arXiv:hep-ph/0701270v1  31 Jan 2007

# Observable Electron EDM and Leptogenesis

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## Abstract

In the context of the minimal supersymmetric seesaw model, the CP-violating neutrino Yukawa couplings might induce an electron EDM. The same interactions may also be responsible for the generation of the observed baryon asymmetry of the Universe via leptogenesis. We identify in a model-independent way those patterns within the seesaw models which predict an electron EDM at a level probed by planned laboratory experiments and show that negative searches on $\tau \rightarrow e\gamma$ decay may provide the strongest upper bound on the electron EDM. We also conclude that a possible future detection of the electron EDM is incompatible with thermal leptogenesis, even when flavour effects are accounted for.

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# 1  Introduction

New physics has to be invoked in order to explain why neutrinos are massive and mix among each other, and also why they turn out to be much lighter than the other known fermions. Among the proposed explanations, the idea that neutrino mass suppression is due to the decoupling of heavy states at a very high energy has prevailed in the last three decades. This principle is behind the formulation of the well-known seesaw mechanism [1].

Besides many experiments aiming to characterize neutrino oscillations, several other present and future experiments are planned to search for alternative signals of lepton flavour violation (LFV), *e.g.* in radiative charged lepton decays. It is therefore of great importance to explore theoretical scenarios where LFV is enhanced to levels at reach of planned experiments. One of the most appealing theoretical frameworks where this may actually occur relies on supersymmetry (SUSY). Within the minimal supersymmetric extension of the standard model (MSSM), LFV can be communicated to the SUSY-breaking sector by the same interactions which participate in the seesaw mechanism producing a misalignment between leptons and sleptons [2]. This turns out to be particularly interesting in the case where the dynamics responsible for breaking SUSY is flavour-blind, like in minimal-supergravity scenarios (mSUGRA).

The phenomenological aspects related to supersymmetric seesaw mechanisms have been widely studied in the literature, especially those which concern LFV decays [3, 4, 5, 6, 7]. In particular, most of the analysis have been based on the extension of the MSSM particle content where right-handed (RH) heavy neutrinos (responsible for the neutrino mass suppression) are added. RH neutrinos may play a role in the generation of the baryon asymmetry. If their couplings to the ordinary charged leptons are complex and therefore violate CP, then their out-of-equilibrium decays may induce the observed baryon asymmetry of the Universe, $Y_B = (0.87 \pm 0.03) \times 10^{-10}$ [8], via the leptogenesis mechanism [9]. In the simplest thermal scenario, the lightest heavy Majorana neutrino is produced after inflation by thermal scatterings and subsequently decays out-of-equilibrium violating CP and lepton number. Flavour effects play also a role in thermal leptogenesis since its dynamics depends on which charged lepton Yukawa interactions are in thermal equilibrium and, therefore, the computation of $Y_B$ depends on which temperature regime the heavy Majorana neutrino decays occur. We will come back to this point later.

Besides flavour violation, also CP-violating (CPV) effects may be induced in the soft SUSY breaking terms by the CPV seesaw interactions. These may lead to relevant contributions to the charged lepton electric dipole moments (EDM), in particular that of the electron [10, 11, 12, 13, 14]. The present upper bound on the electron EDM is $d_e < 1.6 \times 10^{-27}$ e cm at 90% C.L. [15]. Within three years, the Yale group plans to reach a sensitivity of about $10^{-29}$ e cm [16] and hopefully go down to $10^{-31}$ e cm within five years. A more ambitious proposal, based on solid-state physics methods, exists to probe $d_e$ down to $10^{-35}$ e cm [17].

The above discussion shows that, by extending the MSSM particle content with heavy neutrino singlets, a new window is widely opened into the investigation of new effects in flavour physics and cosmology. Ultimately, one expects to be able to relate various phenomena like neutrino oscillations, LFV, leptogenesis and EDMs. This programme is of extreme importance and may shed some light over the origin of neutrino masses and mixing and its relation with possible new physics observations. In what follows, we consider the scenario where the (dominant) LFV and CPV effects in the SUSY soft breaking sector are exclusively generated by the seesaw Yukawa interactions. Under these assumptions, we identify the Yukawa structure leading to an observable $d_e$ and at the same time compatible with the present bounds on LFV decays and show that there is an upper bound on $d_e$ coming from negative searches of the $\tau \to e\gamma$ decay. This indirect bound can be even stronger than the direct-search limit. We will also study the impact on thermal flavoured-leptogenesis for cases where $d_e$ is at hand of future experiments, showing that a positive detection of $d_e$ is incompatible with thermal leptogenesis.

The paper is organized as follows. In Section 2 we review the issue of EDM and LFV within supersymmetric seesaw models and discuss the bound on $d_e$ provided by the $\tau \to e\gamma$. In Section 3 we give a short review of thermal leptogenesis, including flavour effects. In Section 4 we discuss the patterns of neutrino Yukawas giving rise to a large $d_e$, but still compatible with present bounds on $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$. The conclusions are drawn in Section 5.

# 2    EDM and LFV from seesaw Yukawa couplings

We consider the framework of the minimal supersymmetric standard model (MSSM) extended with three heavy Majorana neutrino singlets $N$. Working in the basis where the charged lepton mass matrix $m_\ell$ is real and diagonal, the seesaw [1] interactions at high energy are described by the following superpotential terms

$$\mathcal{W}_{\mathcal{SS}} = N\,Y_\nu\,L\,H_u + \frac{1}{2}N\hat{M}N \;\;,\;\; \hat{M} = \mathrm{diag}(M_1, M_2, M_3)\,, \tag{1}$$

where $Y_\nu$ is the Dirac neutrino Yukawa coupling matrix and $M_i$ stand for the (positive) heavy Majorana neutrino masses ordered as $M_1 < M_2 < M_3$. As usual, $L$ denotes the lepton doublets and $H_u$ the hypercharge $1/2$ Higgs superfield. Integrating out the heavy neutrino singlets, after electroweak symmetry breaking one obtains the effective Majorana neutrino mass matrix

$$m_\nu^{eff} = Y_\nu^T \frac{1}{\hat{M}} Y_\nu v_u^2 = U^* \hat{m}\, U^\dagger \;\;,\;\; \hat{m} = \mathrm{diag}(m_1, m_2, m_3)\,, \tag{2}$$

where $U$ is the leptonic mixing matrix (parameterized in the usual way, see Eq. (22)), $m_i$ are the (positive) effective neutrino masses, $v_u = v \sin\beta$ with $v = 174\,\mathrm{GeV}$ and $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$.

Because of RGE running from high to low energy scales, the seesaw Yukawa couplings potentially induce flavour and CP violations in slepton masses. In the following we review their impact by assuming universal and real boundary conditions at the Planck scale $M_{\mathrm{Pl}}$: $m_X^2 = m_0^2$ for the soft scalar masses, $A_X = a_0 Y_X$ for the trilinear $A$-terms and $\tilde{M}_i = M_{1/2}$ for the gaugino mass parameters. In addition, we assume that the $\mu$-term is real. Under these circumstances, the flavour and CP violations in the low-energy slepton masses arise from those present in the seesaw Yukawa couplings through RGE running.

The most relevant flavour misalignment is induced by the seesaw Yukawas in the slepton doublet mass matrix $m_L^2$ [2]. Following the widely used mass insertion approximation, the flavour-violating entries of $m_L^2$ can be parametrized by

$$\delta_{ij}^{LL} = \frac{m_{Lij}^2}{\bar{m}_L^2} = -\frac{1}{(4\pi)^2} \frac{6m_0^2 + 2a_0^2}{\bar{m}_L^2} \, C_{ij} \quad (i \neq j) \,, \tag{3}$$

where $\bar{m}_L$ is an average doublet slepton mass and $C_{ij}$ encodes the dependence of $\delta_{ij}^{LL}$ on the Yukawa interactions:

$$C_{ij} = \sum_k C_{ij}^k \quad , \quad C_{ij}^k = Y_{\nu ki}^* Y_{\nu kj} \ln \frac{M_{\mathrm{Pl}}}{M_k} \,. \tag{4}$$

For later convenience, we have isolated the contribution of each $N_k$ to $C_{ij}$ denoting it by $C_{ij}^k$. Notice that in Eq. (4) $Y_\nu$ has to be evaluated at $M_{\mathrm{Pl}}$. The induced LFV decays are

$$\mathrm{BR}(\ell_i \to \ell_j \gamma) = \mathrm{BR}(\ell_i \to \ell_j \bar{\nu}_j \nu_i) \, F_B \, \tan^2 \beta \, |\delta_{ij}^{LL}|^2 \,, \tag{5}$$

where $F_B$ is an adimensional function of supersymmetric masses which includes the contributions from chargino and neutralino exchange – see e.g. [6] and references therein.

From the above considerations, it is clear that the present experimental limits on LFV decays can be translated into upper bounds on $|C_{ij}|$, denoted by $C_{ij}^{\mathrm{ub}}$, which depend on $\tan \beta$ and supersymmetric masses [5]. In mSUGRA, once the relation between $a_0$ and $m_0$ has been assigned and radiative electroweak breaking required, the supersymmetric spectrum can be expressed in terms of two masses, e.g. the bino mass $\tilde{M}_1$ and the average singlet charged slepton mass $\bar{m}_R$ at low energy (we recall that $\tilde{M}_1 \approx 0.4 M_{1/2}$ and $\bar{m}_R^2 \approx m_0^2 + 0.15 M_{1/2}^2$). For definiteness, we will take from now on $a_0 = m_0 + M_{1/2}$. Considering in particular the point $P = (\tilde{M}_1, \bar{m}_R) = (200, 500)\,\mathrm{GeV}$, for which the SUSY contribution to the anomalous magnetic moment of the muon $\delta a_\mu$ is within the observed discrepancy between the SM and experimental result for $\tan \beta > 35$, we obtain

$$C_{21}^{\mathrm{ub}} \simeq 5 \times 10^{-3} \frac{50}{\tan \beta} \quad , \quad C_{32}^{\mathrm{ub}} \simeq 0.8 \frac{50}{\tan \beta} \quad , \quad C_{31}^{\mathrm{ub}} \simeq \frac{50}{\tan \beta} \,. \tag{6}$$

The strongest constraint comes from $\mu \to e\gamma$, although also those from $\tau \to e\gamma$ and $\tau \to \mu\gamma$ become non-trivial for $\tan \beta \gtrsim 10$, as they imply that $Y_\nu$ couplings are at most of $\mathcal{O}(1)$. This, in turn, supports the use of the perturbative approach. In Fig. 6 of the Appendix we display the dependence of $C_{ij}^{\mathrm{ub}}$ in the plane $(\tilde{M}_1, \bar{m}_R)$.

