II. MICROWAVE GASEOUS DISCHARGES

Prof. S. C. Brown
Prof. W. P. Allis
Dr. M. P. Madan
S. J. Buchsbaum
A. L. Gilardini
E. Gordon
J. J. McCarthy
R. G. Meyerand, Jr.
W. J. Mulligan

A. HIGHLY IONIZED MICROWAVE PLASMA*

In an effort to determine the optimum conditions for the production of a highly ionized hydrogen plasma, microwave breakdown experiments in hydrogen in the presence of a magnetic field (1) are being extended to low pressures (10^{-3} to 10^{-5} mm Hg). A quartz cylindrical tube, 1 cm in diameter and 6 cm in length, filled with hydrogen to a desired pressure, is placed along the axis of a cylindrical cavity that is oscillating in the TE_{111} mode at S-band. A variable, axial magnetic field of approximately 1000 gauss is provided by an electromagnet. The pole pieces of the magnet are shaped to insure the uniformity of the magnetic field over the central portion of the quartz tube.

The purpose of the magnetic field is twofold. First, by adjusting the cyclotron frequency ω_b so that it is equal to the operating frequency ω, a very efficient means is provided for the transfer of energy from the microwave field to the electron cloud. Second, the strong magnetic field severely limits the loss of electrons by diffusion in the radial direction. Thus far, no effort has been made to limit the loss in the axial direction, but a magnet with a specially shaped magnetic field that will also impede this loss is being designed. The effect of the magnetic field on microwave breakdown is generally exhibited by plotting the breakdown field E_b against the magnetic field for a constant gas pressure. The plot usually exhibits a resonance minimum at $\omega = \omega_b$. The width of the resonance curve is characteristic of the process or processes that impede the transfer of energy from the microwave field to the electron cloud. At pressures of the order of millimeters of mercury, the width is the result of collisions of electrons with neutral atoms or molecules. The relative width of the resonance curve at half-power points is then given by ν_c/ω, where ν_c is the collision frequency (for hydrogen, $\nu_c \approx 5 \times 10^9 p$, where p is the pressure in mm Hg) and ω is the operating frequency. In the present experiment, neither collisions with neutral atoms nor the much more frequent collisions with the end walls of the quartz tube occur sufficiently often to account for the observed widths. The explanation lies partly in the inhomogeneity of the magnetic field in the axial direction. The inhomogeneity, if it exists, limits the volume in which electrons gain energy efficiently to only a fraction of that of the quartz tube. To check this, the inhomogeneity of the magnetic field was varied by extending the shims on the pole pieces of the electromagnet by increments of one-quarter inch. The resultant breakdown curves for three such increments are plotted in Fig. II-1, which

*This work was supported in part by the Atomic Energy Commission under Contract AT (30-1) 1842.

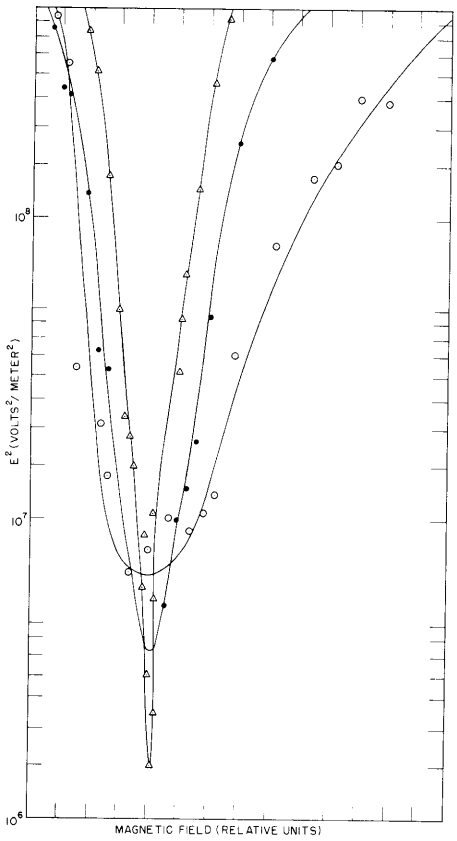

Fig. II-1. Effect of inhomogeneity of magnetic field on microwave breakdown in hydrogen (pressure, 6×10^{-4} mm Hg).

shows distinctly the dependence of the breakdown field on the homogeneity of the magnetic field. In addition, the widest curve shown in Fig. II-1 is asymmetrical. The asymmetry is thought to be the result of an inhomogeneity in the radial direction, which must exist if there is a large inhomogeneity in the axial direction, but this is not yet conclusively determined.

