CERN-PH-TH/2007-015

On the supergravity formulation of mirror symmetry in generalized Calabi-Yau manifolds

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Abstract

We derive the complete supergravity description of the $N = 2$ scalar potential which realizes a generic flux-compactification on a Calabi-Yau manifold (generalized geometry). The effective potential $\mathscr{V}_{eff} = \mathscr{V}_{(\partial_Z \mathscr{V}=0)}$, obtained by integrating out the massive axionic fields of the special quaternionic manifold, is manifestly mirror symmetric, i.e. invariant with respect to $\mathrm{Sp}(2\,h_2 + 2) \times \mathrm{Sp}(2\,h_1 + 2)$ and their exchange, being h_1, h_2 the complex dimensions of the underlying special geometries. $\mathscr{V}_{eff}$ has a manifestly $N = 1$ form in terms of a mirror symmetric superpotential W proposed, some time ago, by Berglund and Mayr.

1. Introduction

Geometries which generalize Calabi-Yau manifolds in the presence of generic fluxes [1, 2, 3, 4, 5, 6] (for comprehensive reviews on flux compactifications see [7]), have received considerable attention, as they realize schemes of compactification which incorporate supersymmetry breaking and moduli stabilization.

On the other hand the scalar potential originating from a compactification on such generalized geometries can be computed, from a supergravity point of view, as a deformation of an $N = 2$ supergravity Lagrangian. This $N = 2$ theory contains hypermultiplets which define a special quaternionic manifold $\mathcal{M}_Q$, obtained by c–map from the complex special geometry $\mathcal{M}_{KS}$ (of dimension h_1) underlying a mirror Calabi-Yau manifold [8]. The deformation of the $N = 2$ theory is effected as an abelian gauging of the $2h_1 + 3$ dimensional Heisenberg algebra of isometries of the special quaternionic manifold [9]. We denote by $h_2 + 1$ the number of vector fields in the model, and by $h_1 + 1$ the number of hypermultiplets, so that $h_1 = h_{11}$, $h_2 = h_{12}$ in Type IIB setting while $h_1 = h_{12}$, $h_2 = h_{11}$ in Type IIA. The resulting potential for generic fluxes ${e_I}^\Lambda$, $e_{I\,\Lambda}$ ($I = 0, \dots h_2$, $\Lambda = 0, \dots h_1$), was determined in [10]. The condition for an abelian gauging of the Heisenberg algebra requires that

$$e_{[I}{}^\Lambda\, e_{J]\,\Lambda} \;=\; 0\,. \tag{1}$$

The generators of the Heisenberg algebra of quaternionic isometries [11] are denoted by X^Λ, X_Λ, $\mathscr{Z}$. It is convenient to group the first $2h_1 + 2$ generators in a symplectic vector $X_A \equiv (X_\Lambda,\, X^\Lambda)$ in terms of which the commutation relations among the Heisenberg generators read

$$[X_A,\, X_B] \;=\; 2\,\mathbb{C}_{AB}\,\mathscr{Z}\,, \tag{2}$$

all the other commutators vanishing. We have denoted by $\mathbb{C}$ the symplectic invariant matrix

$$\mathbb{C} \;=\; \begin{pmatrix} \mathbf{0} & \mathbb{1} \\ -\mathbb{1} & \mathbf{0} \end{pmatrix}. \tag{3}$$

The adjoint action of the remaining quaternionic isometries on the X_A generators preserves this symplectic structure. These isometries comprise those of the special Kähler submanifold $\mathcal{M}_{KS}$ of the quaternionic manifold, of complex dimension h_1. The generators X_A are parametrized by $(2h_1 + 2)$-dimensional $\mathrm{Sp}(2h_1 + 2)$-vector of axions $Z^A = (\zeta^\Lambda,\, \tilde{\zeta}_\Lambda)$, originating from the ten dimensional R-R forms, while the central charge $\mathscr{Z}$ is parametrized by the axion a dual to the Kalb-Ramond antisymmetric 2-form $B_{\mu\nu}$. The electric fluxes ${e_I}^A = ({e_I}^\Lambda,\, e_{I\,\Lambda})$, together with an additional vector c_I, can be viewed as the electric components of and embedding tensor [12] which defines the gauge generators T_I as linear combinations of X_A, $\mathscr{Z}$

$$T_I \;=\; {e_I}^A T_A + c_I\,\mathscr{Z}\,. \tag{4}$$

In what follows we shall suppose that $h_2 < h_1$ and moreover that the rectangular matrix ${e_I}^A$ have maximal rank $h_2 + 1$. The gauge transformation rules for the axionic fields read

$$\delta Z^A \;=\; \xi^I\, {e_I}^A \;\;;\;\; \delta a = \xi^I\, c_I + \xi^I\, {e_I}^\Lambda\, \tilde{\zeta}_\Lambda - \xi^I\, e_{I\,\Lambda}\zeta^\Lambda = \xi^I\, c_I + \xi^I\, {e_I}^A\, \mathbb{C}_{AB}\, Z^B\,, \tag{5}$$

where $\xi^I(x)$ are the gauge parameters: $\delta A^I_\mu = \partial_\mu \xi^I$. In the Type IIA framework the entries ${e_I}^A$ with $I > 0$ can be characterized as geometric fluxes describing a deformation of the Calabi-Yau cohomology and ${e_0}^A$ as the components of the NS-NS 3-form field strength $H^{(3)}$ along the basis of 3-forms labelled by A [5, 9]. The parameters c_I are interpreted as R-R fluxes associated with the forms $F^{(0)}$, $F^{(2)}$, $F^{(4)}$, $F^{(6)}$ in the Type IIA setting, and with the 3-form $F^{(3)}$ in the Type IIB setting.

