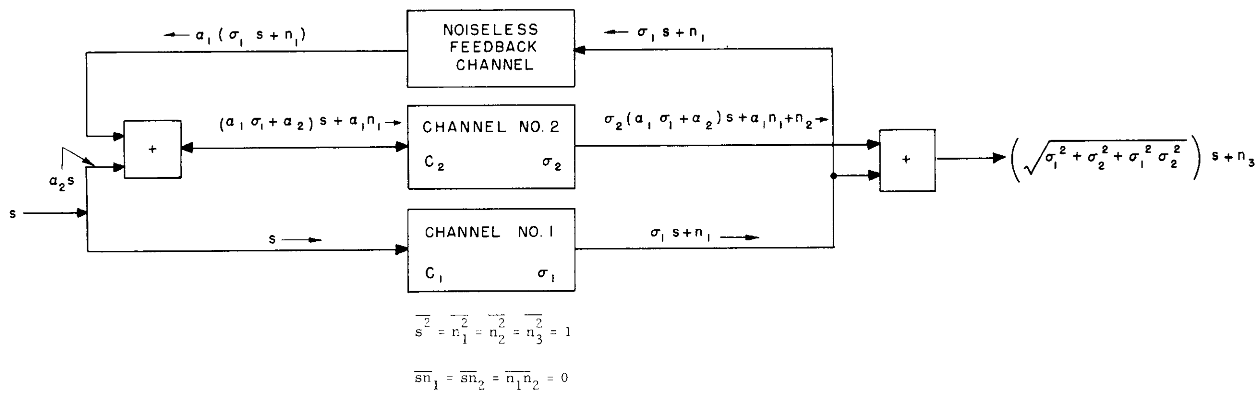

Fig. VIII-1. Combination of two noisy forward channels using noiseless feedback channel.

which approaches the nonfeedback result when σ_3^2 is small, approaches the noiseless case when σ_3^2 is large, and shows that, for any value of σ_3^2, it would always be more advantageous to use the feedback channel as an additional forward channel, if this were possible, and the signal-noise power ratio in it could be kept unchanged. That is, for all σ_1^2, σ_2^2, $\sigma_3^2 > 0$, we have

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 > \sigma_1^2 + \sigma_2^2 + \frac{\sigma_1^2 \sigma_2^2 \sigma_3^2}{\left(1 + \sigma_1^2\right)\left(1 + \sigma_2^2\right) + \sigma_3^2}$$

P. Elias

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B. AMOUNT OF INFORMATION PROVIDED BY A PARTICULAR SYMBOL

Consider a random process that generates sequences of symbols..., u(j-N), u(j-N+1), ..., u(j), ... The symbol u(j), which occurs at time j, is a random variable, taking its values from a finite set or alphabet, X(j), of possible symbols:

$X(j) = \{x_1(j), x_2(j), \ldots, x_{M_j}(j)\}$, where M_j is the number of symbols in the alphabet available at time j, and may vary with j.

The process is determined in a statistical sense if the probabilities of sequences of length N + 1

$$\text{Prob}\{u(j), u(j-1), \ldots, u(j-N)\} \tag{1}$$

are known for all N, all j, and all sets of values of $u(j) \in X(j), \ldots, u(j-N) \in X(j-N)$. If X(j) and all of the probabilities given above are independent of j, the process is called stationary.

Extending the usual definitions to the nonstationary case, we define three informational quantities.

1. The (zeroth order) self-information of the symbol $x_k(j)$ is defined as

$$I[x_k(j)] = -\log \text{Prob}\{u(j) = x_k(j)\} \tag{2}$$

Then $I[x_k(j)]$ is a nonnegative number which is interpreted as the amount of information provided by the event $u(j) = x_k(j)$, when the values of u at other times are not known.

2. The Nth-order conditional self-information of the symbol $x_k(j)$ is defined as

$$I[x_k(j) | u(j-1), \ldots, u(j-N)] = -\log \text{Prob}\{x_k(j) | u(j-1), \ldots, u(j-N)\} \tag{3}$$

This is a nonnegative-valued function of the values of the N preceding symbols $u(j-1), \ldots, u(j-N)$. The function may take on as many as

$$\prod_{k=1}^{N} M_{j-k}$$

different values, since there are that many conceivable past histories, although some of them may have zero probability.