S. J. Buchsbaum, E. Gordon

References

1. B. Lax, W. P. Allis, and S. C. Brown, Technical Report 165, Research Laboratory of Electronics, M.I.T., June 30, 1950.

B. DIFFUSION DECAY IN NEON AFTERGLOWS

In the Quarterly Progress Report of Oct. 15, 1955, page 16, we discussed some of our experimental results concerning the behavior of the diffusion coefficient in the

afterglow of a microwave discharge in the transition region from free to ambipolar diffusion, and our plans for investigating the decay of a discharge in neon contaminated with argon (0.1 per cent) in cavities that have very small diffusion lengths Λ. In this case, we expect an electron density decay that is approximately exponential, with a slowly varying time constant that decreases with increasing postdischarge times, because in going to lower densities, we move from ambipolar to free diffusion conditions. The decay was investigated for a cylindrical metal cavity with $\Lambda = 0.1$ cm, and the effect was found to be present, as shown in the typical curve of Fig. II-2. During the investigation, we also found that, in our experiment, the value of the ambipolar diffusion coefficient is not linearly related to pressure, as we expected; therefore we decided to study this effect first.

In the afterglow of a discharge, in a Penning mixture like neon contaminated with argon, we expect the diffusion to be controlled by the mobility of argon ions in neon; consequently, from the known values of mobilities of alkali ions in different gases (1) we expect a $D_a p$ value of about 310 cm^2 mm Hg/sec. In our experiment, the measured values range from 260 to more than 400 when the pressure varies from 7 to 30 mm Hg. We tentatively explain the values above 310 by assuming the formation of a neon-argon ionized molecule, which, if it exists, is supposed to have a higher $D_a p$ value than the argon ions in neon, and probably will also have a recombination coefficient of the same order as the one for the ionized molecules of the rare gases. It can be shown (2) that in our case we measure an apparent $(D_a p)_a$ which is related to the true $D_a p$ by the relation

$$(D_a p)_a = D_a p + \zeta \Lambda^2 p^3 \tag{1}$$

where (ζp^2) is the frequency of formation of a molecule, according to the reaction,

$$A^+ + 2Ne \rightarrow (ANe)^+ + Ne \tag{2}$$

According to Eq. 1, the measured $(D_a p)_a$ must be a linear function of p^3, and the intercept at zero pressure gives the true $D_a p$. Figure II-3 shows our experimental data plotted in this way, and the graph seems to confirm, within the experimental error, the p^3 dependence and the expected true $D_a p$ value, except for pressures less than 15 mm Hg. We explain the low $D_a p$ values in this low-pressure region by the hypothesis that here the ions are not all argon but partially neon; the $D_a p$ value for neon molecular ions in neon is, in fact, 250 cm^2 mm Hg/sec (see ref. 3). This explanation is consistent with breakdown measurements (4), which show that at low pressures a contamination of 0.1 per cent of argon is not sufficient to produce minimum breakdown.

From the data of Fig. II-3, we obtain for ζ the approximate value 0.5 sec^{-1} $(mm\ Hg)^{-2}$, which is about one-hundredth of the measured value for the conversion of