On the other hand, in order to have a symplectic covariant formulation of this gauging we need to dualize $h_2 + 1$ axions, out of the $h_1 + 1$ Z^A, to antisymmetric tensor fields, along the lines of [13]. This will allow us to introduce the magnetic counterpart m^{IA}, c^I to ${e_I}^A$, c_I. For an interpretation of these parameters in terms of generalized Calabi-Yau geometry see [5]. An other way for introducing magnetic fluxes would be to use the duality covariant formulation in [12] which describes at the same time the scalar fields and their tensor duals, coupled to both electric and magnetic vector fields. This procedure would eventually require a gauge fixing to be made and to solve certain non-dynamic equations. In next section we shall choose a different approach consisting in dualizing axions parametrizing abelian quaternionic isometries while keeping the theory covariant with respect to both the symplectic structures on $\mathcal{M}_{SK}$ (i.e. with respect to the group $\mathrm{Sp}(2h_2 + 2)$ of electric-magnetic duality transformations) and on $\mathcal{M}_{KS}$ (i.e. with respect to the group $\mathrm{Sp}(2h_1 + 2)$ acting on Z^A). It is convenient to group the electric and magnetic fluxes ${e_I}^A$, m^{IA} into a single $(2h_2 + 2) \times (2h_1 + 2)$ rectangular flux matrix Q

$$Q \equiv ({Q_r}^A) = \begin{pmatrix} {e_I}^A \\ m^{IA} \end{pmatrix} \quad (r = 1, \dots, 2h_2 + 2)\,, \tag{6}$$

and introduce the symplectic vector of R-R fluxes $c_r = (c_I,\, c^I)$. [1] These parameters define a $2h_2 + 2$ dimensional symplectic vector of gauge generators $T_r = {Q_r}^A\, X_A + c_r\, \mathscr{Z}$. The abelianity condition $[T_r,\, T_s] = 0$ now implies

$$({Q_r}^A {Q_s}^B \mathbb{C}_{AB}) = Q\, \mathbb{C}\, Q^T = 0\,, \tag{7}$$

while consistency of the theory with electric and magnetic charges requires [12, 13, 14]

$$({Q_r}^A {Q_s}^B \mathbb{C}^{rs}) = Q^T\, \mathbb{C}\, Q = 0\ ; \quad (c_r \mathbb{C}^{rs} {Q_s}^A) = c^T \mathbb{C} Q = 0\,. \tag{8}$$

The above conditions were found in [5, 10, 15]. We shall also use the quantity $\tilde{Q} = \mathbb{C}^T\, Q\, \mathbb{C} = ({Q^r}_A)$. Let us anticipate the main result of the paper, namely the $\mathrm{Sp}(2\,h_2 + 2) \times \mathrm{Sp}(2\,h_1 + 2)$-invariant expression of the $N = 2$ scalar potential $\mathscr{V}$. We shall denote by z^a $(a = 1, \dots, h_1)$ and by w^i $(i = 1, \dots, h_2)$ the complex scalars parametrizing $\mathcal{M}_{KS}$, submanifold of $\mathcal{M}_Q$, and $\mathcal{M}_{SK}$ respectively. Moreover let $V_1^A(z, \bar{z})$ and $V_2^r(w, \bar{w})$ denote the covariantly constant symplectic

[1] Here we shall use the same symbol $\mathbb{C}$ to denote the $\mathrm{Sp}(2h_1 + 2)$-invariant matrix $\mathbb{C}_{AB}$ and the $\mathrm{Sp}(2h_2 + 2)$-invariant matrix $\mathbb{C}_{rs}$, both having the form (3), though different dimensions. Which of the two matrices the symbol $\mathbb{C}$ refers to will be clear from the context, in particular from the dimension of the object it multiplies.

sections on $\mathcal{M}_{KS}$ and $\mathcal{M}_{SK}$ respectively. The scalar potential reads

$$\begin{aligned}\mathscr{V} &= -\frac{1}{8\,\phi^2}\,(c+2\,Q\,\mathbb{C}\,Z)^T\,\mathbb{C}^T\,\mathscr{M}(\mathscr{N}_{SK})\,\mathbb{C}\,(c+2\,Q\,\mathbb{C}\,Z)-\\ &-\frac{2}{\phi}\,\overline{V}_1^{\,T}\tilde{Q}^T\mathscr{M}(\mathscr{N}_{SK})\,\tilde{Q}V_1-\frac{2}{\phi}\,\overline{V}_2^{\,T}\,Q\,\mathscr{M}(\mathscr{N}_{KS})\,Q^T\,V_2-\\ &-\frac{8}{\phi}\,\overline{V_1}^{\,T}\,\mathbb{C}^T\,Q^T\,(V_2\,\overline{V}_2^{\,T}+\overline{V}_2V_2^T)\,Q\,\mathbb{C}\,V_1\,,\end{aligned}\tag{9}$$