Unfortunately, the limit of the function of Eq. 3 need not exist, even for a finite-state stationary source. In order to obtain a limit, it is usual to introduce an average of this function.

3. The Nth-order average conditional information of the process is defined as

$$F_N[X(j)] = -\sum_{u(j)\in X(j)} \cdots \sum_{u(j-N)\in X(j-N)} \text{Prob}\{u(j), \ldots, u(j-N)\}\ I[u(j) | u(j-1), \ldots, u(j-N)] \tag{4}$$

Then for each j, $F_N[X(j)]$ is positive and is monotone nonincreasing in N. This property can be shown by taking the difference $F_N[X(j)] - F_{N+1}[X(j)]$, combining both terms under a common summation, and using the inequality $\log w \leq (w-1) \log e$ to show that the difference is greater than or equal to zero. The function $F_N[X(j)]$ therefore approaches a limit

$$R(j) = F[X(j)] = \lim_{N \to \infty} F_N[X(j)] \geq 0 \tag{5}$$

which may be interpreted as the (ensemble average) rate at which the process is generating information at time j.

The purpose of this note is to point out that it is not necessary to do quite so much averaging as is done in Eq. 4 in order to obtain a function that is positive and monotone nonincreasing in N. Specifically,

4. The Nth-order average conditional information of the symbol $x_k(j)$ is defined as

$$F_N[x_k(j)] = -\sum_{u(j-1)\epsilon X(j-1)} \cdots \sum_{u(j-N)\epsilon X(j-N)} \text{Prob}\left\{u(j-1),\ldots,u(j-N) \mid x_k(j)\right\} I[x_k(j) \mid u(j-1),\ldots,u(j-N)] \tag{6}$$

This may be interpreted as the additional information provided by the symbol $x_k(j)$, on the average, when the values of the N preceding symbols are known. For each j, this function is positive and monotone nonincreasing, like $F_N[X(j)]$, by exactly the same argument, and therefore approaches the limit

$$F[x_k(j)] = \lim_{N\to\infty} F_N[x_k(j)] \geq 0$$

which is the average information provided by $x_k(j)$ when all of the past history is known. We have

$$F_N[X(j)] = \sum_{x_k(j) \,\epsilon\, X(j)} \text{Prob}\left\{u(j) = x_k(j)\right\} F_N[x_k(j)] \tag{7}$$

which remains true when we omit the subscript N from each side of the equation.

P. Elias

C. HAZARDS IN SEQUENTIAL SWITCHING CIRCUITS

Stray delays, and possibly elements with intentional delay, exist in every switching circuit. A proper circuit is one that operates correctly despite limited variations in the magnitudes of these delays. A proper circuit that requires no delay elements is defined as being hazard-free. Other pertinent definitions will be found in reference 1.

Previously (1), it was pointed out that hazard-free realizations of certain sequential switching functions do not exist, and a method was described for identifying these situations. Methods using a minimal number of delay elements for designing proper circuits for such functions have since been developed. Various techniques were found for choosing row assignments for an arbitrary flow table in order to make proper operation possible even when delays are associated with a minimal number of the state variables. For certain classes of flow tables (corresponding to binary counters with n-stages, for example) solutions that required only the irreducible minimum of one delay element (regardless of the value of n) were found.

The methods referred to above have the following characteristics in common:

1. All delay elements are associated with state variables.

2. The necessary number of delays depends on the number and location of the hazards in the flow table and on whether the available delay elements are pure (dead-time) or inertial. (The latter are more powerful in the sense that in many situations pure delays cannot be substituted for inertial delays, although inertial delays can always replace pure delays without ill effect.)

Procedures of the type originally investigated will often yield satisfactory results, but they lack generality and, as will be shown presently, do not always lead to the most efficient solutions.

After abandoning the restriction given above it was discovered that, by suitably associating a pure delay with each input variable, any flow table can be properly realized, regardless of the number of hazards in the table. It is necessary when using this method to employ a "one-step" row assignment; that is, only one state variable is changed in any transition between rows of the table. Such an assignment can always be made (2).