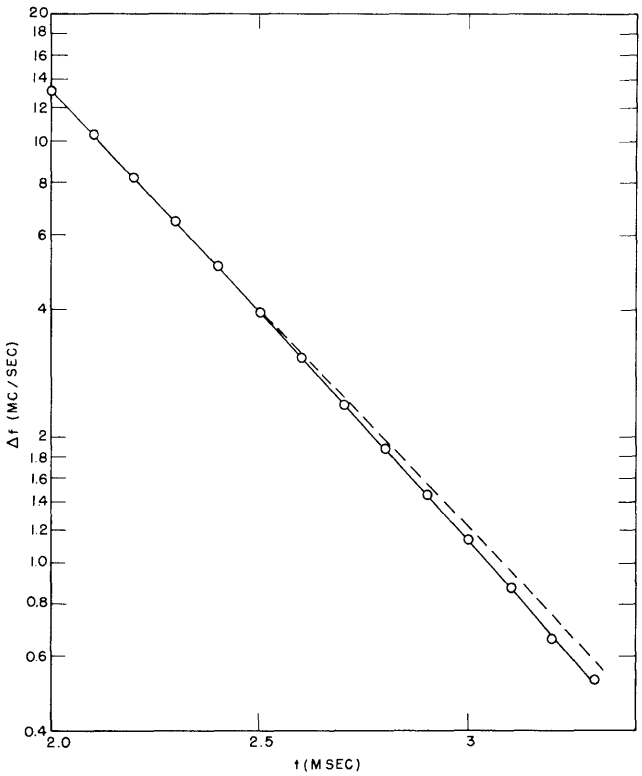

Fig. II-2. Frequency shift Δf versus postdischarge time t.

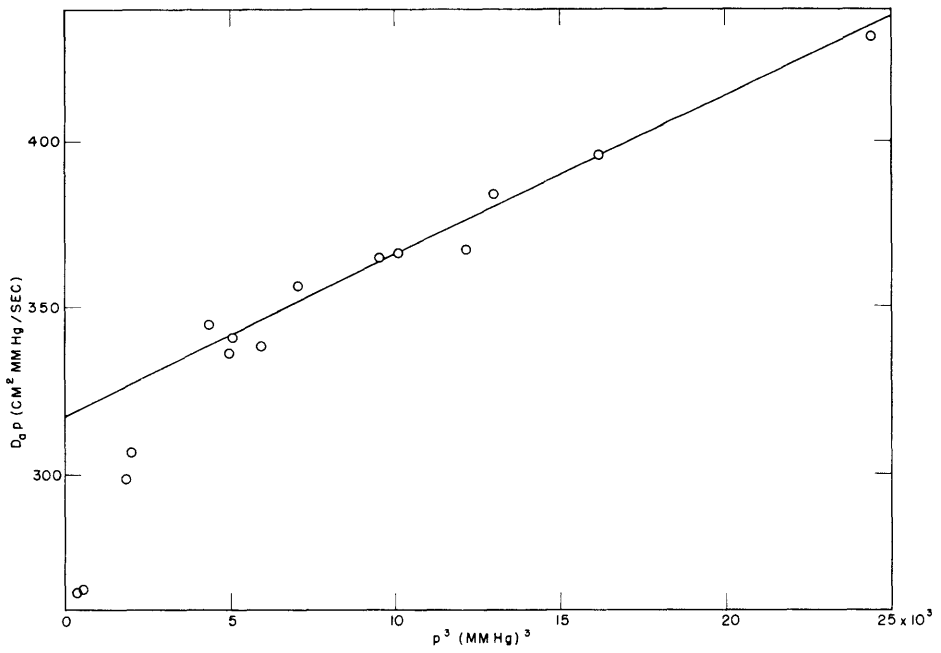

Fig. II-3. Measured $D_a p$ values versus p^3. As usual, the pressures are normalized to 0°C and the diffusion coefficients to 300°K and 1 mm Hg.

ionized atoms of helium into molecules (2); therefore the value seems reasonable.

In the next experiment, we hope to verify the correct dependence of Eq. 1 on Λ and to increase the amount of argon contamination in order to show an increase in $D_a p$ values in the low-pressure region.

A. L. Gilardini

References

1. C. F. Powell and L. Brata, Proc. Roy. Soc. (London) A 138, 117 (1932).
2. A. V. Phelps and S. C. Brown, Phys. Rev. 86, 102 (1952).
3. R. J. Munson and A. M. Tyndall, Proc. Roy. Soc. (London) A 177, 187 (1941); M. A. Biondi and L. M. Chanin, Phys. Rev. 94, 910 (1954).
4. H. J. Oskam, J. Appl. Phys. 27, 848 (1956).