There are two potentially important sources of CP violation induced by the seesaw Yukawa couplings in the doublet slepton mass matrix. One is associated to the flavour-conserving (FC) $A$-terms [10, 11, 12], while the other, generically dominant for $\tan\beta \gtrsim 10$, is mediated by flavour-violating (FV) $\delta$'s [12, 13]. The corresponding contributions to lepton EDMs are given by

$$d_i^{\rm FC}\,[e\,{\rm cm}] = F_d\,m_{\ell_i}\,{\rm Im}(a_i) \qquad , \qquad d_i^{\rm FV}\,[e\,{\rm cm}] = F_d''\,\mu\tan\beta\,{\rm Im}(\delta^{RR}m_\ell\,\delta^{LL})_{ii}\,, \qquad (7)$$

where $F_d$, $F_d''$ (with dimension of mass$^{-2}$) are functions of the slepton, chargino and neutralino masses – see e.g. [6] and references therein. These sources of CPV are

$$ {\rm Im}(a_i) = \frac{8a_0}{(4\pi)^4}\,I_i^{\rm FC} \quad , \quad {\rm Im}(\delta^{RR}m_\ell\delta^{LL})_{ii} = \frac{8m_{\ell_i}}{(4\pi)^6}\frac{(6m_0^2 + 2a_0^2)(6m_0^2 + 3a_0^2)}{\bar m_L^2\,\bar m_R^2}\frac{m_\tau^2\tan^2\beta}{v^2}\,I_i^{\rm FV}\,, \tag{8}$$

where

$$ I_i^{\rm FC} = \sum_{k>k'} I_i^{(kk')\rm FC} \quad , \quad I_i^{\rm FV} = \sum_{k>k'} I_i^{(kk')\rm FV}\,. \tag{9} $$

Adopting a notation which renders more explicit the link with LFV decays and defining $\ln_b^a = \ln(M_a/M_b)$ for short, one has [14]

$$ I_i^{(kk')\rm FC} = \frac{\ln_{k'}^k}{\ln_{k'}^{\rm Pl}}\,{\rm Im}(C^k C^{k'})_{ii} \quad , \quad I_i^{(kk')\rm FV} = \widetilde{\ln}_{k'}^k\,{\rm Im}\left(C^k\,\frac{m_\ell^2}{m_\tau^2}\,C^{k'}\right)_{ii}\,, \tag{10} $$

with $\widetilde{\ln}_2^3 = \ln_2^3$, $\widetilde{\ln}_1^3 = \ln_1^3(1 - 2\ln_2^3\ln_1^2 / \ln_1^3\ln_1^{\rm Pl})$, $\widetilde{\ln}_1^2 = \ln_1^2(1 - 2\ln_3^{\rm Pl}\ln_2^3 / \ln_2^{\rm Pl}\ln_1^{\rm Pl})$. Notice that these contributions arise as an effect of a splitted spectrum of right-handed neutrinos and would vanish in the case of a degenerate spectrum [10, 18]. It turns out that the seesaw-induced contributions to $d_\mu$ and $d_\tau$ are below the planned experimental sensitivities; on the contrary the seesaw-induced contribution to $d_e$ might be at the level of planned experiments. The present experimental upper limit, $d_e^{exp} = 1.4 \times 10^{-27}$ e cm, can correspondingly be translated into upper bounds on $|I_e^{\rm FC}|$ and $|I_e^{\rm FV}|$, with a dependence on $\tan\beta$ and supersymmetric masses - see [12, 14] for more details. Within the mSugra framework, we display these bounds in Fig. 6 of the Appendix. In particular, for the point $P$ introduced before, the upper bounds are

$$ I_e^{\rm ub,FC} \simeq 10^3 \quad , \quad I_e^{\rm ub,FV} \simeq 10^2\,\left(\frac{50}{\tan\beta}\right)^3\,. \tag{11} $$

For comparison, it is useful to estimate the upper allowed values of $I_e^{\rm FC}$ and $I_e^{\rm FV}$ assuming perturbativity: by allowing the relevant Yukawa couplings of $Y_\nu$ to be of $\mathcal{O}(1)$ and the logarithms to be large enough, from Eq. (10) one obtains $I_e^{\rm FC} \lesssim 50$ and $I_e^{\rm FV} \lesssim 300$. This means that in point $P$ the FC-type seesaw-induced contribution, $d_e^{\rm FC}$, is below the level of the present experimental sensitivity; an improvement by two orders of magnitude, pushing $d_e^{exp}$ at the level of $10^{-29}$ e cm, would be needed to start testing it. On the contrary, the FV-type seesaw-induced contribution, $d_e^{\rm FV}$, might have already exceeded

the present experimental bound in $P$ for $\tan\beta \gtrsim 35$ (and for even smaller $\tan\beta$ if the supersymmetric masses are taken to be smaller than those in $P$).

However, as we now turn to discuss, also the experimental limit on $\tau \to e\gamma$ provides an upper bound to the seesaw-induced $I_e^{\rm FC}$ and $I_e^{\rm FV}$. At present, such indirect bounds are even stronger than those from direct searches of the electron EDM. Once the constraint from $\tau \to e\gamma$ is taken into account, it turns out that both the FC and FV-type seesaw contributions have to be sizeably smaller than what would be allowed assuming perturbativity. The argument is based on the fact that, barring cancelations, $|C_{ij}^k| < C_{ij}^{\rm ub}$ for each $k$. In Eq. (10), the strong bound from $\mu \to e\gamma$ makes the terms involving $C_{12}^k$ to be negligibly small. If large, $I_e^{\rm FC}$ and $I_e^{\rm FV}$ are then proportional to the same combination of Yukawas [14]:

$$I_e^{\rm FC} \approx \sum_{k>k'} \frac{\ln_{k'}^k}{\ln_{k'}^{\rm Pl}} {\rm Im}(C_{13}^k C_{13}^{k'*}) \quad , \quad I_e^{\rm FV} \approx \sum_{k>k'} \widetilde{\ln}_{k'}^k {\rm Im}(C_{13}^k C_{13}^{k'*}) . \tag{12}$$

In the discussion of the next sections, it will turn out that the only relevant contribution comes from $(k,k') = (3,2)$. Therefore, in what follows, we will focus on $I_e^{(32){\rm FC}} \ln_2^{\rm Pl} \approx I_e^{(32){\rm FV}} \approx \ln_2^3 {\rm Im}(C_{13}^3 C_{13}^{2*})$. Requiring ${\rm Im}(C_{13}^k C_{13}^{k'*}) \leq C_{13}^{\rm ub^2}$, one thus obtains an indirect upper bound from $\tau \to e\gamma$, to be denoted by $I_e^\tau$. Considering in particular the point $P$, the indirect upper bounds from $\tau \to e\gamma$ are

$$I_e^{\tau{\rm FC}} \ln_2^{\rm Pl} = I_e^{\tau{\rm FV}} = \ln_2^3 \left(\frac{50}{\tan\beta}\right)^2 . \tag{13}$$

As a comparison, allowing the relevant Yukawa couplings $Y_\nu$ to be (at most) of $\mathcal{O}(1)$ and the logarithms involved to be large, one would have ${\rm Im}(C_{13}^3 C_{13}^{2*}) \lesssim 50$. This means that, for $\tan\beta \gtrsim 5$, both $I_e^{\tau{\rm FC}}$ and $I_e^{\tau{\rm FV}}$ are smaller than what allowed by perturbativity. Typically, $10\, I_e^{\tau{\rm FC}} \sim I_e^{\tau{\rm FV}} \sim 500$ for $\tan\beta = 5$, while $10\, I_e^{\tau{\rm FC}} \sim I_e^{\tau{\rm FV}} \sim 5$ for $\tan\beta = 50$. The indirect bounds from $\tau \to e\gamma$ are thus stronger than those from direct searches of $d_e$, shown in Eq. (11). Taking $P$ and representative splittings between $M_3$ and $M_2$, we display in Fig. 2 the ratio $d_e^\tau/d_e^{exp} = I_e^{\tau{\rm FC}}/I_e^{\rm ub,FC} + I_e^{\tau{\rm FV}}/I_e^{\rm ub,FV}$ as a function of $\tan\beta$. It turns out that the indirect upper bound from $\tau \to e\gamma$, $d_e^\tau$, is stronger than the present direct one, $d_e^{\rm exp}$, by about two orders of magnitude[1]. Planned experiments lowering the bounds on $d_e$ by about three orders of magnitude will then provide sensible tests of the seesaw-induced effects. Clearly they will test the seesaw models that maximize $I_e^{\rm FV}$ (hence also $I_e^{\rm FC}$), i.e. the models where: 1) $M_3 \gtrsim 10\, M_2$; 2) ${\rm Im}(C_{13}^3 C_{13}^{2*}) \sim C_{13}^{\rm ub^2}$. The latter of course also implies that $\tau \to e\gamma$ is close to the experimental bound.

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[1]As a consequence, a potential discovery of the electron EDM within an order of magnitude from the present sensitivity should not to be interpreted as due to the seesaw-induced effects - this of course holds for mSugra and point $P$.
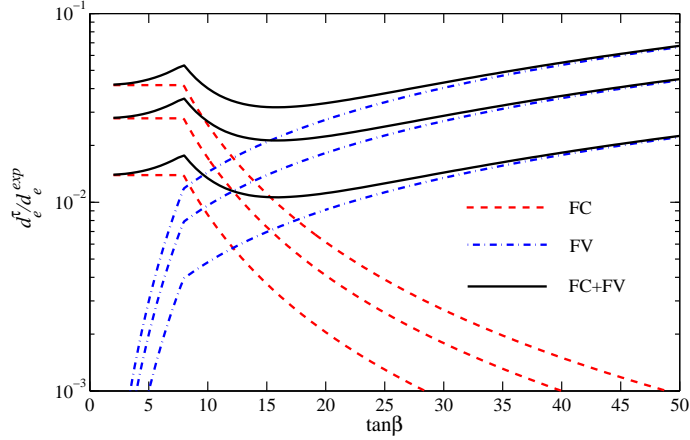
Figure 1: Ratios $\min(I_e^{\tau\text{FC}}, \ln_2^3 50/\ln_2^{\text{Pl}})/I_e^{\text{ubFC}}$ (dashed) and $\min(I_e^{\tau\text{FV}}, \ln_2^3 50)/I_e^{\text{ubFV}}$ (dash-dotted) evaluated in $P$ as a function of $\tan\beta$. Their sum is $d_e^\tau/d_e^{exp}$ (solid). We take $M_2 = 5 \times 10^{14}$ GeV and $M_3/M_2 = 10^3, 10^2, 10$ from top to bottom, respectively.