Further work along these lines led to the somewhat surprising result that one pure delay is sufficient to combat all possible hazards in any flow table. Once this discovery was made, several different design methods and standard circuit forms were devised for achieving such "single-delay" circuits. The one that will be described here requires the use of one-step row assignments, but other methods have been found that permit designs based on the $2S_0 + 1$ or $2S_0$ assignments (2). Apparently, two delays are required in conjunction with the $2S_0 - 1$ assignment.

Figure VIII-2 shows one of the single-delay circuits mentioned above. The Y and Z circuits are ordinary, hazard-free (3), excitation and output circuits that correspond to a one-step row assignment for the desired function. The D-network is the special hazard-proofing feature. It has the following properties:

1. In the steady state, $y_i = Y_i$ ($i = 1, 2, \ldots, n$, where n is the number of state variables).

2. If only one Y_i is changed while the D-circuit is in a quiescent condition, then, after a delay T_D, y_i will change to agree with Y_i again. For correct operation, no

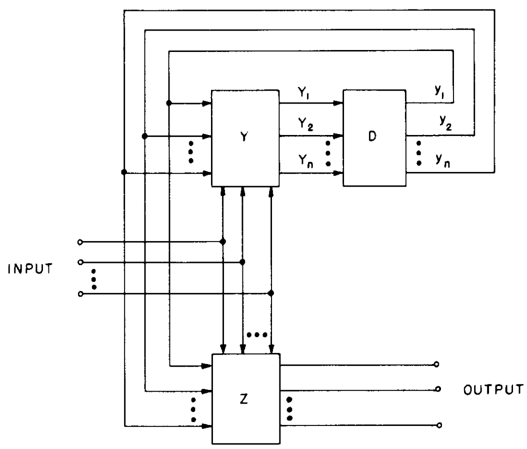

Fig. VIII-2. Block diagram of proper sequential switching circuit.

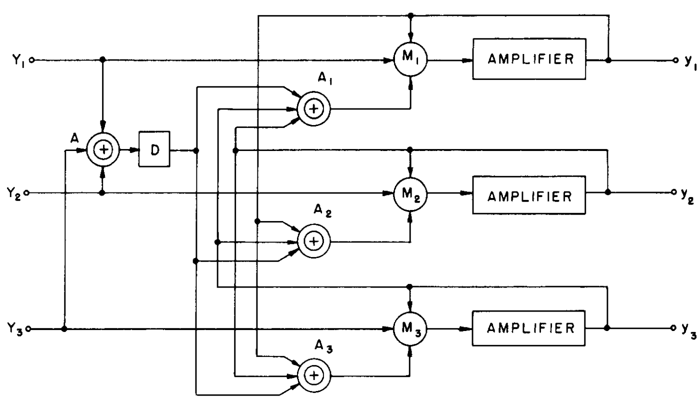

Fig. VIII-3. D-network, using one delay element to do the work of three.

further Y changes should occur until the circuit has again settled down.

3. Only one delay element is used in the D-network.

4. There is an amplifier between each Y_i and y_i. (The necessity for having these amplifiers will be discussed later.)

From the preceding statements, it follows that if only one Y at a time is changed, and if these changes are properly spaced in time, then, the D-network will behave precisely as if each Y_i were directly connected to y_i through a delay of value T_D and an amplifier. But, as was stated in reference 1, a sequential circuit whose Y's and y's are so related will always be proper. It thus remains to show how such a D-circuit can be constructed.

The proposed D-circuit is shown in Fig. VIII-3. The elements denoted by circled plus signs are modulo two adders (the output is the modulo two sum of the inputs), the

M's are majority elements (the output is one if and only if at least two of the inputs are one), and the D is a pure or inertial delay element. Such a circuit can be constructed for any number of input-output pairs, although only three are shown in the diagram.