C. MICROWAVE DETERMINATIONS OF FREQUENCY OF IONIZATION AND COEFFICIENT OF FREE DIFFUSION

A direct method of determining the significant quantities, ν_i, the coefficient of ionization, and D_-, the coefficient of free diffusion, was described in the Quarterly Progress Report of July 15, 1956, page 8. The present report deals with the theory of the method, necessary corrections to be applied to experimental data, and the results obtained.

The equation for the build-up of the electron density, in the absence of space charge, is written as

$$\frac{\partial n}{\partial t} = \nu_i n + D\nabla^2 n \tag{1}$$

where $\nu_i n$ is the production of electrons per second by ionization and $D\nabla^2 n$ is the loss of electrons per second by diffusion. For a cylindrical cavity of length L and radius R, when ν_i and D_- are independent of position, the solution of Eq. 1, for the case of a single electron entering the cavity at t = 0, at the point $Z = Z_o$, r = 0, and the boundary condition that n = 0 at the walls of the cavity, is

$$n(\bar{r}, Z, t) = \frac{2}{v_c} \sum_{m\ell} \sin \frac{m\pi}{L} Z_o \sin \frac{m\pi Z}{L} \frac{J_o}{J_1^2} \frac{\left(\beta_{o\ell} \frac{r}{R}\right)}{\left(\beta_{o\ell}\right)} \exp\left(\gamma_{m\ell} t\right) \qquad m = 1, 3, 5 \ldots \tag{2}$$

where

$$\gamma_{m\ell} = \nu_i - \frac{D_-}{\Lambda_{m\ell}^2} = \nu_i - D_- \left[\left(\frac{m\pi}{L}\right)^2 + \left(\frac{\beta_{o\ell}}{R}\right)^2\right]$$

and m, ℓ are positive integers, and $B_{o\ell}$ is the root of the zeroth-order Bessel function.

The fundamental mode is represented by $\ell = 1$, $m = 1$. It can be shown that higher modes in the Z-direction are important only at very short times and thus can be neglected. The contribution of higher modes in the radial direction is dependent on the radius of the cavity, pressure, and time. By writing the ratio of the first two terms, it can be seen that at large R and small t (of the order of a few microseconds in the present experiment), many higher modes are present, and the electron density distribution that results from all these modes is concentrated within a narrow central region. For this case, the electron density does not have time enough to diffuse out of the narrow central region, which can be seen by writing an alternative expression for n, obtained by the use of the Born approximation.

$$n(\bar{r}, Z, t) = \frac{2}{L} \sin \frac{\pi Z_o}{L} \sin \frac{\pi Z}{L} \frac{\exp(-r^2/4Dt)}{4\pi Dt} \exp\left[\nu_i - D\left(\frac{\pi}{L}\right)^2\right] t \qquad (3)$$

Equation 3 satisfies the postulated initial and boundary conditions. In this expression, a term of the order of

$$\frac{\exp(-R^2/4Dt)}{4\pi Dt}$$

necessary to satisfy the boundary condition at $r = R$, is omitted, since $R^2/4Dt > p > 15$ mm Hg.

Since the quantity obtained experimentally is essentially $\int_v n(\bar{r}, Z, t)\, E^2\, dv$, let us define

$$N = \frac{1}{E_o^2} \int_v n\, E^2\, dv \qquad (4)$$

where N is the total number of electrons in the cavity averaged over the electric energy. The cavity oscillates in the TM_{010} mode; therefore, if we write $E = E_o J_o(\beta_{o\ell}\, r/R)$, we obtain

$$N = \left[\int_0^L \frac{2}{L} \sin \frac{\pi Z_o}{L} \sin \frac{\pi Z}{L}\, dz \int_0^R 2\pi r\, dr \frac{\exp(-r^2/4Dt)}{4\pi Dt} J_o^2\left(\beta_{o\ell} \frac{r}{R}\right)\right]$$

$$\times \exp\left[\nu_i - D\left(\frac{\pi}{L}\right)^2\right] t \qquad (5)$$

Small error is made by allowing R to go to ∞, since the exponential for $r > R$ is less than e^{-20}. This yields