# 3 Thermal leptogenesis including flavours

In this section we provide a short review of leptogenesis which will be useful to establish the link between the high energy CP-violation responsible for the generation of the baryon asymmetry through thermal leptogenesis and the one responsible for the EDM of the electron.

Thermal leptogenesis [19, 20, 21] takes place through the decay of the lightest of the heavy Majorana neutrinos if these are present in the early Universe. The out-of-equilibrium decays occur violating lepton number and CP, thus satisfying the Sakharov's conditions [22]. In grand unified theories (GUT) the heavy Majorana neutrino masses are typically smaller than the scale of unification of the electroweak and strong interactions, $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV, by a few to several orders of magnitude. This range coincides with the range of values of the heavy Majorana neutrino masses required for a successful thermal leptogenesis.

We will account for the flavour effects which appear for $M_1 \lesssim 10^{12}$ GeV and have been recently investigated in detail in [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34] including the quantum oscillations/correlations of the asymmetries in lepton flavour space [25, 34]. The Boltzmann equations describing the asymmetries in flavour space have additional terms which can significantly affect the result for the final baryon asymmetry. The ultimate reason is that realistic leptogenesis is a dynamical process, involving the production and destruction of the heavy RH neutrinos, and of a lepton asymmetry that is distributed among *distinguishable* lepton flavours. Contrarily to what is generically assumed in the one-single flavour approximation, the $\Delta L = 1$ inverse decay processes which wash-out the net lepton number are flavour dependent, that is the lepton asymmetry carried by, say, electrons can be washed out only by the inverse decays involving the electron flavour. The

asymmetries in each lepton flavour, are therefore washed out differently, and will appear with different weights in the final formula for the baryon asymmetry. This is physically inequivalent to the treatment of wash-out in the one-flavour approximation, where the flavours are taken indistinguishable, thus obtaining the unphysical result that, *e.g.*, an asymmetry stored in the electron lepton charge may be washed out by inverse decays involving the muon or the tau charges.

When flavour effects are accounted for, the final value of the baryon asymmetry is the sum of three contributions. Each term is given by the CP asymmetry in a given lepton flavour $\ell$, properly weighted by a wash-out factor induced by the same lepton number violating processes. The wash-out factors are also flavour dependent.

Of course, since we are dealing with the MSSM, we have to consider the presence of the supersymmetric partners of the RH heavy neutrinos, the sneutrinos $\widetilde{N}_i$ ($i = 1, 2, 3$), which also give a contribution to the flavour asymmetries, and of the supersymmetric partners of the lepton doublets, the slepton doublets. Since the effects of supersymmetry breaking may be safely neglected, the flavour CP asymmetries in the MSSM are twice those in the SM and double is also the possible channels by which a lepton flavour asymmetry is reproduced. However, the $\Delta L = 1$ scatterings washing out the asymmetries are also doubled and the number of relativistic degrees of freedom is almost twice the one for the SM case. As a result, introducing new degrees of freedom and interactions does not appreciably change the flavour asymmetries. There are however two other and important differences with respect to the nonsupersymmetric thermal leptogenesis SM case. First, in the MSSM, the flavour-independent formulae can only be applied for temperatures larger than $(1+\tan^2\beta) \times 10^{12}$ GeV. Indeed, the squared charged lepton Yukawa couplings in the MSSM are multiplied by $(1+\tan^2\beta)$. Consequently, the $\mu$ and $\tau$ lepton Yukawa couplings are in thermal equilibrium for $(1+\tan^2\beta) \times 10^5$ GeV $\ll T \ll (1+\tan^2\beta) \times 10^9$ GeV and all flavours in the Boltzmann equations are to be treated separately. For $(1+\tan^2\beta) \times 10^9$ GeV $\ll T \ll (1+\tan^2\beta) \times 10^{12}$ GeV, only the $\tau$ Yukawa coupling is in equilibrium and only the $\tau$ flavour is treated separately in the Boltzmann equations, while the $e$ and $\mu$ flavours are indistinguishable. Secondly, the relation between the baryon asymmetry $Y_B$ and the lepton flavour asymmetries has to be modified to account for the presence of two Higgs fields. Between $(1+\tan^2\beta) \times 10^5$ and $(1+\tan^2\beta) \times 10^9$ GeV, the relation is

$$Y_B \simeq -\frac{10}{31g_*} \left[ \epsilon_e \eta \left( \frac{93}{110}\widetilde{m}_e \right) + \epsilon_\mu \eta \left( \frac{19}{30}\widetilde{m}_\mu \right) + \epsilon_\tau \eta \left( \frac{19}{30}\widetilde{m}_\tau \right) \right], \tag{14}$$

where the flavour lepton asymmetries are computed including leptons and sleptons. Between $(1+\tan^2\beta) \times 10^9$ and $(1+\tan^2\beta) \times 10^{12}$ GeV, the relation is

$$Y_B \simeq -\frac{10}{31g_*} \left[ \epsilon_2 \eta \left( \frac{541}{761}\widetilde{m}_2 \right) + \epsilon_\tau \eta \left( \frac{494}{761}\widetilde{m}_\tau \right) \right], \tag{15}$$

where the number of relativistic degrees of freedom is counted by $g_* = 228.75$. The observed value is $Y_B = (8.7 \pm 0.3) \times 10^{-11}$ [8]. Let us explain the various terms entering

in Eqs. (14) and (15). The CP-asymmetry in each flavour is given by [23, 26, 25, 28]

$$\epsilon_\ell = -\frac{3M_1}{8\pi v_u^2} \frac{\text{Im}\left(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U_{\ell\beta}^* U_{\ell\rho} R_{1\beta} R_{1\rho}\right)}{\sum_\beta m_\beta |R_{1\beta}|^2}, \quad \ell = e, \mu, \tau, \tag{16}$$

where the (in general complex) orthogonal matrix $R$ [4] is defined as

$$Y_\nu = \frac{\sqrt{\hat{M}}}{v_u} R \sqrt{\hat{m}} U^\dagger. \tag{17}$$

Similarly, one defines a "wash-out mass parameter" for each flavour $\ell$ [23, 26, 25, 28]:

$$\left(\frac{\widetilde{m}_\ell}{2 \sin^2\beta \times 10^{-3}\,\text{eV}}\right) \equiv \frac{\Gamma(N_1 \to H\,\ell)}{H(M_1)}, \quad \widetilde{m}_\ell \equiv \frac{|Y_{\nu 1\ell}|^2 \, v_u^2}{M_1} = \left|\sum_k R_{1k} m_k^{1/2} U_{\ell k}^*\right|^2. \tag{18}$$

The quantity $\widetilde{m}_\ell$ parametrizes the decay rate of $N_1$ to the leptons of flavour $\ell$. Furthermore, in Eq. (15), $\epsilon_2 = \epsilon_e + \epsilon_\mu$ and $\widetilde{m}_2 = \widetilde{m}_e + \widetilde{m}_\mu$. Finally,

$$\eta\left(\widetilde{m}_\ell\right) \simeq \left[\left(\frac{\widetilde{m}_\ell}{8.25 \times 10^{-3}\,\text{eV}}\right)^{-1} + \left(\frac{0.2 \times 10^{-3}\,\text{eV}}{\widetilde{m}_\ell}\right)^{-1.16}\right]^{-1} \tag{19}$$

parametrizes the wash-out suppression of the asymmetry due to $\Delta L = 1$ inverse decays and scatterings. Notice that the wash-out masses $\widetilde{m}_2$ and $\widetilde{m}_\tau$ in Eq. (14) are multiplied by some numerical coefficients which account for the dynamics involving the lepton doublet asymmetries and the asymmetries stored in the charges $\Delta_\ell = (1/3)B - L_\ell$ [28].

# 4  Maximizing $d_e$ with the constraint from $\mu \to e\gamma$ and impact on thermal leptogenesis

In this section our goal is to identify those patterns within the seesaw models which predict $d_e$ at hand of future experiments, while keeping the prediction for LFV decays, in particular $\mu \to e\gamma$, below the present bound. In this respect, the analysis we perform is model-independent. From the discussion of the previous section, it is clear that to have $d_e$ at hand of future experiments, the quantities $I_e$ defined in Eq. (9) have to be maximized. Our aim is also to understand whether these models can explain the observed baryon asymmetry of the universe via $N_1$ decays. Our work differs from that of Ref. [11] in two respects. First, our analysis includes also the FV contribution to the electron EDM, which is dominant for large values of $\tan\beta$ and, secondly, we include flavour effects in computing the baryon asymmetry from leptogenesis.

We choose to work with not too heavy RH neutrino $N_1$, say $M_1$ in the range $(10^{10} - 10^{11})$ GeV. This choice has the advantage that, barring accidental cancelations, the couplings of $N_1$ are rather small, $Y_{\nu 1j} \lesssim x = \mathcal{O}(10^{-2})$. This has important consequences: i) $C_{21}^1$ does not exceed the $\mu \to e\gamma$ bound; ii) the eventual contributions to $d_e$ from $I_e^{(31),(21)}$ (both FV and FC) are much smaller than the limit inferred from experiment and can be neglected in first approximation. The only potential source of a large $d_e$ is therefore

$$I_e^{(32)\mathrm{FC}} \ln_2^{\mathrm{Pl}} \approx I_e^{(32)\mathrm{FV}} \approx I_4 \ln_3^{\mathrm{Pl}} \ln_2^{\mathrm{Pl}} \ln_2^3 \quad , \qquad I_4 = \mathrm{Im}(Y_{\nu 31}^* Y_{\nu 33} Y_{\nu 21} Y_{\nu 23}^*) , \qquad (20)$$

which is maximized requiring that:

1) the splitting between $M_3$ and $M_2$ is large;
2) the four Yukawas appearing in Eq. (20) are of $\mathcal{O}(1)$, but satisfy the bound from $\tau \to e\gamma$.

Barring conspiracies between $C_{21}^2$ and $C_{21}^3$, to suppress $\mu \to e\gamma$ below the present bound one must require both to be smaller than $C_{21}^{\mathrm{ub}}$. Since $C_{21}^k \propto Y_{\nu k2}^* Y_{\nu k1}$, in order to keep $I_e^{(32)}$ large, one has to impose $|Y_{\nu 32}|, |Y_{\nu 33}| \lesssim y \approx C_{21}^{\mathrm{ub}}/5$. Explicitly, we are looking for textures of the form

$$Y_\nu = \begin{pmatrix} \lesssim x & \lesssim x & \lesssim x \\ \mathcal{O}(1) & \lesssim y & \mathcal{O}(1) \\ \mathcal{O}(1) & \lesssim y & \mathcal{O}(1) \end{pmatrix} , \qquad (21)$$

where, as already mentioned, $x = \mathcal{O}(10^{-2})$ ensures that $C_{21}^1$ satisfies the upper bound on $\mu \to e\gamma$. Notice also that this pattern enhances $\tau \to e\gamma$ while suppressing $\tau \to \mu\gamma$.