The circuit operates as follows. Assume that $y_i = Y_i$ for all i's and that the Y's have all been constant for at least T_D seconds. Then two inputs to each M_i are equal to Y_i, so that the output of each $M_i = Y_i$ and hence the y_i values are all stable. The output of each of the adders A_i is

$$\sum_{\text{all } j\neq i} y_j \oplus D \sum_{j=1}^{n} Y_j$$

(All sums are modulo two; DX refers to the value of X delayed by T_D seconds.) But since all Y's have been fixed for a time at least equal to T_D, the last term of the sum can be replaced by

$$\sum_{j=1}^{n} Y_j$$

Since $Y_i = y_i$ (by hypothesis),

$$\sum_{j=1}^{n} Y_j = \sum_{j=1}^{n} y_j$$

The output of adder A_i is thus

$$\sum_{\text{all } j\neq i} y_j \oplus \sum_{j=1}^{n} y_j = y_i$$

Hence all three inputs of each majority element agree in this state. Note that y_i can be changed only if two inputs to M_i change. Since the input from y_i to M_i cannot change spontaneously, it follows that both the Y_i input and the output of D must change before y_i will switch. Thus no y_i can be switched if the corresponding Y_i is held constant.

Now suppose that Y_k is changed (all other Y's being held constant). We see at once that the only y that can possibly respond to this action is y_k. One input to M_k immediately changes and the input to the delay D also changes. Nothing further happens until T_D seconds have elapsed. At this time, the output of D changes, causing one input to each A_i to switch. This changes the outputs of all the A's. The effect of the change in the output of A_k is to switch a second input to M_k (the first to be altered was the one from Y_k) and this is enough to switch y_k. The effect of the latter occurrence is to change the third input to M_k, bringing it to the same value as the other two, and to switch a second input of each of the A_i's other than A_k. (Note that y_k is not connected to A_k.) But if two inputs to a modulo two adder are altered, the net result is no change at the output. Thus all inputs to the M's other than M_k are the same at the end of the process as at the beginning. Only y_k changed. As soon as the effects of the switching of y_k propagate to the terminals of all the M's, the D-network is ready to respond correctly to another Y change. Hence, if the D-network of Fig. VIII-3 is inserted in the circuit of Fig. VIII-2, the result will be a proper circuit with only one delay element.

It is interesting to note that if only Y_k is allowed to vary, then the portion of the D-network pertaining to Y_k and y_k behaves like the smoother described in reference 4. A consequence of this behavior (which was not predicted in the original design) is that the outputs of the D-network will not respond to those changes in the Y-inputs which persist for a time less than T_D. It follows, then, that the Y-network of Fig. VIII-2 need not be hazard-free and also that several input changes may be permitted to occur simultaneously without the risk of having the system get into the wrong state. (But transient output hazards will then exist.)

The principal results of the research on hazards in sequential switching circuits can be briefly summarized: All sequential switching functions fall into one of two classes. Proper circuit realizations of functions in one class can be constructed without using any delay elements, and members of the second class can always be synthesized in the form of proper circuits that contain only one delay element (but not less).

S. H. Unger

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D. PITCH-SYNCHRONOUS CHOPPING OF SPEECH

An investigation of the pitch-synchronous chopping of speech has been completed, and a doctoral thesis on the subject is in preparation for submission to the Department of Electrical Engineering, M.I.T. The work and preliminary results can be briefly summarized as follows.

An apparatus was constructed which (a) determines the pitch period of a voiced sound,