where $\mathscr{M}(\mathscr{N})$ denotes the (negative definite) symplectic matrix constructed in terms of the real and imaginary part of the period matrix $\mathscr{N}$ on a special Kähler manifold [16]. It then follows that the terms in the first two lines of (9) are non-negative. Note that scalar potential depends on Z^A only through the combinations $Q\mathbb{C}Z\equiv(Q_r{}^A\mathbb{C}_{AB}Z^B)$ which do not contain h_2+1 axions, since it is gauge invariant, provided the matrix Q satisfies (7). These are precisely the axions that are dualized to antisymmetric tensor fields which acquire mass, in virtue of the anti-Higgs mechanism, by eating the vector fields. The combinations $Q\mathbb{C}Z$ turn out to depend only on h_2+1 of the undualized axions, which then acquire mass from the potential and can be integrated out. The remaining $2(h_1-h_2)$ R-R scalars are flat directions. They are absent for a self-mirror manifold, characterized by having $h_1=h_2$. In this case Q is a square matrix. The condition which fixes the h_2+1 axions at the extremum value is $c+2\,Q\,\mathbb{C}\,Z=0$. After the massive axions Z^A are integrated out we find the effective potential

$$\begin{aligned}\mathscr{V}_{eff}(\phi,w,\bar{w},z,\bar{z}) &= \mathscr{V}_{|\frac{\partial\mathscr{V}}{\partial Z^A}=0}=\\ &-\frac{2}{\phi}\,\overline{V}_1^{\,T}\tilde{Q}^T\mathscr{M}(\mathscr{N}_{SK})\,\tilde{Q}V_1-\frac{2}{\phi}\,\overline{V}_2^{\,T}\,Q\,\mathscr{M}(\mathscr{N}_{KS})\,Q^T\,V_2-\\ &-\frac{8}{\phi}\,\overline{V_1}^{\,T}\,\mathbb{C}^T\,Q^T\,(V_2\,\overline{V}_2^{\,T}+\overline{V}_2V_2^T)\,Q\,\mathbb{C}\,V_1\,.\end{aligned}\tag{10}$$

This potential is manifestly mirror symmetric, namely symmetric if we exchange $\mathcal{M}_{SK}$ with $\mathcal{M}_{KS}$ and replace Q by $\tilde{Q}^T$. It is now possible to show, and we shall do it in the last section, that V_{eff} has an $N=1$ form with superpotential given by

$$W = e^{-\frac{K_{SK}+K_{KS}}{2}}\,V_2(w,\bar{w})^T\,Q\,\mathbb{C}\,V_1(z,\bar{z})\,,\tag{11}$$

which coincides with the expression proposed in [17], and Kähler potential of the form

$$\begin{aligned}K_{tot} &= K_S+K_{SK}+K_{KS}\,,\\ K_S &= -\log(i(S-\bar{S}))\;\;;\;\;K_{SK}=-\log(i\,\overline{V}_1^{\,T}\mathbb{C}V_1)\;;\;\;K_{KS}=-\log(i\,\overline{V}_2^{\,T}\mathbb{C}V_2)\,,\end{aligned}\tag{12}$$

K_{SK} and K_{KS} being the Kähler potentials on $\mathcal{M}_{SK}$ and $\mathcal{M}_{KS}$ respectively.

The paper is organized as follows. In section 2. we perform the dualization of the axion a and of those components of Z^A which transform non trivially under the gauge group. We then introduce the magnetic components of the embedding tensor in the resulting Lagrangian. In section 3. we extend the results of [10], using the general formulae of [13, 12], to write the full $\mathrm{Sp}(2\,h_2+2)\times\mathrm{Sp}(2\,h_1+2)$-invariant scalar potential. Finally in section 4. we make contact with the $N=1$ potential proposed in [17]. We end with some conclusions.

2. Dualization with electric and magnetic charges

Let us start by introducing the notations. We consider a special quaternionic manifold $\mathcal{M}_Q$ of real dimension $4\,(h_1+1)$, which is parametrized by the scalars

$$q^u \;=\; \{\phi,\, a,\, \zeta^\Lambda,\, \tilde{\zeta}_\Lambda,\, z^a\}\,, \tag{13}$$

where, from Type IIB point of view, a is the scalar dual to the 2–form NS tensor $B_{\mu\nu}$, $\zeta^0 = C_{(0)}$, $\zeta^\Lambda = C^\Lambda_{(2)}$, $(\Lambda > 0)$, $\tilde{\zeta}_0$ is dual to $C_{\mu\nu}$, $\tilde{\zeta}_\Lambda = C_{(4)\,\Lambda}$, $(\Lambda > 0)$, ϕ describes the four–dimensional dilaton and the complex scalars z^a are the Kähler moduli of the Calabi-Yau and span the special Kähler submanifold $\mathcal{M}_{KS}$ of complex dimension h_1. In the Type IIA description the axions ζ^Λ, $\tilde{\zeta}_\Lambda$ arise as the components of the R-R 3-form along a basis α_Λ, β^Λ of the third chomology group $H^{(3)}$ of the Calabi-Yau, while z^a describe its complex structure moduli. We can introduce on $\mathcal{M}_{KS}$ the projective coordinates $\mathcal{X}^\Lambda$ which define the upper components of a holomorphic symplectic section: $\mathcal{X}^0 = 1$, $\mathcal{X}^a = z^a$. As anticipated in the introduction, there exists a subgroup of the isometry group generated by a Heisenberg algebra $(X_A,\, \mathscr{Z}) \equiv (X_\Lambda,\, X^\Lambda,\, \mathscr{Z})$, whose action of the hyperscalars has the following form:

$$\begin{aligned}
\delta\zeta^\Lambda &= \alpha^\Lambda\,,\\
\delta\tilde{\zeta}_\Lambda &= \beta_\Lambda\,,\\
\delta a &= \gamma + \alpha^\Lambda \tilde{\zeta}_\Lambda - \beta_\Lambda \zeta^\Lambda\,,
\end{aligned} \tag{14}$$