$$N = \frac{4}{\pi} \sin \frac{\pi Z_o}{L} \exp\left(-\frac{\beta_{o\ell}^2}{2}\frac{4Dt}{R^2}\right) J_o\left(j\frac{\beta_{o\ell}^2}{2}\frac{4Dt}{R^2}\right) \exp\left[\nu_i - D\left(\frac{\pi}{L}\right)^2\right] t$$

$$\approx \frac{4}{\pi} \sin \frac{\pi Z_o}{L} \left[1 - \left(\frac{\beta_{o\ell}^2}{2}\frac{4Dt}{R^2}\right)\right]\left[1 + \frac{1}{4}\left(\frac{\beta_{o\ell}^2}{2}\frac{4Dt}{R^2}\right)\right] \tag{6}$$

Experimentally, we seek the minimum time t_m in order to achieve a given density N. This corresponds to an initial electron appearing at $Z_o \approx L/2$, which makes $\sin \pi Z_o/L$ approximately unity. Furthermore, since the quantities of interest occur in the exponential and

$$\frac{\beta_{o\ell}^2}{2}\frac{4Dt}{R^2} < \frac{1}{10}$$

and N is of the order of 10^9, an error of less than 1 per cent is made by writing

$$N_b \approx \exp\left[\nu_i - D\left(\frac{\pi}{L}\right)^2\right] t_m \tag{7}$$

With the help of the method discussed in the Quarterly Progress Report of April 15, 1956, variation of the electric field E with the total electron density N can be computed for the present case, with ν_c/ω as a parameter for the 1/8-inch cavity (see Fig. II-4). This slight variation of N with ν_c/ω can be taken care of by writing $N_b^{f(\nu_c/\omega)}$, taking $N_b = 5 \times 10^8$. Since ν_i/p and $D_- p$ are functions of E/p only, Eq. 6 can be written in the form

$$\frac{f}{t_m p} = \frac{1}{(\ln N_b)}\left(\frac{\nu_i}{p} - \frac{Dp}{(p\Lambda)^2}\right) \tag{8}$$

where N_b is the total electron density at which the power begins to be reflected from the cavity and corresponds to the minimum time t_m.

Experimental measurements of the effective field E_e and t_m, with p_o, the pressure at 0°C, as a parameter, are plotted in Fig. II-5 for the 1/8-inch and 1/16-inch cavities. With the help of this plot, we can draw another curve of f/tp against $(p\Lambda)^{-2}$, with E_e/p_o as a parameter. But before this can be done, t_m should be corrected for the effective diffusion coefficient D_s, which includes the diffusion caused by space charge. This is necessary because, until now, we have assumed that the controlling

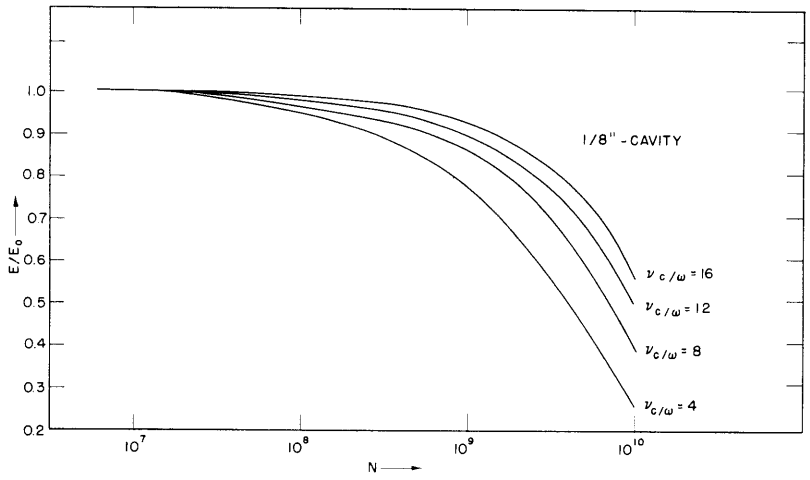

Fig. II-4. Variation of electric field with total electron density N.