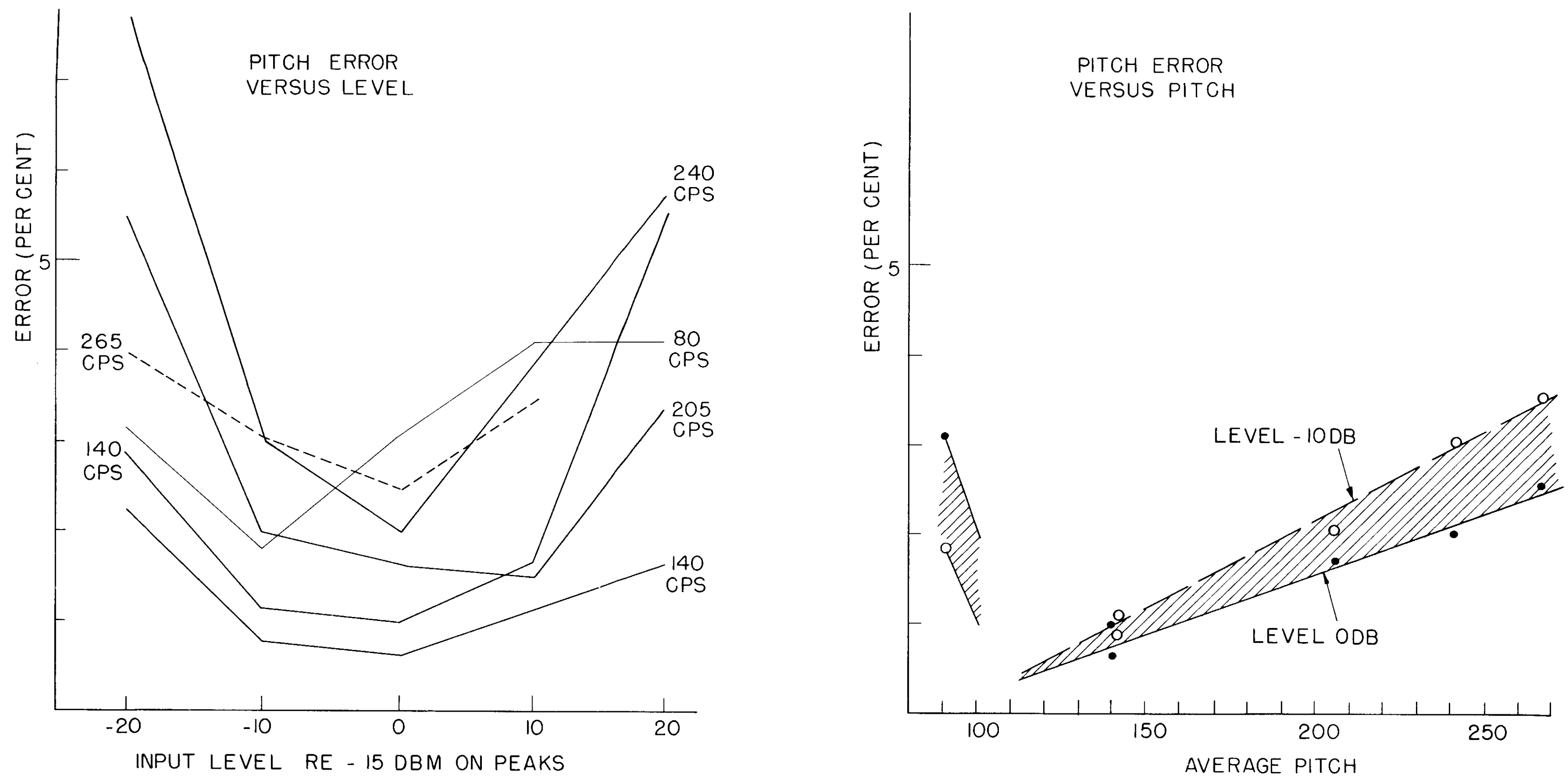

Fig. VIII-4. Pitch-finder performance.

(b) selects and stores a single cycle of a voiced sound, (c) repeats this stored sample a predetermined number of times, and (d) on unvoiced sounds, selects an arbitrary sample for storage and repetition. By alternating between steps (b/d) and (c) an output is obtained, from an ordinary speech input, which simulates the output of a speech-compression system based on transmitting one out of N cycles of the input speech, in which N is predetermined.

The device for determining the pitch period is based on the analytic representation of a signal (1)

$$\overrightarrow{f(t)} = x(t) + j\, y(t)$$

in which x(t) is the observed signal, and y(t) is the Hilbert transform of x(t) – that is, x(t) with all Fourier components shifted 90° in phase without change in amplitude. If x(t) and y(t) drive the horizontal and vertical deflection of an oscilloscope, the resulting pattern is the locus of the tip of the vector $\overrightarrow{f(t)}$. If x(t) is a damped sine wave, this locus is a spiral. Since voiced speech is the sum of several damped oscillations (the formant resonances) repeatedly shock-excited by impulses from the vocal cords, an oscilloscope display of the speech f(t) should be roughly spiral in form. The result is the spiral speech patterns reported previously (2). A display unit was used at the Boston University Speech Clinic in an attempt to determine its usefulness in teaching correct speech to persons with nonpsychological speech impediments. The results so far have been negative (3).