and which close the algebra (2). Using the notations of [11], we introduce the following one forms

$$\begin{aligned}
v &= e^{\tilde{K}}\,[d\phi - i\,(da + \tilde{\zeta}^T\,d\zeta - \zeta^T\,d\tilde{\zeta})]\,,\\
u &= 2i\,e^{\frac{\tilde{K}+\hat{K}}{2}}\,\mathcal{X}^T\,(d\tilde{\zeta} - \mathcal{N}_{KS}\,d\zeta)\,,\\
E &= i\,e^{\frac{\tilde{K}-\hat{K}}{2}}\,P\,N^{-1}\,(d\tilde{\zeta} - \mathcal{N}_{KS}\,d\zeta)\,,\\
e &= P\,d\mathcal{X}\,,
\end{aligned} \tag{15}$$

where

$$e^{\tilde{K}} \;=\; \frac{1}{2\phi} = \frac{e^{2\varphi}}{2}\,,\;\;;\;\; e^{\hat{K}} = \frac{1}{2\overline{\mathcal{X}}^T N \mathcal{X}} = \frac{e^{K_{KS}}}{2}\;\;;\;\;(\phi > 0)\,, \tag{16}$$

where φ denotes the four dimensional dilaton and K_{KS} is the Kähler potential on $\mathcal{M}_{KS}$ defined in (12).

The metric on the quaternionic manifold reads:

$$\begin{aligned}
ds^2 &= \bar{v}\,v + \bar{u}\,u + \bar{E}\,E + \bar{e}\,e = \\
&\quad K_{a\bar{b}}\,dz^a\,d\bar{z}^{\bar{b}} + \frac{1}{4\,\phi^2}\,(d\phi)^2 + \frac{1}{4\,\phi^2}\,(da + dZ^T\mathbb{C}Z)^2 - \frac{1}{2\,\phi}\,dZ^T\,\mathscr{M}(\mathcal{N}_{KS})\,dZ\,,
\end{aligned} \tag{17}$$

where $\mathcal{N}_{KS}$ is the period matrix on $\mathcal{M}_{KS}$ [2], the symplectic matrix $\mathcal{M}(\mathcal{N})$ is defined as follows:

$$\mathcal{M}(\mathcal{N}) = \begin{pmatrix} 1\!\!1 & -\mathrm{Re}\mathcal{N} \\ 0 & 1\!\!1 \end{pmatrix} \begin{pmatrix} \mathrm{Im}\mathcal{N} & 0 \\ 0 & \mathrm{Im}\mathcal{N}^{-1} \end{pmatrix} \begin{pmatrix} 1\!\!1 & 0 \\ -\mathrm{Re}\mathcal{N} & 1\!\!1 \end{pmatrix}, \qquad (18)$$

and the axion vector $Z^A = \begin{pmatrix} \zeta^\Lambda \\ \tilde{\zeta}_\Lambda \end{pmatrix}$ was defined in the introduction.

The Killing vectors associated with the abelian gauge algebra generators T_I defined in (4) read:

$$k_I = (c_I + e_I{}^\Lambda \tilde{\zeta}_\Lambda - e_{I\Lambda} \zeta^\Lambda) \frac{\partial}{\partial a} + e_I{}^\Lambda \frac{\partial}{\partial \zeta^\Lambda} + e_{I\Lambda} \frac{\partial}{\partial \tilde{\zeta}_\Lambda}. \qquad (19)$$

Let us start with the deformation [9] of the quaternionic Lagrangian (17) which corresponds to the chosen gauging of the Heisenberg isometry algebra:

$$\begin{aligned} \mathscr{L} &= -K_{a\bar{b}}\, dz^a \wedge \star\, d\bar{z}^{\bar{b}} - \frac{1}{4\,\phi^2} (Da - Z^A\, \mathbb{C}_{AB}\, DZ^B) \wedge \star (Da - Z^A\, \mathbb{C}_{AB}\, DZ^B) + \\ &\quad + \frac{1}{2\,\phi} DZ^A\, \mathcal{M}(\mathcal{N}_{KS})_{AB} \wedge \star DZ^B, \end{aligned} \qquad (20)$$

where the covariant derivatives are defined as follows:

$$\begin{aligned} Da &= da - c_I\, A^I - e_I{}^A\, \mathbb{C}_{AB}\, Z^B\, A^I, \\ DZ^A &= dZ^A - e_I{}^A\, A^I, \end{aligned} \qquad (21)$$

The electric charges $e_I{}^A$ satisfy the cocycle condition (1) corresponding to the requirement that the gauge algebra be abelian:

$$e_I{}^A\, e_J{}^B\, \mathbb{C}_{AB} = 0. \qquad (22)$$

As a consequence of the above condition the charges $e_I{}^A$ select an abelian "section" of the Heisenberg algebra to be gauged. Using $e_I{}^A$, we can split the RR scalar fields in two orthogonal sets Z^I, $\hat{Z}^A$, as follows:

$$Z^A = e_I{}^A\, Z^I + \hat{Z}^A. \qquad (23)$$

It is also useful to define the scalars $Z_I \equiv e_I{}^A \mathbb{C}_{AB} Z^B = e_I{}^A \mathbb{C}_{AB} \hat{Z}^B$. We may define the above splitting in a more formal way by introducing a matrix $\tilde{e}_A{}^I$ satisfying the conditions

$$\tilde{e}_A{}^I\, e_I{}^B = P^{(+)}{}_A{}^B\ ;\ \tilde{e}_A{}^I\, e_J{}^A = \delta_I{}^J, \qquad (24)$$

where $P^{(+)}{}_A{}^B$ is the projector on the $h_2 + 1$ dimensional subspace corresponding to the non vanishing minor of $e_I{}^A$. We also define the orthogonal projector $P^{(-)}{}_A{}^B = \delta_A^B - P^{(+)}{}_A{}^B$. Using

