II. MICROWAVE GASEOUS DISCHARGES

Prof. S. C. Brown
Prof. W. P. Allis

Dr. M. P. Madan
S. J. Buchsbaum

E. Gordon
J. J. McCarthy

A. PLASMA OSCILLATIONS

The usual approach to the study of plasma oscillations is to postulate a potential wave which propagates through the plasma and interacts with the plasma and beam electrons. The analysis is similar to that used for double or slipping-stream amplifiers. The usual result of the analysis is a dispersion relation which predicts, under certain circumstances, a growing wave that will give rise to amplification, and, if feedback is provided, to oscillations (1).

It was recently pointed out that the growing wave is not real; its apparent reality is caused by a misinterpretation of the dispersion relation. Piddington (2) states that the apparent growth appears because the observer views a cut-off wave emanating from a boundary moving with respect to the observer. The wave seems to grow because it is viewed progressively closer to the moving boundary and hence has less attenuation. The following analysis will show that it is possible to explain plasma oscillations without recourse to a wave theory.

Pines and Bohm (3) analyze the plasma by expanding the electron density, which is periodic in a unit volume, in a Fourier series. They show that the fluctuations in density consist of two components. One is a collective oscillation associated with the long-range part of the Coulomb force whose net effect is to shield the particle so that the field extends only to the Debye radius. The other is a random part which represents the unorganized motion of the plasma electrons. These fluctuations determine the forces on a given electron.

Making use of this approach, they calculate the response of the plasma to an electron moving with a velocity v ($v > \langle v^2 \rangle^{1/2}$, the rms velocity of the plasma electrons). If the electron starts at z = 0 at time t = 0 and moves along the positive z-axis, the fluctuation density, $q(\bar{x}, v, t)$, is given by

$$q(\bar{x}, v, t) = -\frac{\omega_p^2}{4\pi^2(v^2 - \langle v^2 \rangle)} \iint \frac{dk_x \, dk_y}{k_z^S} \exp i(k_x x + k_y y) \sin k_z^S(z - vt)$$

$$k_z^{S2} = \left(\omega_p^2 + \left(k_x^2 + k_y^2\right) \langle v^2 \rangle\right) / (v^2 - \langle v^2 \rangle)$$

The integral is zero for z > vt; the limits are

$$0 < k_x^2 + k_y^2 < 2/3 \; 1/\lambda_o^2 = 2\omega_p^2 / \langle v^2 \rangle$$

where λ_o is the Debye radius.

If we assume that a beam of electrons of current i_0 is moving along the z-axis and that an electron starting at time $t = t_0$ has a velocity

$$v(t_0) = v_0 + v_1 \cos \omega t_0$$

the total response of the plasma is given by

$$Q(\bar{x}, t) = i_0/e \int_{-\infty}^{t' = t - \frac{z}{v(t')}} dt_0 \, q\left(\bar{x}, v(t_0), t - t_0\right) \exp\left[-a\left(t - \frac{z}{v(t_0)} - t_0\right)\right]$$

where a is a small damping caused by collisions. The result of performing this integration is too lengthy to write here. However, the essentials are easily outlined. $Q(\bar{x}, t)$ is sinusoidal in space and time. The coefficients of the terms $\sin \omega\left(t - \frac{z}{v_0}\right)$ and $\cos \omega\left(t - \frac{z}{v_0}\right)$ are of the order v_1/v_0, which is taken to be small, and one of the coefficients is proportional to z. The coefficients are large only when

$$\omega^2 = \omega_p^2 + \omega^2 \langle v^2 \rangle / v_0^2$$

It appears that a premodulated beam of electrons entering a plasma can drive the plasma oscillations, if the driving frequency satisfies the above relation which is identical to the relation derived by Bohm and Gross (4). It is hoped that this result can be checked quantitatively. A tube that allows a premodulated beam of electrons to enter the plasma is nearing completion.

E. Gordon

References

1. E. Gordon, Quarterly Progress Report, Research Laboratory of Electronics, M. I. T., Jan. 15, 1956, pp. 15-16.
2. J. H. Piddington, Phys. Rev. 101, 14 (1956).
3. D. Pines and D. Bohm, Phys. Rev. 85, 338 (1952).
4. D. Bohm and E. Gross, Phys. Rev. 75, 1851 (1949).