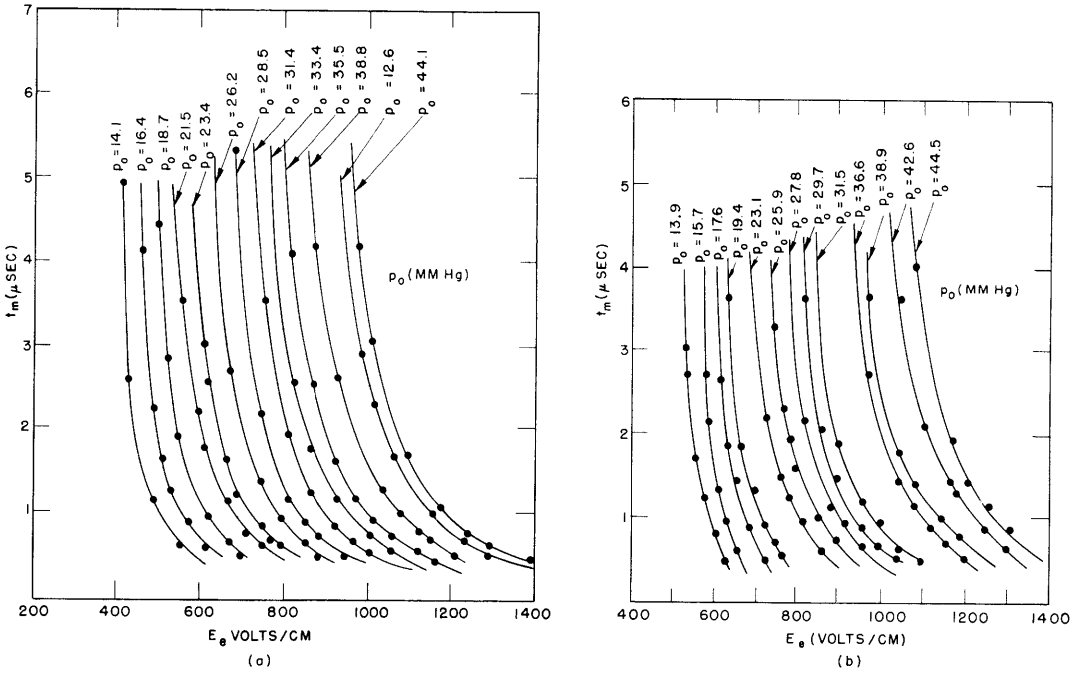

Fig. II-5. Experimental measurements of effective electric field versus minimum time t_m, with p_o, the pressure at 0°C, as parameter. (a) 1/8-inch cavity. (b) 1/16-inch cavity.

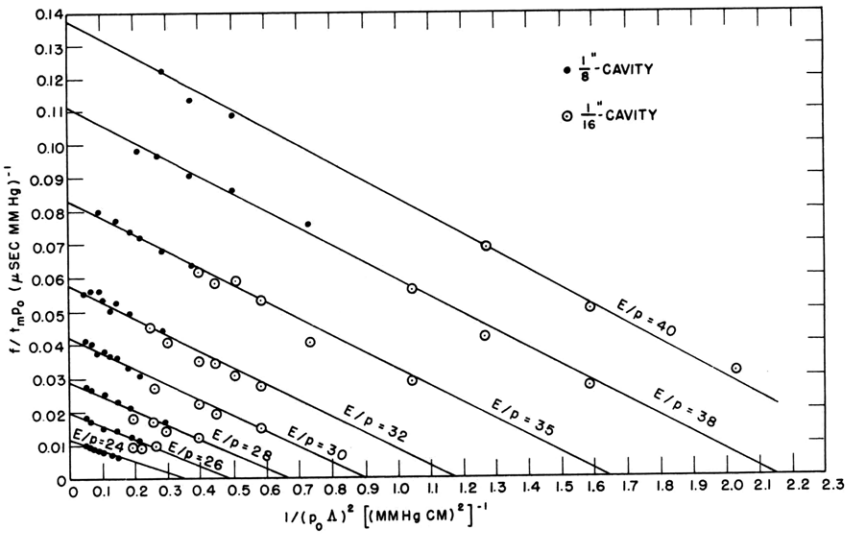

Fig. II-6. $f/t_m p$ against $(p_o\Lambda)^{-2}$ with E_e/p_o as parameter.