[2] In our conventions $\mathcal{N}_{KS} = i\, \mathcal{N}_s$ where $\mathcal{N}_s$ is the period matrix used in [11].

these projectors we can define $Z^I = \tilde{e}_A{}^I\, P^{(+)}{}_B{}^A\, Z^B$ and $\hat{Z}^A = P^{(-)}{}_B{}^A\, Z^B$. Note that under gauge transformations

$$\delta Z^I \;=\; \xi^I \;;\; \delta\hat{Z}^A = 0\,, \tag{25}$$

namely the $\hat{Z}^A$ components are gauge invariant. In other words the embedding tensor $e_I{}^A$, c_I defines an abelian subalgebra of the Heisenberg algebra spanned by the axions a, Z^I. Our aim is to dualize these scalars. We start from rewriting the vielbein along the $\mathscr{Z}$ direction on the tangent space, in the following form

$$da + dZ^T\mathbb{C}Z \;=\; da + Z_I\, dZ^I - Z^I\, dZ_I - \hat{Z}^A\mathbb{C}_{AB}\, d\hat{Z}^B\,. \tag{26}$$

From the above expression we see that, if we make the redefinition $a \to a + Z_I\, Z^I$, all the scalars Z^I in eq. (26), and therefore also in (20), can be covered by derivatives and thus a and Z^I can be dualized into closed 3–forms $H = dB$, $H_I = dB_I$. To this end we introduce a set of unconstrained 1–forms η, U^I replacing the differentials da, dZ^I in the Lagrangian (20) and add the 3–forms H, H_I as Lagrange multipliers. Note that the H_I can be expressed as combinations of $2\,(h_1+1)$ 3-forms H_A and similarly the corresponding antisymmetric tensors B_I can be expressed as combinations of $2\,(h_1+1)$ 2-forms B_A:

$$H_I \;=\; e_I{}^A\, H_A \;\;;\;\; B_I = e_I{}^A\, B_A \;\;;\;\; H_A = dB_A\,. \tag{27}$$

The resulting first order Lagrangian reads:

$$\begin{aligned}\mathscr{L}_Q \;=\;& -K_{a\bar{b}}\, dz^a\wedge\star d\bar{z}^{\bar{b}} - \frac{1}{4\,\phi^2}\,(\eta + 2\,Z_I\, U^I - R)\wedge\star(\eta + 2\,Z_I\, U^I - R) + \\ & +(U^I - A^I)\,\Delta_{IJ}\wedge\star(U^J - A^J) + 2\,(U^I - A^I)\, e_I{}^A\,\Delta_{AB}\wedge\star d\hat{Z}^B + d\hat{Z}^A\,\Delta_{AB}\wedge\star d\hat{Z}^B + \\ & +H\wedge(\eta - da) + H_I\wedge(U^I - dZ^I)\,,\end{aligned} \tag{28}$$

where we have used the following notation:

$$\begin{aligned} R \;&=\; 2\,Z_I\, A^I + c_I\, A^I + \hat{Z}^A\,\mathbb{C}_{AB}\, d\hat{Z}^B\,, \\ \Delta_{AB} \;&=\; \frac{1}{2\,\phi}\,\mathscr{M}(\mathscr{N}_{KS})_{AB}\;;\;\; \Delta_{IJ} = e_I{}^A e_J{}^B\,\Delta_{AB}\,.\end{aligned} \tag{29}$$

By varying the Lagrangian with respect to a and Z^I we obtain $H = dB$, $H_I = dB_I$. The field equations from the variations with respect to U^I and η are:

$$\begin{aligned} &\frac{\delta\mathscr{L}}{\delta\eta} = 0 \;\;\Rightarrow\;\; \eta + 2\,Z_I\, U^I - R = -2\phi^2\star H\,, \\ &\frac{\delta\mathscr{L}}{\delta U^I} = 0 \;\;\Rightarrow\;\; Z_I\,(\eta + 2\,Z_J\, U^J - R) = 2\,\Delta_{IJ}\,\phi^2\,(U^J - A^J) + 2\,\phi^2\, e_I{}^A\,\Delta_{AB}\, d\hat{Z}^B - \\ &\qquad\qquad\qquad -\phi^2\star H_I\,.\end{aligned} \tag{30}$$

Solving the above equations with respect to η, U_I and substituting in the first order Lagrangian we obtain the dual Lagrangian:

$$\begin{aligned}\mathscr{L}_{QD} \;=\;& -K_{a\bar{b}}\, dz^a\wedge\star d\bar{z}^{\bar{b}} - (\phi^2 - \Delta^{IJ}\, Z_I\, Z_J)\, H\wedge\star H + \frac{1}{4}\,\Delta^{IJ}\, H_I\wedge\star H_J - \Delta^{IJ}\, H\wedge\star H_I\, Z_J - \\ & -(H_I - 2\,H\, Z_I)\,\Delta^{IJ}\, e_J{}^A\,\Delta_{AB}\wedge d\hat{Z}^B + H\wedge\hat{Z}^A\,\mathbb{C}_{AB}\, d\hat{Z}^B + (H_I + c_I\, H)\wedge A^I + \\ & +d\hat{Z}^A\,\tilde{\Delta}_{AB}\wedge\star d\hat{Z}^B\,,\end{aligned} \tag{31}$$

where

$$\Delta^{IK}\,\Delta_{KJ} \;=\; \delta^I_J\ ;\ \ \tilde{\Delta}_{AB} = \Delta_{AB} - \Delta^{IJ}\,e_J{}^C\,\Delta_{CA}\,e_I{}^D\,\Delta_{DB} \qquad (32)$$