It is convenient to exploit the complex orthogonal matrix $R$ [4] introduced in Eq. (17) and adopt a standard parameterization for the MNS mixing matrix

$$U = O_{23}(\theta_{23}) \, \Gamma_\delta \, O_{13}(\theta_{13}) \, \Gamma_\delta^* \, O_{12}(\theta_{12}) \times \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1) , \qquad (22)$$

where $\Gamma_\delta = \mathrm{diag}(1, 1, e^{i\delta})$, $O_{ij} = [(c_{ij}, s_{ij})(-s_{ij}, c_{ij})]$ with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. The Dirac and Majorana type phases were denoted by $\delta$ and $\alpha_{1,2}$, respectively. We adopt for $R$ a parameterization in terms of three complex angles $\theta_{ij}^R$:

$$R = O_{12}(\theta_{12}^R) \, O_{13}(\theta_{13}^R) \, O_{23}(\theta_{23}^R) = \begin{pmatrix} c_{12}^R c_{13}^R & s_{12}^R c_{23}^R - c_{12}^R s_{23}^R s_{13}^R & s_{12}^R s_{23}^R + c_{12}^R c_{23}^R s_{13}^R \\ -s_{12}^R c_{13}^R & c_{12}^R c_{23}^R + s_{12}^R s_{23}^R s_{13}^R & c_{12}^R s_{23}^R - s_{12}^R c_{23}^R s_{13}^R \\ -s_{13}^R & -s_{23}^R c_{13}^R & c_{23}^R c_{13}^R \end{pmatrix} . \qquad (23)$$

As already discussed, the $\mu \to e\gamma$ constraint is implemented by requiring $|Y_{\nu 32}|, |Y_{\nu 22}| \lesssim y$. In the approximation $y = 0$ (which turns out to be very satisfactory), this means that $R$ must satisfy respectively the conditions

$$R_{i3}\sqrt{m_3}U_{23}^* + R_{i2}\sqrt{m_2}U_{22}^* + R_{i1}\sqrt{m_1}U_{21}^* = 0 \quad , \qquad i = 3, 2 \; . \qquad (24)$$

Simultaneously we want to enhance

$$I_4 = \frac{M_3 M_2}{v_u^4} \mathrm{Im}\left[(R\sqrt{\hat{m}}\,U^\dagger)_{31}^* (R\sqrt{\hat{m}}\,U^\dagger)_{33} (R\sqrt{\hat{m}}\,U^\dagger)_{21} (R\sqrt{\hat{m}}\,U^\dagger)_{23}^*\right] . \qquad (25)$$

9

A potentially interesting contribution to the electron EDM requires $M_3$ and $M_2$ to be as large as possible and, as already stressed, enough splitted to ensure a large $\ln_2^3$. This also explains why we cannot rely here on resonant leptogenesis [35]. One could have, for instance, $M_1 \simeq M_2 \ll M_3$. But since $M_1$ has to be small (for $\mu \to e\gamma$), this would imply small $M_2$ and consequently a suppressed $I_4$. Instead, the pattern $M_1 \ll M_2 \simeq M_3$ implies $\ln_2^3 \ll 1$. Moreover, any baryon asymmetry resulting from the resonant decays of $N_2$ and $N_3$ can be washed out by $N_1$ (see however [36] and especially [37]). Finally, the situation $M_1 \simeq M_2 \simeq M_3$ would lead to a very suppressed $d_e$ [10, 18].

As we are going to discuss in the next section, the structure of $Y_\nu$ in Eq. (21) determines, to some extent, the form of $R$. From a qualitative point of view, one can already put forward some guesses. The requirement of a large atmospheric angle implies that for some $N_j$ it should be $Y_{\nu j2} \approx Y_{\nu j3}$, but from Eq. (21), this can happen only for $j = 1$. In the case of hierarchical light neutrinos, this implies dominance of $m_3$ by $M_1$, while $M_3$ and $M_2$ will be mainly associated to the lighter masses. Since we aim at $M_2 << M_3$, this means that $M_2$ and $M_3$ will dominate respectively $m_2$ and $m_1$. According to the interpretation of $R$ as a dominance matrix [38], these considerations in particular determine the first row of $R$, which is relevant for leptogenesis, to be approximately $(0, \sqrt{m_2/m_3}, 1 - \mathcal{O}(m_2/m_3))$.

In the following we treat separately the cases of normal and inverted hierarchy for light neutrinos.

## 4.1  Normal hierarchy for light neutrinos

For normal hierarchy (NH) we take $m_3 \approx m_@ = \sqrt{m_3^2 - m_2^2}$, $m_2 \approx m_\odot = \sqrt{m_2^2 - m_1^2}$ and leave $m_1 \ll m_2$ undetermined. It is convenient to rewrite the conditions (24) assuming $R_{i3} \neq 0$,

$$-\frac{R_{i2}}{R_{i3}} = \bar{t} + \frac{R_{i1}}{R_{i3}}\tilde{t} \ , \tag{26}$$

where the quantities $\bar{t}$ and $\widetilde{t}$ depend only on the neutrino parameters at low energy,

$$
\begin{aligned}
\bar{t} &= \sqrt{\frac{m_3}{m_2}}\frac{U_{23}^*}{U_{22}^*} = \sqrt{\frac{m_3}{m_2}}\frac{e^{i\frac{\alpha_2}{2}}t_{23}}{c_{12}} \ [1 + \mathcal{O}(s_{13})] \ , \\[2mm]
\widetilde{t} &= \sqrt{\frac{m_1}{m_2}}\frac{U_{21}^*}{U_{22}^*} = -\sqrt{\frac{m_1}{m_2}} \, t_{12} \, e^{i\frac{\alpha_2 - \alpha_1}{2}} \ [1 + \mathcal{O}(s_{13})] \ ,
\end{aligned}
\tag{27}
$$

with $t_{ij} = \tan\theta_{ij}$. In particular, for a light neutrino spectrum with NH and taking into account the present neutrino oscillation data, $|\bar{t}| \approx 3$ while $|\widetilde{t}| \ll 1$.

Using the above expressions and introducing the reference scale $M_@ = v_u^2/m_@ \approx 5 \times 10^{14}$ GeV for the right-handed neutrino masses, the Yukawa couplings of the second and third rows of $Y_\nu$ are given by

$$Y_{\nu 2i} = \sqrt{\frac{M_2}{M_@}}\left(R_{23}\mathbb{U}_i^* + R_{21}\sqrt{\frac{m_1}{m_3}}\mathbb{W}_i^*\right) \ ,$$

$$Y_{\nu 3i} = \sqrt{\frac{M_3}{M_@}} \left( R_{33} \mathbb{U}_i^* + R_{31} \sqrt{\frac{m_1}{m_3}} \mathbb{W}_i^* \right) , \tag{28}$$

with

$$\mathbb{U}_i = U_{i3} - \frac{U_{23}}{U_{22}} U_{i2} \quad , \quad \mathbb{W}_i = U_{i1} - \frac{U_{21}}{U_{22}} U_{i2} . \tag{29}$$

Notice that $\mathbb{U}_2$ and $\mathbb{W}_2$ correctly vanish, as required by the conditions $Y_{\nu 22}, Y_{\nu 32} = 0$. We also anticipate that, for $m_1 = 0$, one has $I_4 \propto \text{Im}(|\mathbb{U}_1|^2 |\mathbb{U}_3|^2) = 0$ which, in the present framework, leads to a very suppressed $d_e$. For the sake of the following discussion, we report the approximate expressions for $\mathbb{U}_{1,3}$ and $\mathbb{W}_{1,3}$ at first order in $s_{13}$:

$$\mathbb{U}_1 = -t_{23} t_{12} (1 + t_{23} t_{12} s_{13} e^{i\delta}) + s_{13} e^{-i\delta} \quad , \quad \mathbb{W}_1 = e^{i\frac{\alpha_1}{2}} \frac{1}{c_{12}} (1 + s_{13} e^{i\delta} t_{23} t_{12}) ,$$

$$\mathbb{U}_3 = \frac{1}{c_{23}} (1 + s_{13} e^{i\delta} t_{23} t_{12}) \quad , \quad \mathbb{W}_3 = -s_{13} e^{i(\delta + \frac{\alpha_1}{2})} \frac{1}{c_{23} c_{12}} . \tag{30}$$

As an example, for tri-bimaximal mixing one has: $\mathbb{U}_3 = \sqrt{2}$, $\mathbb{U}_1 = -1/\sqrt{2}$, $\mathbb{W}_3 = 0$ and $\mathbb{W}_1 = \sqrt{3/2} \, e^{i\alpha_1/2}$.

From the above expressions, and taking into account the smallness of $\mathbb{W}_3 = \mathcal{O}(U_{e3})$, it turns out that large $Y_{\nu 23}$ requires $R_{23} = \mathcal{O}(1)$ and $M_2 = \mathcal{O}(M_@)$. Then, the condition of large splitting between $M_2$ and $M_3$ implies $M_3 >> M_@$ while large $Y_{\nu 33}$ requires $R_{33} = \mathcal{O}(\sqrt{M_@/M_3}) < 1$. Explicitly, $R_{33} = c_{23}^R c_{13}^R$ but, since the condition for $i = 3$ in Eq. (26), $t_{23}^R = \bar{t} - t_{13}^R \tilde{t}/c_{23}^R$, naturally suggests $c_{23}^R$ to be large[2], the parameter to be suppressed is rather $c_{13}^R$. We then define $c_{13}^R = \chi < 1$ and expand at first order in $\chi$. In particular, from Eq.(28), we have $c_{23}^R \chi \approx \sqrt{M_@/M_3} Y_{\nu 33}/\mathbb{U}_3^*$. The conditions in Eq. (26) get now simplified:

$$\frac{c_{12}^R c_{23}^R + s_{12}^R s_{23}^R}{-c_{12}^R s_{23}^R + s_{12}^R c_{23}^R} + \mathcal{O}\left(\chi \tilde{t}\right) = \bar{t} = t_{23}^R + \frac{1}{\chi c_{23}^R} \tilde{t}. \tag{31}$$

One can then envisage two relevant cases according to the value of $\theta_{12}^R$:

- If $s_{12}^R = \epsilon < 1$, Eq. (31) gives $t_{23}^R \approx -1/\bar{t} + \epsilon$ and $\tilde{t} \approx (\bar{t} + 1/\bar{t}) \chi c_{23}^R$. The latter can be rewritten as

$$\sqrt{\frac{m_1}{m_3}} \sqrt{\frac{M_3}{M_@}} \approx \frac{s_{23}}{s_{12}} |Y_{\nu 33}| = \mathcal{O}(1) . \tag{32}$$

Here, $m_1$ cannot be arbitrarily small, otherwise $M_3$ would exceed the GUT or Planck scale. At first order in $\epsilon$ one has $s_{23}^R \approx -\bar{c} + \epsilon \bar{s}$, $c_{23}^R \approx \bar{s} + \epsilon \bar{c}$, so that

$$R = \begin{pmatrix} \chi & \bar{c} & \bar{s} \\ 0 & \bar{s} & -\bar{c} \\ -1 & \chi \bar{c} & \chi \bar{s} \end{pmatrix} + \mathcal{O}(\epsilon^2, \epsilon\chi, \chi^2) . \tag{33}$$

_____

[2]Clearly, $c_{23}^R$ is small only when $\theta_{23} \approx \pi/2$, but in this case the condition for $i = 3$ in Eq. (26) forces $c_{13}^R$ to be still small, $c_{13}^R \approx -\tilde{t}$.