B. HIGH-DENSITY MICROWAVE PLASMA

A method has been developed for measuring high-density plasmas by an extension of the microwave method. It is based on the fact that if an electromagnetic field has its electric vector everywhere parallel to the plasma boundary and perpendicular to the density gradient, no disturbing space-charge fields will be induced in the plasma. Space-charge field is the factor that limits the usual microwave method to measurements of relatively low electron densities. If, in addition, the plasma occupies a small region
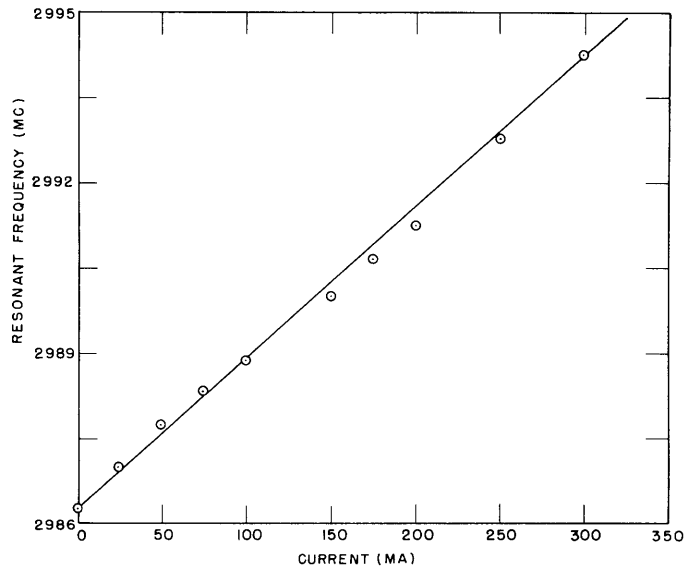
Fig. II-1.   Resonant frequency of the cavity as a
function of the plasma current.

where the field is weak, the field is perturbed very little, and a perturbation calculation may be used.

These conditions are satisfied by a narrow cylindrical plasma with axially symmetric density distribution, introduced coaxially into a cylindrical cavity that resonates in the TE_{011} mode. The field of this mode is given by $E_z = 0$, $E_r = 0$, $E_\phi = E_o J_1(\chi r/a)$ $\sin(\pi z/L)$, where a is the radius of the cavity, L is its length, and $\chi = 3.832$. To illustrate this method, measurements were made on a cylindrical plasma with a diameter of 1.30 cm. The diameter of the cavity was 13.6 cm and it resonated at 3000 mc. Figure II-1 shows the measured shift in the resonant frequency as a function of the current through the plasma tube. Since the voltage across the tube remained approximately constant, the electron density averaged over the cross section of the tube is proportional to the current. It is estimated that the highest electron density reached in this experiment was about 5×10^{11} cm^{-3}. Even for these high densities, corresponding to the plasma frequency $\omega_p \approx 2\omega$, the frequency shift is directly proportional to the electron density in the plasma.

S. J. Buchsbaum

C. PULSE BREAKDOWN IN GASES

To understand the various fundamental processes that occur during the breakdown of a pulse, it is important to know how the electric field varies with the electron

density of the discharge in a resonant cavity. A detailed discussion of the significance of several quantities has been given in three technical reports (1).

The normalized impedance of the cavity and the discharge terminating the line is

$$Z = \frac{1}{g_s} + \frac{1}{g' + jb'} \tag{1}$$

where $g' = g + g_d$, and $b' = b + b_d$. The power absorbed is $P_a = P_i(1 - |\Gamma|^2)$, where $|\Gamma|$ is the magnitude of the complex reflection coefficient given by

$$|\Gamma| = \left| \frac{1 - y}{1 + y} \right| \tag{2}$$

From Eq. 1, we have for the admittance:

$$y = \frac{g_s(g' + jb')}{(g' + g_s) + jb'} \tag{3}$$

which can be written as

$$y = \frac{g'(1 + g'/g_s) + b'^2/g_s + jb'}{(1 + g'/g_s)^2 + b'^2/g_s^2} \tag{4}$$