The dual Lagrangian is invariant under the following gauge transformations:

$$\delta A^I \;=\; d\xi^I\ \ ;\ \ \delta B_I = d\Xi_I\ ;\ \ \delta B = d\Xi\,, \qquad (33)$$

where the 1-forms Ξ_I, Ξ parametrize the tensor-gauge transformations. We can complete the Lagrangian (28) by adding the kinetic and theta term of the vector fields:

$$\mathscr{L}_{vec} \;=\; \mathrm{Im}(\mathscr{N}_{SK})_{IJ}\,F^I\wedge\star F^J + \frac{1}{2}\,\mathrm{Re}(\mathscr{N}_{SK})_{IJ}\,F^I\wedge\,F^J\,. \qquad (34)$$

It is straightforward to generalize the above construction by including magnetic charges m^{IA}, c^I, according to the following prescription [13]:

- In $\mathscr{L}_{vec}$ substitute F^I by $\hat{F}^I \equiv F^I + m^{IA}\,B_A + c^I\,B$.
- In $\mathscr{L}_{QD}$ substitute the topological term $H_I\wedge\,A^I \;=\; e_I{}^A\,H_A\wedge\,A^I \;=\; -e_I{}^A\,B_A\wedge F^I$ by $-e_I{}^B\,B_B\wedge(\hat{F}^I - \frac{1}{2}\,m^{IA}\,B_A - \frac{1}{2}\,c^I\,B)$. The same for the term $-c_I\,B\wedge F^I$.

In conclusion the final Lagrangian describing scalar, tensor and vector fields coupled to each other by means of electric and magnetic charges reads:

$$\begin{aligned}\mathscr{L}_D \;=\;& \mathrm{Im}(\mathscr{N}_{SK})_{IJ}\,\hat{F}^I\wedge\star\hat{F}^J + \frac{1}{2}\,\mathrm{Re}(\mathscr{N}_{SK})_{IJ}\,\hat{F}^I\wedge\,\hat{F}^J - \\ & -K_{a\bar{b}}\,dz^a\wedge\star d\bar{z}^{\bar{b}} - (\phi^2 - \Delta^{IJ}\,Z_I\,Z_J)\,H\wedge\star H + \frac{1}{4}\,\Delta^{IJ}\,H_I\wedge\star H_J - \Delta^{IJ}\,H\wedge\star H_I\,Z_J - \\ & -(H_I - 2\,H\,Z_I)\,\Delta^{IJ}\,e_J{}^A\,\Delta_{AB}\wedge d\hat{Z}^B + H\wedge\hat{Z}^A\,\mathbb{C}_{AB}\,d\hat{Z}^B - \\ & -(B_I + c_I\,B)\,\wedge\,(\hat{F}^I - \frac{1}{2}\,m^{IA}\,B_A - \frac{1}{2}\,c^I\,B) + d\hat{Z}^A\,\tilde{\Delta}_{AB}\,\wedge\star d\hat{Z}^B\,. \end{aligned} \qquad (35)$$

The above Lagrangian enjoys the extra tensor–gauge invariance:

$$\delta B_I = d\Xi_I\ ;\ \ \delta B = d\Xi\ \ ;\ \ \delta A^I = -m^{IA}\,\Xi_A - c^I\,\Xi\,, \qquad (36)$$

provided the following conditions are met:

$$e_I{}^A\,m^{IB} - e_I{}^B\,m^{IA} \;=\; 0\ ;\ \ c_I\,m^{IB} - e_I{}^B\,c^I = 0\,, \qquad (37)$$

which are equivalent to (8). The form of Lagrangian (35) is consistent with the construction given in [13][3] as far as the kinetic metric of the tensors and the tensor–scalar couplings are concerned. This is the case since, although we introduce $2\,h_1 + 2$ tensors B_A formally corresponding to all of the symplectic scalars Z^A, only the combination $B_I = e_I{}^A B_A$ and B are actually propagating and they mirror the scalars Z^I, a which parametrize an abelian subalgebra of the Heisenberg algebra, due to condition (22). A related observation is the fact that in paper [13] the choice

[3] In [13] to role of the indices I, Λ is exchanged.

of dualizing the parameters of an abelian algebra was made from the very beginning so that condition (22) was not needed. Let us note that also the combination $m^{IA}\,B_A$ can be expressed in terms of the only propagating tensors B_I. Indeed we can write

$$m^{IA}\,B_A \;=\; m^{JA}\,e_J{}^B\,\tilde{e}_B{}^I\,B_A = m^{JB}\,e_J{}^A\,\tilde{e}_B{}^I\,B_A = m^{JB}\,\tilde{e}_B{}^I\,B_J\,, \tag{38}$$

where the first of conditions (37) has been used.