The magnitude of $\overrightarrow{f(t)}$, $|f(t)|$, is the envelope of the waveform x(t).

$$|f(t)| = [x^2(t) + y^2(t)]^{1/2}$$

In a single damped oscillation that is repeatedly shock-excited, the magnitude has an abrupt increase at the time of shock, and decays exponentially to the time of the next shock. The superposition of several damped oscillations modifies the smoothness of the envelope decay, but the abrupt increases are preserved; f(t) for speech has these characteristics, which are the basis of the procedure by which the shock times (the pitch rate) are determined. Since 1951 a variety of methods for obtaining and processing f(t) have been tried with roughly equal steady-state success; the superiority of the present equipment lies in its dynamic behavior on ordinary speech as opposed to essentially sung tones. This performance is indicated by the preliminary data in the curves of

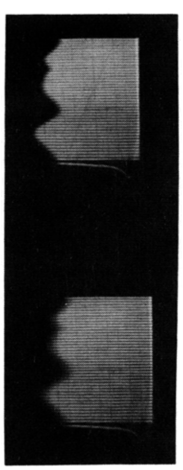

Fig. VIII-5. Memory patterns.

Fig. VIII-4. In the present equipment the speech is filtered to emphasize the first formant before being processed, but this filtering is not critical. The output is in the form of a pulse at the beginning of each pitch period. Double-beam oscillograph film recordings were made of speech input and pitch output. An error was charged when, according to the eye, a pulse was omitted, an extra one inserted, or a pulse was unevenly spaced between its neighbors. The breakdown at low frequencies is attributable in part to incidental circuits; a further evaluation with these omitted pulses is under way.

The storage system that is used is a Williams storage tube modified for analog use. The speech is sampled at an 8.1-kc rate, and each sample is stored as the length of a line on the storage tube face. The resulting pattern is shown in the upper part of Fig. VIII-5. As this pattern is read out, it is regenerated with a fixed displacement to the right, as is evidenced by the shading in the picture, which contains 10 recyclings. This displacement gives rise to a keying voltage in the output, which, if it is uniform across the tube face, can be balanced out.

For objective evaluation of the over-all system performance, intelligibility tests were conducted in which standard PB monosyllabic lists were used. Preliminary results are given in Fig. VIII-6, in which the "reduction ratio" is the total number of times each sample cycle is repeated. These results are similar to those obtained by Kodman (4) for nonsynchronous chopping, when allowance is made for the differences in size of test vocabulary and type of listener training.

Subjectively, synchronously chopped speech sounds more natural than speech chopped

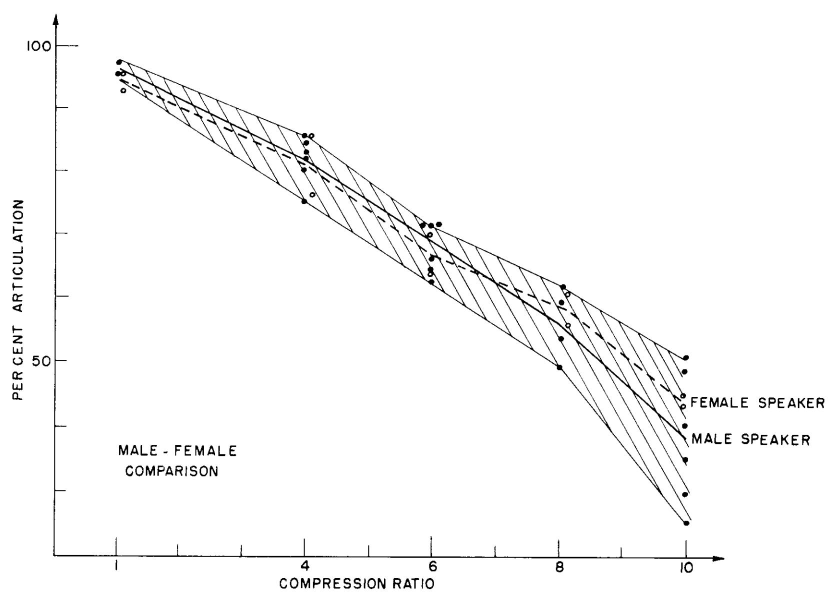

Fig. VIII-6. System performance.

at a fixed rate, in that it has natural pitch cadence and does not sound banjo-like at high reduction ratios; however, the present synchronous chopper has a disagreeable harshness in its output. The harshness is, in part, the result of a nonuniform keying voltage from the memory, which results in an annoying pitch-synchronous noise; in part, a result of the waveform mismatch at the end of each sequence of stored cycle repetitions. The harshness is also attributable to the fact that the formants and the pitch in the synthesizer output must move in the same direction during the repetition interval, whereas they may be moving in arbitrary directions in the input speech.