As expected, since $\bar{c} \approx 1/\bar{t} < 1$ and $\bar{s} \approx 1 - 1/(2\bar{t}^2)$, this structure means that $m_3 \approx m_@$ is dominated by the lightest right-handed neutrino $N_1$, while $m_2 \approx m_\odot$ by $N_2$. Instead, $m_1$ is dominated by the heaviest, $N_3$, which decouples the more $\chi$ is small. Notice that such a precise determination of the first row of $R$ allows to predict leptogenesis, as we are going to discuss. The Yukawa couplings relevant for $d_e$ are

$$Y_{\nu 3i} = \sqrt{\frac{M_3}{M_@}} \left( \bar{s}\chi \mathbb{U}_i^* - \sqrt{\frac{m_1}{m_3}} \mathbb{W}_i^* \right) \;,\; Y_{\nu 2i} = \sqrt{\frac{M_2}{M_@}} \left( -\bar{c}\, \mathbb{U}_i^* + \mathcal{O}(\chi)\sqrt{\frac{m_1}{m_3}} \mathbb{W}_i^* \right),$$
(34)

so that

$$
\begin{aligned}
I_4 &= \frac{M_2}{M_@} \frac{M_3}{M_@} |\bar{c}|^2 \sqrt{\frac{m_1}{m_3}} \operatorname{Im}\left( -\bar{s}\chi \mathbb{W}_1 \mathbb{U}_1^* |\mathbb{U}_3|^2 + \bar{s}\chi \mathbb{W}_3 \mathbb{U}_3^* |\mathbb{U}_1|^2 + \sqrt{\frac{m_1}{m_3}} \mathbb{W}_3^* \mathbb{W}_1 \mathbb{U}_1^* \mathbb{U}_3 \right) \\
&= |Y_{\nu 33} Y_{\nu 31} Y_{\nu 21} Y_{\nu 23}| \left[ -\frac{m_2}{m_3} \frac{1}{t_{23}^2} \sin\alpha_2 + \mathcal{O}(s_{13}\sin\delta) \right].
\end{aligned}
$$
(35)

The first term is the only one present in the limit $|U_{e3}| = 0$ and displays a suppression by a factor $m_\odot/m_@ \simeq 0.17$. For large values of $|U_{e3}|$ and $\delta \sim \pm\pi/2$ (large Dirac-type CP violation at low-energies), the second and third terms can be dominant.

- If $c_{12}^R = \epsilon < 1$, Eq. (31) gives $t_{23}^R \approx \bar{t}\,(1 - \epsilon\,\bar{t})$ and $\widetilde{t} \approx \epsilon\,\bar{t}\,(\bar{t} + 1/\bar{t})\chi c_{23}^R$. The latter can be rewritten as

$$\sqrt{\frac{m_1}{m_3}}\sqrt{\frac{M_3}{M_@}} \approx |\epsilon\,\bar{t}|\;\frac{s_{23}}{s_{12}} |Y_{\nu 33}| = \mathcal{O}(\epsilon).$$
(36)

In this case $m_1$ is allowed to be small and the limit $m_1 \to 0$ can be applied. At first order in $\epsilon$ one has $s_{23}^R \approx \bar{s} - \epsilon\bar{c}$, $c_{23}^R \approx \bar{c} + \epsilon\bar{s}$, leading to

$$R = \begin{pmatrix} 0 & \bar{c} & \bar{s} \\ -\chi & \bar{s} & -\bar{c} \\ -1 & -\chi\bar{s} & \chi\bar{c} \end{pmatrix} + \mathcal{O}(\epsilon^2, \epsilon\chi, \chi^2).$$
(37)

Again, dominance of $N_1$ and $N_2$ has been obtained respectively for $m_@$ and $m_\odot$, while $m_1$ is associated to $N_3$, which decouples the more $\chi$ is small. The Yukawa couplings are now

$$Y_{\nu 3i} = \sqrt{\frac{M_3}{M_@}} \left( \bar{c}\chi \mathbb{U}_i^* - \sqrt{\frac{m_1}{m_3}} \mathbb{W}_i^* \right) \;,\; Y_{\nu 2i} = \sqrt{\frac{M_2}{M_@}} \left( -\bar{c}\, \mathbb{U}_i^* + \mathcal{O}(\chi)\sqrt{\frac{m_1}{m_3}} \mathbb{W}_i^* \right),$$
(38)

and, consequently,

$$
\begin{aligned}
I_4 &= \frac{M_2}{M_@} \frac{M_3}{M_@} |\bar{c}|^2 \sqrt{\frac{m_1}{m_3}} \operatorname{Im}\left( -\bar{c}\chi \mathbb{W}_1 \mathbb{U}_1^* |\mathbb{U}_3|^2 + \bar{c}\chi \mathbb{W}_3 \mathbb{U}_3^* |\mathbb{U}_1|^2 + \sqrt{\frac{m_1}{m_3}} \mathbb{W}_3^* \mathbb{W}_1 \mathbb{U}_1^* \mathbb{U}_3 \right) \\
&= \epsilon\, |Y_{\nu 33} Y_{\nu 31} Y_{\nu 21} Y_{\nu 23}| \left[ -\frac{t_{23}}{c_{12}^3} \sin\frac{\alpha_2}{2} + \mathcal{O}(s_{13}\sin\delta) \right],
\end{aligned}
$$
(39)

which is suppressed the more $\epsilon$ is small. Indeed, as already mentioned, $I_4$ vanishes in the limit $m_1 = 0$ (namely $\epsilon = 0$), as a consequence of the alignment between the second and third rows of $Y_\nu$, which ensures a vanishing eigenvalue. The remaining contributions from $I_e^{(31),(32)}$ do not vanish but, as stressed before, are negligible even considering future sensitivities.

Summarising, it turns out that a large $d_e$ is obtained only under the conditions of the first case, namely if $M_2 \sim M_@$ and the relation $m_1/m_@ \sim M_@/M_3$ holds. The contribution induced by $\delta$ could even exceed the one associated to $\alpha_2$; the phase $\alpha_1$ plays no role. Notice in particular that values of $m_1 \lesssim 0.5 \times 10^{-3} m_3$ are incompatible with an experimentally relevant $d_e$.

We remark that in all the expressions above, the Yukawa couplings, the elements of the matrix $U$ and the light neutrino mass eigenstates are consistently evaluated at $M_{\rm Pl}$. For instance, in the case of NH, the effect of running is such that $\hat{m}$ is rescaled by a numerical factor, while the parameters of $U$ do not change significantly. We illustrate this point by considering a numerical example for the first case discussed previously. If we take the low energy neutrino spectrum consistent with the present-day neutrino oscillation data, $\hat{m} = (0.96 \times 10^{-2}, 0.17, 1) m_@$, then, for $\tan\beta = 30$, $(M_3, M_2) = (10^2, 2) M_@$, $M_1 = 3 \times 10^{11}$ GeV, $\alpha_2 = \pi/2$, $\delta = \pi/2$ and any $\alpha_1$, at the Planckian scale we obtain[3] $\hat{m} = 1.8(10^{-2}, 0.166, 1) m_@$. Correspondingly, the mixing angles change from $\theta_{23} = 47°$, $\theta_{12} = 35.2°$, $\theta_{13} = 9.7°$ to $\theta_{23} = 45°, \theta_{12} = 35°, \theta_{13} = 10°$. With these choices one obtains: $I_e^{\rm FV} = 10.2$, $I_e^{\rm FC} = 1.35$, $C_{31} = 1.5$, $C_{32}/8 = C_{21} = 10^{-3}$. These values are slightly changed taking different values of $\tan\beta$, which enters only through the running effects. This means that, in the point $P$ and for any $\tan\beta$, the present example gives $d_e^{\rm FC} = 1.3 \times 10^{-3} d_e^{exp}$, which may escape detection. Since $d_e^{FV}$ and LFV decays explicitly depend on $\tan\beta$, we need to specify it. Taking for instance $\tan\beta = 30$, one has $d_e^{\rm FV} = 0.022\, d_e^{exp}$, which is at hand of future experiments. With $\tan\beta = 30$, the BRs of $\tau \to e\gamma$, $\tau \to \mu\gamma$ and $\mu \to e\gamma$ turn out to be smaller than their corresponding experimental limits by factors of 0.8, $4 \times 10^{-5}$ and 0.02, respectively.