3. Scalar potential with electric and magnetic fluxes

The general form of the $\mathscr{N}=2$ scalar potential is [18]:

$$\mathscr{V} = 4\,h_{uv}k^u_I k^v_J\,L^I\,\overline{L}^J + g_{r\bar{s}}\,k^r_I k^{\bar{s}}_J\,L^I\,\overline{L}^J + (U^{IJ} - 3\,L^I\overline{L}^J)\mathscr{P}^x_I\,\mathscr{P}^x_J\,, \tag{39}$$

where the second term does not contribute to the gauging we are considering, which involves quaternionic isometries only since it is abelian. The vectors L^I denote the upper part of the covariantly holomorphic symplectic section V on the special Kähler manifold $\mathcal{M}_{SK}$ parametrized by the vector multiplet scalars w^i, $\bar{w}^{\bar{\imath}}$. The expression for the momentum maps $\mathscr{P}^x_J$ is:

$$\mathscr{P}^x_I \;=\; k^u_I\,\omega^x_u\,, \tag{40}$$

where ω^x is the SU(2) connection. This form is Heisenberg–invariant and so is therefore the SU(2) curvature. This justifies the absence of a compensator on the right hand side of eq. (40).

It is useful to rewrite the scalar potential in two equivalent ways:

$$\mathscr{V} \;=\; 4\,h_{uv}\,k^u_I\,k^v_J\,L^I\,\overline{L}^J + (U^{IJ} - 3\,L^I\overline{L}^J)\,k^u_I\,k^v_J\,\omega^x_u\,\omega^x_v\,, \tag{41}$$

$$\mathscr{V} \;=\; -\frac{1}{2}\,(\mathrm{Im}\mathscr{N}_{SK})^{-1IJ}\,k^u_I k^v_J\,\omega^x_u\,\omega^x_v + 4\,(h_{uv} - \omega^x_u\,\omega^x_v)\,k^u_I\,k^v_J\,L^I\,\overline{L}^J\,, \tag{42}$$

where we have used the special geometry identity:

$$U^{IJ} \;=\; -\frac{1}{2}\,(\mathrm{Im}\mathscr{N}_{SK})^{-1IJ} - \overline{L}\,L^T\,. \tag{43}$$

In order to evaluate the expression on the right hand side of eq. (42) it is useful to compute the following quantity [11]:

$$G_{IJ} \;=\; k^u_I\,k^v_J\,(h_{uv} - \omega^x_u\,\omega^x_v) = k^u_I\,k^v_J\,[\bar{v}\,v + \bar{u}\,u + \bar{E}\,E - (\bar{v}\,v + 4\,\bar{u}\,u)]_{uv}\,. \tag{44}$$

Using the following notation:

$$r_I \;=\; c_I + 2\,(e_I{}^\Lambda\,\tilde{\zeta}_\Lambda - e_{I\Lambda}\,\zeta^\Lambda)\;\;;\;\; s_{I\Lambda} = e_{I\Lambda} - e_I{}^\Sigma\,(\mathscr{N}_{KS})_{\Sigma\Lambda}\,, \tag{45}$$

we can express G_{IJ} as follows:

$$G_{IJ} \;=\; 2\,e^{\tilde{K}}\,\bar{s}_{I\Lambda}\,s_{J\Sigma}\,\left(\mathscr{U} - 3\,\overline{\mathcal{L}}\,\mathcal{L}^T\right)^{\Lambda\Sigma}\;\;;\;\;\mathscr{U} = -\frac{1}{2}\,(\mathrm{Im}\mathscr{N}_{KS})^{-1} - \overline{\mathcal{L}}\,\mathcal{L}^T\;\;;\;\;\mathcal{L} = e^{\frac{K_{KS}}{2}}\,\mathcal{X} \tag{46}$$

In deriving the above expression for G_{IJ} we made use of the following properties:

$$\begin{aligned} N^{-1}P^\dagger P N^{-1} &= e^K\,(-N^{-1}+\mathcal{L}\,\overline{\mathcal{L}}^T)\,,\\ -\frac{1}{2}\,(\mathrm{Im}\mathcal{N}_{KS})^{-1} &= -N^{-1}+\mathcal{L}\,\overline{\mathcal{L}}^T+\overline{\mathcal{L}}\,\mathcal{L}^T\,. \end{aligned} \tag{47}$$

Now we can evaluate the two equivalent expressions for the scalar potential given in eqs. (41) and (42) [10]:

$$\begin{aligned} \mathscr{V} &= \overline{L}^I\,L^J\left[\frac{1}{\phi^2}\,(c_I+2\,e_I\mathbb{C}Z)\,(c_J+2\,e_J\mathbb{C}Z)-\frac{2}{\phi}\,e_I\,\mathscr{M}(\mathcal{N}_{KS})\,e_J^T\right]+\\ &\quad \frac{1}{2\,\phi}\,(U-3\,\bar{L}L^T)^{(IJ)}\left(\frac{1}{2\,\phi}\,r_I\,r_J+8\,\bar{s}_{I\Lambda}\,s_{J\Sigma}\,\overline{\mathcal{L}}^\Lambda\mathcal{L}^\Sigma\right)\,, \end{aligned} \tag{48}$$

$$\begin{aligned} \mathscr{V} &= -\frac{1}{4\,\phi}\,(\mathrm{Im}\mathcal{N}_{SK})^{-1IJ}\left(\frac{1}{2\,\phi}\,r_I\,r_J+8\,\bar{s}_{I\Lambda}\,s_{J\Sigma}\,\overline{\mathcal{L}}^\Lambda\mathcal{L}^\Sigma\right)+\\ &\quad \frac{4}{\phi}\,\overline{L}^I\,L^J\,\bar{s}_{(I|\Lambda}\,s_{J)\Sigma}\,\left(\mathscr{U}-3\,\bar{\mathcal{L}}\,\mathcal{L}^T\right)^{\Lambda\Sigma}\,, \end{aligned} \tag{49}$$