R. M. Lerner

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E. NONLINEAR OPERATIONS ON VIDEO SIGNALS

A commercially available noise-suppression system for record players uses a variable filter which suppresses the high-frequency components of the signal when the signal amplitude is low (thereby eliminating needle scratch) but passes all frequencies when the signal amplitude is high enough to mask the needle scratch. The use of an operation of similar character on pictures was investigated.

The operation consisted of comparing the intensity I(x, y) at a point in the picture with the average intensity

$$\overline{I}(x, y) = \frac{1}{A} \iint_A I(x + \xi, y + \eta)\, d\xi\, d\eta$$

and generating an output signal which was equal to I(x, y) when the difference $I - \overline{I}$ was large in magnitude but was equal to the average $\overline{I}$ when the difference was small in magnitude. The effect is, essentially, to suppress high-frequency components (abrupt intensity changes) that are below some threshold in amplitude and to admit abrupt intensity changes that are large and, presumably, correspond to an outline in the picture rather than to a small textural variation or noise.

One application of this kind of picture processing is the smoothing of pictures that have been received over a pulse code modulation system that uses only a few levels, let us say eight. In looking at such pictures it is evident that in regions of detail – faces, for example – the picture looks quite satisfactory, but in regions of gradual intensity change, like the sky, annoying steps are visible as the picture shifts from level to level over a long boundary. An operation of the kind described, in which the threshold was set at one quantization level, might reasonably be expected to smooth out the "steps in the sky," but not to affect the detail.

A preliminary investigation of this operation was carried out with a flying-spot scanning system. Two copies of the picture were scanned, one with a large spot, the other one with a small spot, to provide the two signals $I(x, y)$ and $\bar{I}(x, y)$. An electronic circuit was then designed to produce an output that varied smoothly from the first of these to the second as the difference between them decreased in magnitude.

The experimental program involved a considerable amount of equipment construction. The few results that were obtained in the allotted time were highly suggestive, but not conclusive. This work is discussed in detail in a thesis submitted by A. J. Osborne to the Department of Electrical Engineering, M.I.T., in partial fulfillment of the requirements for the degree of Master of Science.

P. Elias

F. RUNS OF CONSTANT INTENSITY IN PICTURE SIGNALS

In the usual method of transmitting a black-and-white picture through a communication system, a picture can be considered as divided into a large number of elementary areas, or cells. The light intensities of these cells are transmitted in succession by rows. Statistical independence of the light intensities of all cells would preclude the possibility of any bandwidth reduction without improvement of the signal-to-noise ratio of the transmission system.

Even a casual inspection of a photograph indicates, however, that there should be rather strong statistical constraints among the cells of a picture. One approach to the problem of bandwidth reduction for these signals, then, is to attempt to find means of generating a signal that takes advantage of the statistical constraints among the cells and uses a single coded message for a large block of cells.

One such method of coding would involve considering a run of adjacent cells with equal intensities as a block and then transmitting information relating to the value of the intensity and the block length. Although, in practice, runs are too short to make such a scheme worth-while when the light intensity is quantized to a sufficiently large number of levels, they might become significantly long when a considerably smaller number of levels is used, together with a smoothing operation, such as the one described in Section VIII-E. For eight-level quantization, the run-lengths of constant intensity are appreciable.

Equipment for the measurement of statistics of runs of constant intensity was constructed and operated by Mr. F. F. Tung at this Laboratory. His results indicate some reduction in the entropy of a picture signal. For one test, in which a picture with considerable detail was quantized to 32 levels, the average run length was 1.95 cells and the entropy was 3.26 bits per cell. For a different test, in which a picture with little detail was quantized to 20 levels, the average run-length was 4.9 cells and the entropy was 1.53 bits. This work is covered fully in a thesis submitted by F. F. Tung to the Department of Electrical Engineering, M.I.T., in partial fulfillment of the requirements for the degree of Master of Science, July 1956.

Further work regarding runs determined on the basis of other signal characteristics will be carried out.

W. A. Youngblood