The left-hand side of Fig. 2 shows the dependence of $I_e^{\rm FV}$ and $Y_B$ on $\alpha_2$ for a given set of values for $\delta$, keeping for the remaining parameters of $U$ and light and heavy neutrino masses the same set as before - in particular we recall that $M_1 = 3 \times 10^{11}$ GeV and $\tan\beta = 30$, but the dependence of $Y_\nu(M_{\rm Pl})$ on $\tan\beta$ is mild. On the right-hand side of the same figure, we show the behaviour of $I_e^{\rm FV}$ and $Y_B$ in the $(\alpha_2, \delta)$-plane. Hence, $I_e^{\rm FV}$ displayed in Fig. 2 slightly changes considering other values of $\tan\beta$. The same applies separately for each asymmetry $\epsilon_\ell$, but not to $Y_B$, for which two different regimes can be identified according to the value of $\tan\beta$. Indeed, having $M_1 = 3 \times 10^{11}$ GeV, in the case that $\tan\beta > 10$ the baryon asymmetry is generated in the range of temperatures

---

[3]In practice we do the opposite, namely we assign $U$ and rescale $\hat{m}$ by a numerical factor at high energy and check that, when evolved at low energy, the parameters of $U$ and the neutrino spectrum are still within the experimental window.
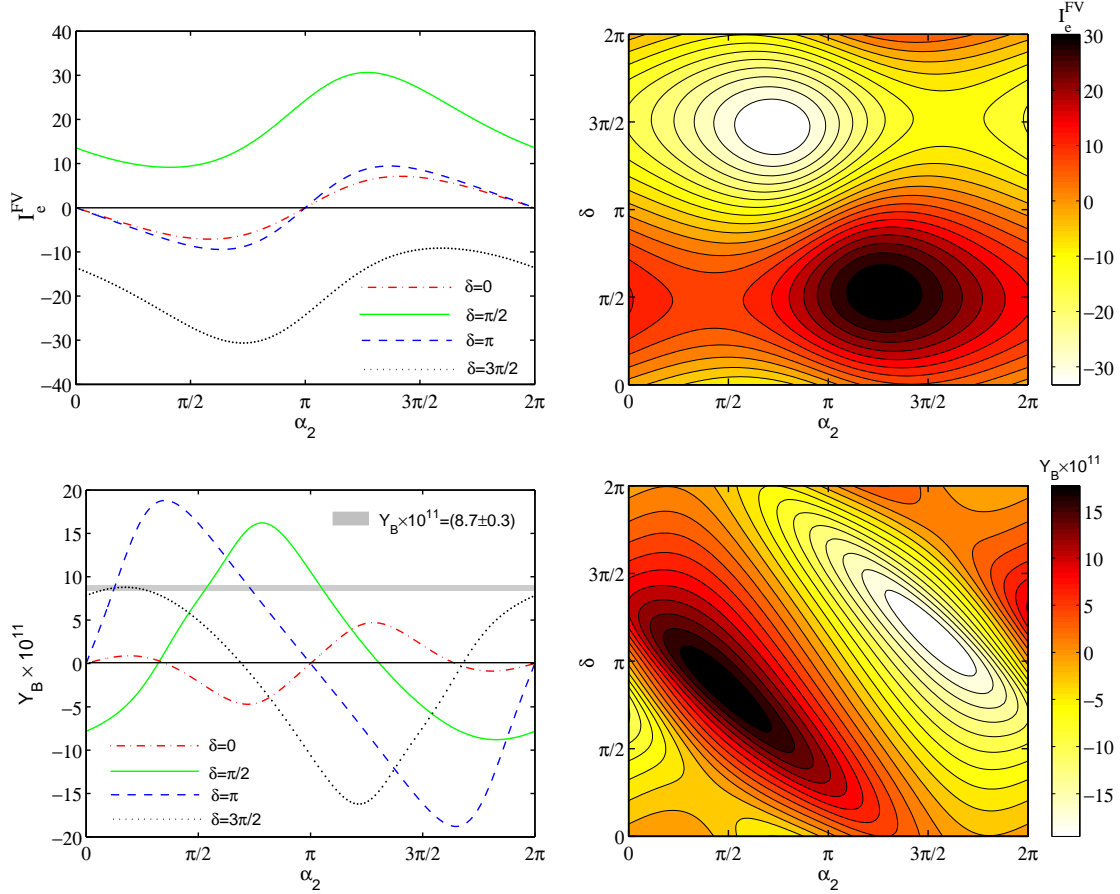
Figure 2: Dependence of $I_e^{\mathrm{FV}}$ (left upper-plot) and $Y_B$ (left lower-plot) on $\alpha_2$ for a given set of values for $\delta$ and $\tan\beta > 10$ ($\mu$ and $\tau$ in equilibrium): we take $\delta = 0, \pi/2, \pi, 3\pi/2$. On the right-hand side we show the behaviour of $I_e^{\mathrm{FV}}$ (upper plot) and $Y_B$ (lower-plot) in the $(\alpha_2, \delta)$-plane. For the choice of the remaining parameters, see the text.

for which all the lepton flavours, but the electron one, are in thermal equilibrium and Eq. (14) applies, see Fig. 2; in the case that $\tan\beta < 10$ thermal leptogenesis takes place in a range of temperatures where only the tau flavour is in thermal equilibrium and, in this case, one has to apply Eq. (15), see Fig. 3. From the comparison of the upper and lower plots of Fig. 2 one can see that there are regions in the $(\alpha_2, \delta)$ parameter space where the seesaw-induced effects can lead to values of $d_e$ within future experimental sensitivities and $Y_B$ is compatible with its observed value. For $\tan\beta < 10$ the baryon asymmetry is on the contrary below its observed value (the horizontal grey line) and one would need slightly larger values of the mass $M_1$ to obtain a value of the baryon asymmetry consistent with observation. This seems to be a generic conclusion. Also notice that, in this temperature regime, $Y_B$ does not depend strongly on $\delta$.

Fig. 5 shows the dependence of $I_e^{FV}$ and $Y_B$ on $m_1/m_3$ (left and right panel respectively), keeping $m_1/m_3 = M_\circledcirc/M_3$ as in the previous example, as well as the same set for the remaining parameters. We also display (dashed line) the baryon asymmetry com-
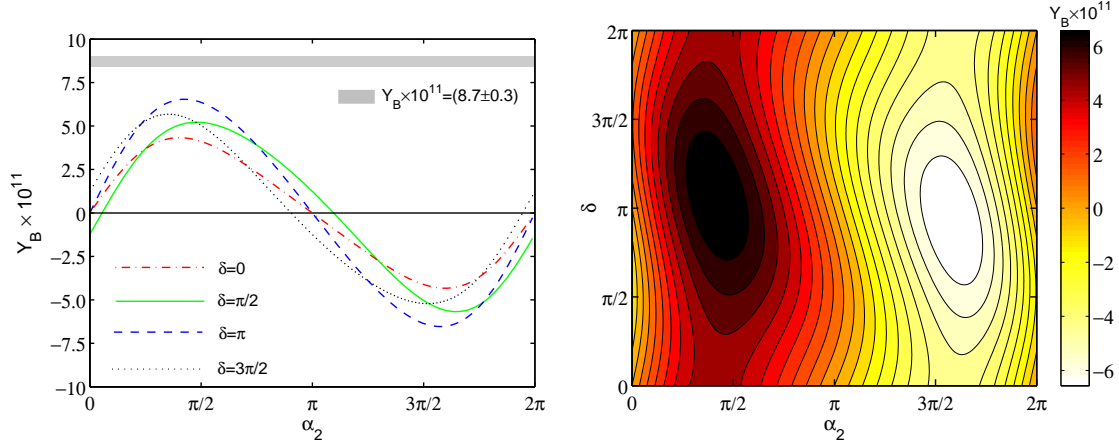
Figure 3: The same as in Fig. 2 for $Y_B$ taking $\tan\beta < 10$ (only $\tau$ in equilibrium).

puted in the one-single flavour approximation. We see that there is almost one order of magnitude difference.

## 4.2 Inverted hierarchy for light neutrinos

For inverted hierarchy (IH), we take $m_{2,1} \approx m_{@}$ and leave $m_3$ undetermined. It is convenient to redefine the matrix $R$ according to: $R = O_{12}O_{13}O_{23} \times \text{antidiag}(1,1,1)$. This simple exchange of the first and third columns of $R$ allows to carry on the discussion in a parallel way to what done before. Indeed, assuming also $R_{i1} \neq 0$, the conditions (24) become

$$-\frac{R_{i2}}{R_{i1}} = \widetilde{t} + \frac{R_{i3}}{R_{i1}} \bar{t}, \tag{40}$$

where $\widetilde{t}$ and $\bar{t}$ are defined as in Eq. (27), but now we have $|\widetilde{t}| \approx 1/\sqrt{2}$ while $|\bar{t}| \ll 1$.

Using these expressions, the Yukawa couplings of the second and third rows are given by

$$Y_{\nu 2i} = \sqrt{\frac{M_2}{M_{@}}} \left( R_{21} \mathbb{W}_i^* + R_{23}\sqrt{\frac{m_3}{m_1}} \mathbb{U}_i^* \right) ,$$

$$Y_{\nu 3i} = \sqrt{\frac{M_3}{M_{@}}} \left( R_{31} \mathbb{W}_i^* + R_{33}\sqrt{\frac{m_3}{m_1}} \mathbb{U}_i^* \right) , \tag{41}$$

which, as before, correctly vanish for $i = 2$. Large $Y_{\nu 21}$ requires $M_2 = \mathcal{O}(M_{@})$, so that to maximize $d_e$ we require $M_3 \gg M_{@}$. Instead, large $Y_{\nu 31}$ is obtained provided $R_{13} = \mathcal{O}(\sqrt{M_{@}/M_3}) < 1$. Explicitly, $R_{31} = c_{23}^R c_{13}^R$, but the condition for $i = 3$ of Eq. (40), $t_{23}^R = \widetilde{t} - t_{13}^R \bar{t}/c_{23}^R$, suggests that $c_{23}^R$ is quite large. We rather suppress $c_{13}^R = \chi < 1$ and expand at first order in $\chi$. In particular, now $c_{23}^R \chi \approx \sqrt{M_{@}/M_3}Y_{\nu 31}/\mathbb{W}_1^*$ and the conditions in (40) get simplified:

$$\frac{c_{12}^R c_{23}^R + s_{12}^R s_{23}^R}{-c_{12}^R s_{23}^R + s_{12}^R c_{23}^R} + \mathcal{O}(\chi\bar{t}) = \widetilde{t} = t_{23}^R + \frac{1}{\chi c_{23}^R} \bar{t} . \tag{42}$$

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It is useful to distinguish between two relevant cases.