The susceptance b is given by

$$b = \omega c - \frac{1}{\omega l}$$

and the susceptance b' by

$$b' = \omega c - \frac{1}{\omega l} - \frac{1}{\omega l_d} = \omega c - \frac{1}{\omega l'} \tag{5}$$

if we assume that the discharge susceptance for the present case is inductive. The presence of the discharge susceptance b_d changes the resonant frequency ω_o to a new value ω_o' given by $\omega_o' c = 1/\omega_o' l'$, where $\omega_o' - \omega_o = \Delta\omega \ll \omega_o$. Equation 5 may be written as

$$b' = \omega_o' c(\omega/\omega_o' - \omega_o'/\omega)$$

which at $\omega = \omega_o$ becomes

$$b' = \omega_o' c \left[-2 \frac{\Delta\omega}{\omega_o} \right] \tag{6}$$

The Q_u of the cavity at empty resonance is given by

$$Q_u = \frac{\omega_o c}{g(1 + g/g_s)}$$

Substituting this in Eq. 6, we get

$$b' = -2Q_u \, g(1 + g/g_s) \frac{\Delta f}{f} \tag{7}$$

Assuming that the discharge produces a negligible change in η and that there is suffi-
cient padding between the magnetron and the load, we have

$$(E/E_o)^2 = \frac{(1 - |\Gamma|^2) \, Q_u'}{(1 - |\Gamma_o|^2) \, Q_u}$$

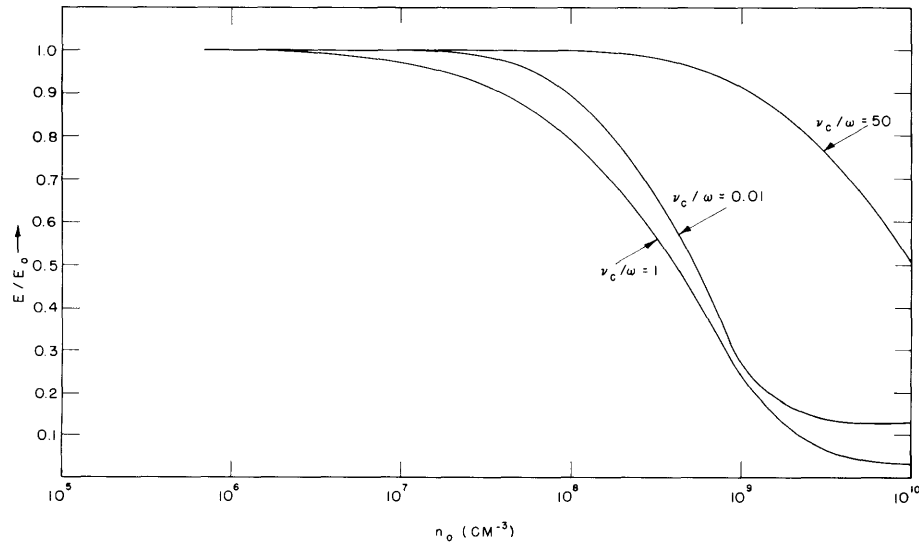where E_o is the field at the resonant frequency ω_o for the empty cavity, and E is the

Fig. II-2.   Variation of the electric field with electron density.

field when the discharge is present, at the same frequency. When the cavity is per-
fectly matched to the line, that is $\Gamma = 0$ and $g = 1$, we have

$$(E/E_o)^2 = \frac{(1 - |\Gamma|^2)(1 + 1/g_s)}{g'(1 + g'/g_s)} \tag{8}$$

The discharge admittance g_d can be computed from the equation (ref. 1)

$$g_d = \frac{n_o Q_u (1 + 1/g_s) \lambda^2 (\nu_c/\omega)}{2.44 \times 10^{13} \left[1 + (\nu_c/\omega)^2\right]} \qquad \text{in cgs units} \qquad (9)$$

and the shift in resonant frequency from

$$n_o = 4.87 \times 10^{13} \left[1 + (\nu_c/\omega)^2\right] \Delta\lambda/\lambda^3 \ \text{cm}^{-3} \qquad (10)$$

Equations 10 and 11 show that the discharge admittance g_d and the shift in resonant frequency are dependent upon the ratio of the collision frequency ν_c and the angular frequency ω. This dependence is most marked at a value of $\nu_c/\omega = 1$. With the use of Eq. 8, the theoretical values of the ratio E/E_o for various values of the electron density are plotted in Fig. II-2, with ν_c/ω as a parameter. Computations were made with assumed values of $Q_u = 2000$ and $\lambda = 10.7$ cm.

We wish to point out that the computations given above are valid for values of $n < 10^{10}$ electrons cm^{-3}.

M. P. Madan, J. J. McCarthy

References

1. S. C. Brown and D. J. Rose, Technical Reports 222, 223, and 230, Research Laboratory of Electronics, M.I.T., Jan. 22, 1952, Jan. 22, 1952, May 6, 1952.