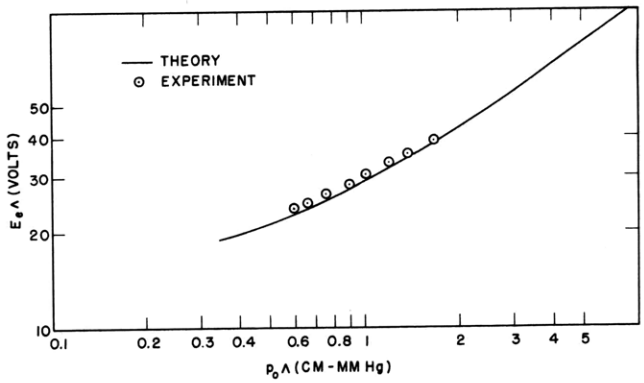

Fig. II-7. Steady-state breakdown voltages as a function of $p\Lambda$. The line is theoretical, the points experimental.

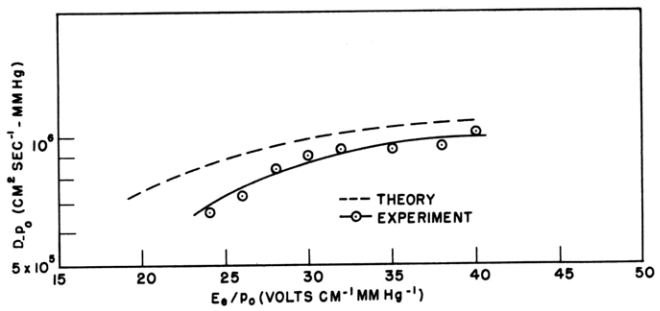

Fig. II-8. Comparison of experimentally determined ($D_- p_o$) with the theory of Allis and Brown.

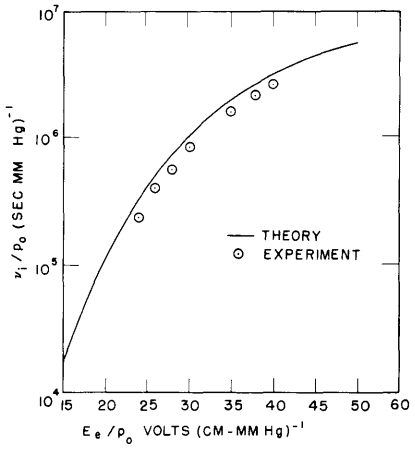

Fig. II-9. Comparison of experimentally determined ν_i/p_o with the theory of Allis and Brown.

factor over the period t_m is the free diffusion D_-.

With the help of Eq. 6, this correction can be written as

$$p\Delta t = \frac{p}{(p\Lambda)^2\left(\frac{\nu_i}{p} - \frac{Dp}{(p\Lambda)^2}\right)^2}\int (D_- - D_s)\frac{dn}{n} \qquad (9)$$

in order to yield the corrected values of $t_m p$. In this equation, D_s can be written in terms of D_-, by fitting an analytical expression to the curves computed by Allis and Rose (1). Figure II-6 is a plot of $f/t_m p_o$ against $(p_o\Lambda)^{-2}$, with E_e/p_o as a parameter. Each E/p-value line intersects the abscissa at different values of $(p\Lambda)^{-2}$, giving the pressure to which that particular value of E/p will correspond for the case of steady-state breakdown. Steady-state breakdown voltages as a function of $(p_o\Lambda)$ are plotted in Fig. II-7. The slope gives the value of D_-p, while the intercept with the ordinate gives ν_i/p. These values are plotted in Figs. II-8 and II-9 (see also ref. 2).

These experimentally determined ν_i/p-values require a correction, on account of the radial variation of the applied field, and can be treated by solving the diffusion equation for the electron density distribution for the case of a nonuniform field. This will be reported later.

M. P. Madan

References

1. W. P. Allis and D. J. Rose, Phys. Rev. 93, 84 (1954).
2. W. P. Allis and S. C. Brown, Phys. Rev. 87, 419 (1952).