- If $s_{12}^R = \epsilon < 1$, Eq. (42) gives $t_{23}^R = -1/\tilde{t} + \epsilon$ and $\bar{t} \approx (\tilde{t} + 1/\tilde{t})\chi c_{23}^R$. The latter can be rewritten as

$$\sqrt{\frac{m_3}{m_2}}\sqrt{\frac{M_3}{M_{@}}} = \mathcal{O}(1). \tag{43}$$

Notice that in this case $m_3$ cannot be arbitrarily small, otherwise $M_3$ would exceed the GUT or Planck scale. In addition

$$R = \begin{pmatrix} \tilde{s} & \tilde{c} & \chi \\ -\tilde{c} & \tilde{s} & 0 \\ \tilde{s}\chi & \chi\tilde{c} & -1 \end{pmatrix} + \mathcal{O}(\epsilon^2, \epsilon\chi, \chi^2). \tag{44}$$

The large masses $m_2$ and $m_1$ are dominated by $N_1$ and $N_2$ with competitive strenght[4]. In particular, the first row of $R$ determines leptogenesis, as we are going to discuss. The smallest mass $m_3$ is instead dominated by $N_3$, which decouples the more $\chi$ is small. The Yukawas are

$$Y_{\nu 3i} = \sqrt{\frac{M_3}{M_{@}}}\left(\tilde{s}\chi\mathbb{W}_i^* - \sqrt{\frac{m_3}{m_1}}\mathbb{U}_i^*\right) \quad , \quad Y_{\nu 2i} = \sqrt{\frac{M_2}{M_{@}}}\left(-\tilde{c}\mathbb{W}_i^* + \mathcal{O}(\chi\epsilon)\sqrt{\frac{m_3}{m_1}}\mathbb{U}_i^*\right), \tag{45}$$

so that

$$I_4 = \frac{M_2}{M_{@}}\frac{M_3}{M_{@}}|\tilde{c}|^2\sqrt{\frac{m_3}{m_1}}\,\text{Im}\left(-\tilde{s}\chi\mathbb{U}_1\mathbb{W}_1^*|\mathbb{W}_3|^2 + \tilde{s}\chi\mathbb{U}_3\mathbb{W}_3^*|\mathbb{W}_1|^2 + \sqrt{\frac{m_3}{m_1}}\mathbb{U}_3^*\mathbb{U}_1\mathbb{W}_1^*\mathbb{W}_3\right), \tag{46}$$

which turns out to be proportional to $U_{e3}$.

- If $c_{12}^R = \epsilon < 1$, one has $t_{23}^R = \tilde{t}(1 - \epsilon\tilde{t})$ and $\bar{t} \approx \epsilon\tilde{t}(\tilde{t} + 1/\tilde{t})\chi c_{23}^R$, which in turn implies

$$\sqrt{\frac{m_3}{m_2}}\sqrt{\frac{M_3}{M_{@}}} = \mathcal{O}(\epsilon) \tag{47}$$

In this case $m_3$ is allowed to be small and the limit $m_3 \to 0$ can be applied. In addition

$$R = \begin{pmatrix} \tilde{s} & \tilde{c} & 0 \\ -\tilde{c} & \tilde{s} & -\chi \\ \tilde{c}\chi & -\chi\tilde{s} & -1 \end{pmatrix} + \mathcal{O}(\epsilon^2, \epsilon\chi, \chi^2). \tag{48}$$

The Yukawas are

$$Y_{\nu 3i} = \sqrt{\frac{M_3}{M_{@}}}\left(\tilde{c}\chi\mathbb{W}_i^* - \sqrt{\frac{m_3}{m_1}}\mathbb{U}_i^*\right) \quad , \quad Y_{\nu 2i} = \sqrt{\frac{M_2}{M_{@}}}\left(-\tilde{c}\mathbb{W}_i^* - \mathcal{O}(\chi)\sqrt{\frac{m_3}{m_1}}\mathbb{U}_i^*\right), \tag{49}$$

---

[4]Assuming for instance $\alpha_1 = \alpha_2$ and tri-bimaximal mixing one has $\tilde{c} = \sqrt{2/3}$, $\tilde{s} = \sqrt{1/3}$; while for $\alpha_2 - \alpha_1 = \pi$ one has $\tilde{c} = \sqrt{2}$, $\tilde{s} = -i$.

so that

$$I_4 = \frac{M_2}{M_@} \frac{M_3}{M_@} |\tilde{c}|^2 \sqrt{\frac{m_3}{m_1}} \mathrm{Im} \left( -\tilde{c}\chi \mathbb{U}_1 \mathbb{W}_1^* |\mathbb{W}_3|^2 + \tilde{c}\chi \mathbb{U}_3 \mathbb{W}_3^* |\mathbb{W}_1|^2 + \sqrt{\frac{m_3}{m_1}} \mathbb{U}_3^* \mathbb{U}_1 \mathbb{W}_1^* \mathbb{W}_3 \right),$$

$$(50)$$

which is again proportional to $U_{e3}$ and suppressed by a factor of $\epsilon$ with respect to (46). Hence, it vanishes in the limit $m_3 = 0$ due to the fact that the second and third rows of $Y_\nu$ are aligned. The contributions from $I_e^{(31),(32)}$ are negligible even for the future sensitivities.

Summarizing, in the inverted-hierarchical case $d_e$ can be at hand of future experiments only if $M_2 \sim M_@$, $m_3/m_@ \sim M_@/M_3$ and $|U_{e3}|$ is large. The dependence on $\alpha_2$ and $\delta$ is quite complicated and also $\alpha_1$ plays a role.

With an IH light neutrino spectrum, the RGE effects might be important, in particular for the solar angle and mass squared difference. Nevertheless, the above conclusions remain valid. In practice, one has just to rescale $\hat{m}$ and $m_\odot$ at high energy by a numerical factor, and check whether the parameters of $U$, when evolved at low energy, fall within the experimental window. It is well known [39] that this is always the case if $|\alpha_2 - \alpha_1| = \pi$, namely if the solar pair of eigenstates are of pseudo-Dirac type. Then, $I_e^{\mathrm{FV}}$ and all the $C_{ij}$ are mildly dependent on $\alpha_2$ and essentially depend only on $\delta$. This will turn out to be the case also for $Y_B$.

We illustrate this considering an example of the first case where the RGE effects turn out to be quite small. We consider $\tan \beta = 30$ and take at $M_{\mathrm{Pl}}$: $\hat{m} = 1.5(1, 0.97, 10^{-2})m_@$, $(M_3, M_2) = (8, 0.1)M_@$, $M_1 = 10^{11}$ GeV, $\theta_{23} = 45°, \theta_{12} = 35°, \theta_{13} = 10°$, $\delta = 3\pi/2$, and any $\alpha_2 = \alpha_1 + \pi$. At low energy, the neutrino spectrum is viable, $\hat{m} = (1, 0.98, 10^{-2})m_@$, and the angles of the MNS are $\theta_{23} = 44.4°$, $\theta_{12} = 35°$, $\theta_{13} = 10.4°$. We obtain $I_e^{\mathrm{FV}} = -4.8$, $I_e^{\mathrm{FC}} = -0.45$, $C_{31} = 1.4$, $C_{32}/4 = C_{21} = 3 \times 10^{-3}$. Hence, for the point $P$, $d_e^{\mathrm{FC}} = -5 \times 10^{-4} d_e^{exp}$. With $\tan \beta = 30$, the FV contribution to $d_e$ is at hand of future experiments $d_e^{\mathrm{FV}} = 0.01 d_e^{exp}$; as for the BR of $\tau \to e\gamma$, $\tau \to \mu\gamma$, $\mu \to e\gamma$, they are smaller than their corresponding experimental limits by factors of $0.7$, $4 \times 10^{-7}$, $0.15$, respectively.

Fig. 4 shows the dependence of $I_e^{FV}$ and $Y_B$ on $\delta$, keeping for the remaining parameters the same set as before. The curves are absent for $\delta$ very small or close to $2\pi$, where $C_{31} > 10$ because some of the Yukawas in $Y_\nu$ blow up. Since both $Y_B$ and $I_e^{\mathrm{FV}}$ are directly proportional to $|U_{e3}|$, the plot can be adapted correspondingly to other values of $\theta_{13}$. For a sizeable $d_e$ and $Y_B$, $\theta_{13}$ cannot be smaller than a few degrees. The value chosen for $M_1$ is $10^{11}$ GeV which corresponds to the regime of temperatures where all lepton flavours, but the electron one, are in thermal equilibrium, since $\tan \beta = 30$. In this case, Eq. (14) applies. We have checked that for smaller values of $\tan \beta$, where only the tau is in equilibrium, the final baryon asymmetry does not change significantly.

In Fig. 5 we show the dependence of $I_e^{\mathrm{FV}}$ and $Y_B$ on $m_3/m_2$ (left and right panels, respectively), while keeping $m_3/m_2 = 0.08 \, M_@/M_3$ as in the previous example and assuming
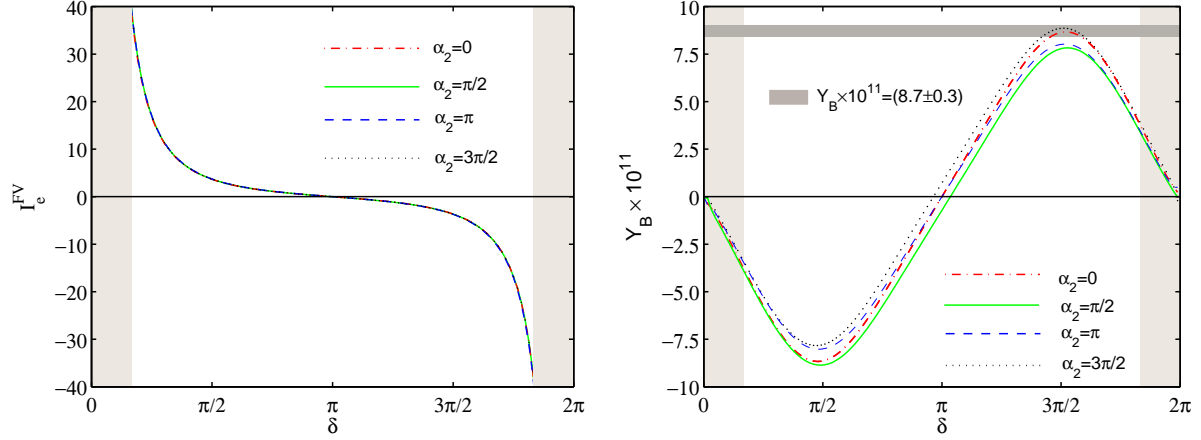
Figure 4: Dependence of $I_e^{\mathrm{FV}}$ and $Y_B$ on $\delta$ for $\alpha_2 = \alpha_1 + \pi$: we take $\alpha_2 = 0, \pi/2, \pi, 3\pi/2$. The vertical light-grey regions are excluded by requiring perturbative $Y_\nu$. For the choice of the other parameters, see the text.
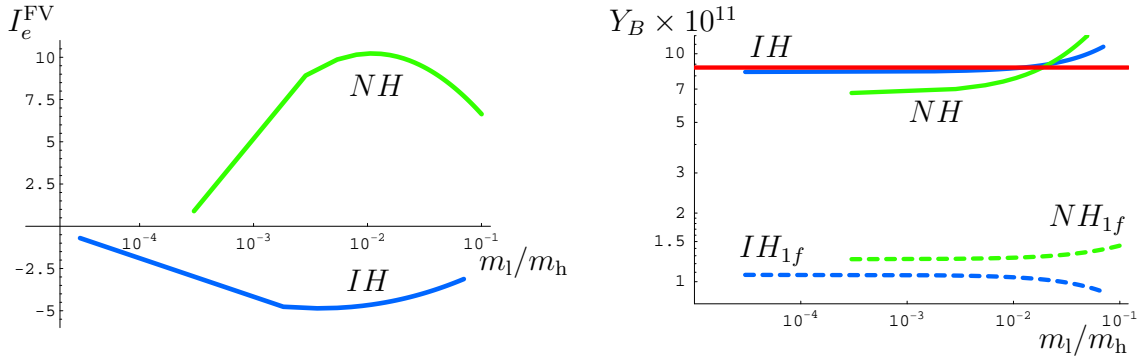


Figure 5: Dependence of $I_e^{\mathrm{FV}}$ and $Y_B$ on $m_{\mathrm{l}}/m_{\mathrm{h}}$. For normal hierarchy $m_{\mathrm{l}}/m_{\mathrm{h}} = m_1/m_3$, while inverted hierarchy $m_{\mathrm{l}}/m_{\mathrm{h}} = m_3/m_2$. For the choice of the other parameters, see the text. The dashed lines are the result of the 1-flavour approximation.

the same set for the remaining parameters (in particular we selected $\alpha_2 = 0$, $\alpha_1 = \pi$). On the right-hand side we compare the baryon asymmetry computed with the flavour effects included to the baryon asymmetry computed in the one-single flavour approximation. We see that there is almost one order of magnitude difference, both in the normal and in the inverted hierarchical case. The main reason is that in the one-flavour case the total lepton asymmetry is strongly washed out since the wash-out parameter $\widetilde{m} = \widetilde{m}_e + \widetilde{m}_\mu + \widetilde{m}_\tau$ is larger than $m_@$, yielding a suppressed baryon asymmetry. On the contrary, in the flavour approximation, the asymmetries in the electron and muon flavours are only weakly washed out. Furthermore, in the inverted hierarchy case, the one-flavour approximation leads to a suppression in the CP asymmetry that goes like $m_\odot^2/m_@$, while including flavours the individual CP asymmetries go as $m_@$.

# 5 Conclusions

The predictions for the electron EDM in the context of the supersymmetric seesaw and mSUGRA have been investigated. First, we showed the existence of an indirect upper bound on $d_e$ from negative searches for $\tau \to e\gamma$, within the supersymmetric seesaw. This indirect bound may be even stronger than the present experimental direct upper limit. Planned searches for $d_e$, improving the sensitivity by about three orders of magnitude, will be able to supersede the indirect bound from $\tau \to e\gamma$ and provide considerable tests of the seesaw-induced effects. We identified in a model-independent way the patterns of seesaw models that lead to a potentially observable electron EDM, considering in turn the case of normal and inverted light neutrino spectra. A widely splitted spectrum for right-handed neutrinos is a crucial ingredient, as well as the relation $m_l/m_h \sim M_@/M_3$, where $M_@ = v_u^2/m_@$. Indeed, in the limit of a vanishing lightest neutrino mass, the electron EDM drops much below the planned sensitivities.

The seesaw interactions may also be responsible for the generation of the baryon asymmetry of the Universe via the mechanism of leptogenesis. The patterns of seesaw models identified requiring an electron EDM within future experimental sensitivities allow to extract the value of $M_1$ required to generate a sufficiently large baryon asymmetry via thermal leptogenesis. The importance of taking into account flavour effects has been emphasized. Our findings show that a large enough baryon asymmetry may be achieved through thermal leptogenesis for those patterns which give rise to a large electric EDM and suppressed $\tau \to e\gamma$, $\tau \to \mu\gamma$, $\mu \to e\gamma$ branching ratios. However, a sufficiently large baryon asymmetry is reached only for large values of $M_1$, at least larger than about $10^{11}$ GeV. Since we are dealing with a supersymmetric leptogenesis set-up, we should face the problem arising from the so-called gravitino bound. The latter is posed by the possible overproduction of gravitinos during the reheating stage after inflation, see for instance [40]. Being only gravitationally coupled to the SM particles, gravitinos may decay very late jeopardising the successfull predictions of Big Bang nucleosynthesis. This does not happen, however, if gravitinos are not efficiently generated during reheating, that is if the reheating temperature $T_{RH}$ is bounded from above, $T_{RH} \lesssim 10^{10}$ GeV [40]. The severe bound on the reheating temperature makes the generation of the RH neutrinos problematic (for complete studies see [19, 32]), if the latter are a few times heavier than the reheating temperature, rendering the thermal leptogenesis scenario unviable.

In view of the above, if a large $d_e$ is measured in the near future and assuming that its main contribution comes from CP-violating effects induced by the see-saw Yukawas, either the baryon asymmetry is not explained within the thermal leptogenesis scenario or leptogenesis occurs in a non-thermal way. This second alternative stems from the fact that the RH neutrinos might be generated not through thermal scatterings, but by other mechanisms, for example during the preheating stage [41], from the inflaton decays [41, 42] or quantum fluctuations [43]. In these cases, the baryon asymmetry depends

crucially on the abundance of RH neutrinos and sneutrinos generated non-thermally. For instance, these heavy states may be produced very efficiently during the first oscillations of the inflaton field during preheating up to masses of order $(10^{17} - 10^{18})$ GeV [41]; the final baryon asymmetry is generated of the right-order of magnitude if the flavour lepton asymmetries satisfy the mild condition $\epsilon_\ell \gtrsim 10^{-8}(10^{10}\,\text{GeV}/T_{RH})(M_1/10^{11}\,\text{GeV})$, as one can readily deduce from Ref. [41].

## Acknowledgements

# A    Constraints from LFV e EDM

In Fig. 6 we display $C_{ij}^{\mathrm{ub}}$, $I_e^{\mathrm{ubFC}}$ and $I_e^{\mathrm{ubFV}}$ from the present 90% C.L. limits [44], $\mathrm{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11}$ , $\mathrm{BR}(\tau \to \mu\gamma) < 6.8 \times 10^{-8}$ , $\mathrm{BR}(\tau \to e\gamma) < 1.1 \times 10^{-7}$. The upper bounds $I_e^{\mathrm{ubFC}}$ and $I_e^{\mathrm{ubFV}}$ are also shown taking $d_e < 1.6 \times 10^{-27}$ e cm. We consider mSugra with $a_0 = m_0 + M_{1/2}$. For better sensitivities, the values of $C_{ij}^{\mathrm{ub}}$ have to be multiplied by a factor $\sqrt{\mathrm{BR^{fut}/BR^{pr}}}$, while those of $I_e^{\mathrm{FCub}}$ and $I_e^{\mathrm{FVub}}$ by a factor $d_e^{\mathrm{fut}}/d_e^{\mathrm{pr}}$. We recall that $\tilde{M}_1 \simeq 0.4 M_{1/2}$ and $\bar{m}_R^2 \simeq m_0^2 + 0.15 M_{1/2}^2$. The plots are adapted from those in [14], where the reader can find more details and references to the literature.
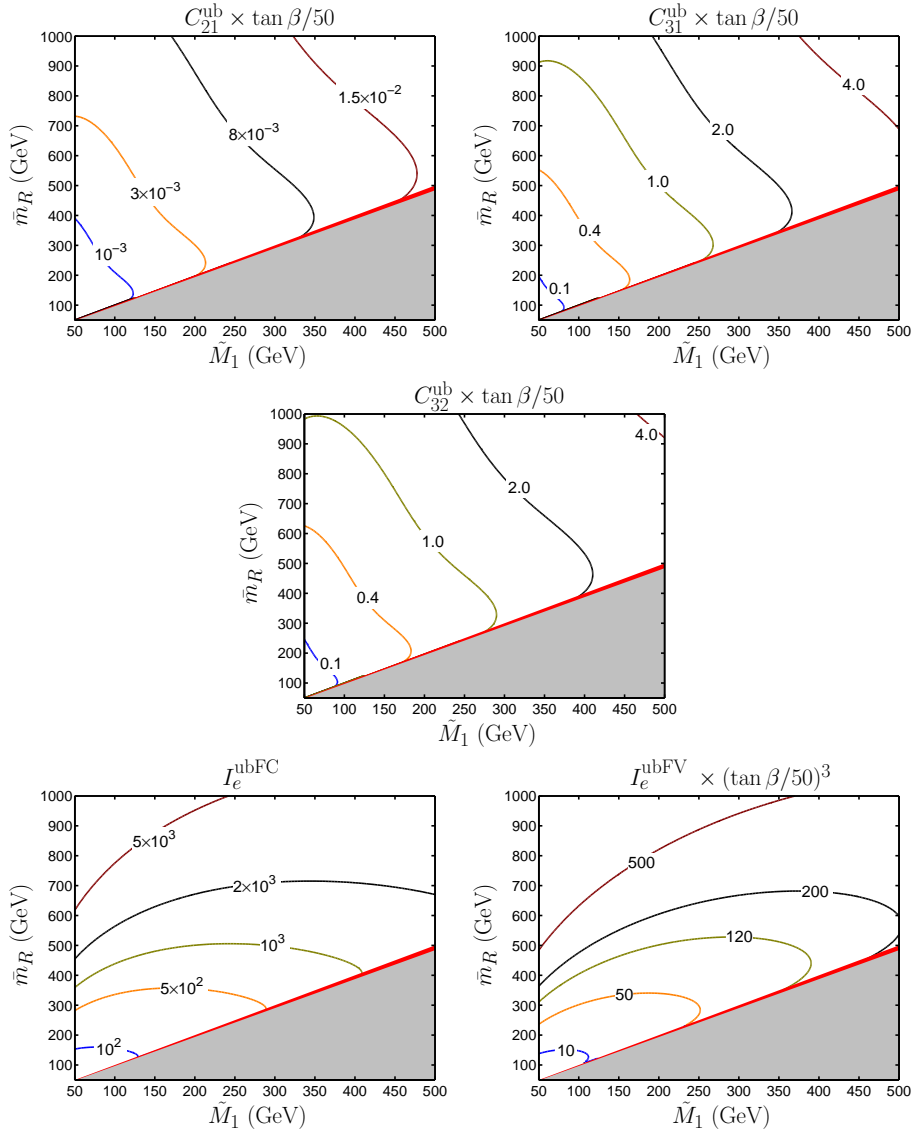


Figure 6: Contours of $C_{ij}^{\mathrm{ub}}$ (upper plots), from the present 90% C.L. limits [44]. The lower plots show the contours of $I_e^{\mathrm{ubFC}}$ and $I_e^{\mathrm{ubFV}}$ taking $d_e < 1.6 \times 10^{-27}$ e cm.

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