where we have introduced the following vectors: $e_I=\begin{pmatrix} e_I{}^\Lambda \\ e_{I\Lambda}\end{pmatrix}$. The first equation (48) is useful for those gaugings which involve just the graviphoton A^0_μ, e.g. Type IIA with NS flux or Type IIB on a half–flat "mirror" manifold [1]. Indeed in these cases the term in the second line of (48) does not contribute for cubic special geometries in the vector multiplet sector since:

$$(U-3\,\overline{L}L^T)^{00} \;=\; 0\,. \tag{50}$$

Similarly the expression (49) is of particular use for those gaugings which involve only isometries $\Lambda=0$, like for instance Type IIA on a half–flat manifold or Type IIB on the "mirror" manifold with NS flux since, for cubic special quaternionic geometries:

$$\left(\mathscr{U}-3\,\overline{\mathcal{L}}\,\mathcal{L}^T\right)^{00} \;=\; 0 \;\Rightarrow\; e^{K_{KS}}=-\frac{1}{8}\,(\mathrm{Im}\mathcal{N}_{KS})^{-1\,00}. \tag{51}$$

Let us now rewrite the scalar potential $\mathscr{V}$ as a symplectic covariant form in terms of the electric and magnetic charge matrix $Q\equiv(Q_r{}^A)$ defined in the introduction. To this end we use the covariantly holomorphic symplectic sections V_2 and V_1, associated with $\mathcal{M}_{SK}$ and $\mathcal{M}_{KS}$ respectively:

$$V_2 \;=\; (V_2^r)=\begin{pmatrix} L^I \\ M_I\end{pmatrix}\;\;;\;\; V_1=(V_1^A)=\begin{pmatrix} \mathcal{L}^\Lambda \\ \mathcal{M}_\Lambda\end{pmatrix}\,. \tag{52}$$

Using the properties

$$\begin{aligned} \bar{s}_{I\Lambda}\,(\mathrm{Im}\mathcal{N}_{KS})^{-1\Lambda\Sigma}\,s_{I\Sigma} &= e_I{}^A\,\mathscr{M}(\mathcal{N}_{KS})_{AB}\,e_I{}^B\,,\\ s_{I\Lambda}\,\mathcal{L}^\Lambda &= -e_I{}^A\,\mathbb{C}_{AB}\,V_1^B\,, \end{aligned} \tag{53}$$

the scalar potential $\mathscr{V}$ in (48), or equivalently in (49), has the following $\mathrm{Sp}(2\,h_2+2)$ invariant extension

$$\mathscr{V} \;=\; -\frac{1}{8\,\phi^2}\,(c+2\,Q\,\mathbb{C}\,Z)^T\,\mathbb{C}^T\,\mathscr{M}(\mathcal{N}_{SK})\,\mathbb{C}\,(c+2\,Q\,\mathbb{C}\,Z)\,-$$

$$-\frac{2}{\phi}\,\overline{V}_1{}^T\tilde{Q}^T\mathscr{M}(\mathscr{N}_{SK})\,\tilde{Q}V_1-\frac{2}{\phi}\,\overline{V}_2{}^T\,Q\,\mathscr{M}(\mathscr{N}_{KS})\,Q^T\,V_2-$$
$$-\frac{8}{\phi}\,\overline{V_1}{}^T\,\mathbb{C}^T\,Q^T\,(V_2\,\overline{V}_2{}^T+\overline{V}_2V_2^T)\,Q\,\mathbb{C}\,V_1\,, \tag{54}$$

where c denotes the symplectic vector of R-R electric and magnetic charges defined in the introduction: $c\equiv(c_I,\,c^I)$. Note that $\mathscr{V}$ depends only on the gauge invariant component $\hat{Z}^A$ of Z^A and not on the Z^I which have been dualized to tensor fields, in virtue of the property (7)

$$Q_r{}^A\,\mathbb{C}_{AB}\,Z^B \;=\; Q_r{}^A\,\mathbb{C}_{AB}\,e_I{}^B\,Z^I+Q_r{}^A\,\mathbb{C}_{AB}\hat{Z}^A=Q_r{}^A\,\mathbb{C}_{AB}\hat{Z}^A\,. \tag{55}$$

The equation of motion for $\hat{Z}$ imply the following condition

$$c+2\,Q\,\mathbb{C}\,\hat{Z} \;=\; 0\,, \tag{56}$$

which fixes part of the undualized $\hat{Z}$ axions. To illustrate which of these axions are fixed and which are flat directions let us choose a basis for Z^A so that, if we split the upper index Λ in $\Lambda=(I,\lambda)$: $\det(e_I{}^J)\neq0$, $e_I{}^\lambda=e_{I\,\Lambda}=0$. Conditions $Q\mathbb{C}Q^T=Q^T\mathbb{C}Q=0$ then imply that the only non vanishing components of $m^{I\,A}$ are described by the non singular matrix m^{IJ} satisfying the condition $m^{I[J}\,e_I{}^{K]}=0$. The combinations $Q\mathbb{C}\hat{Z}$ then single out the only scalars $\tilde{\zeta}_I$, which therefore are the only components of the vector Z^A entering the potential, and thus fixed by condition (56). Therefore in this case the fate of the original Z^A scalars is summarized as follows

$$\begin{aligned}
(h_2+1)\;\; Z^I\equiv\zeta^I &\;\;\longrightarrow\;\; \text{dualized to tensor fields }\; B_{\mu\nu I}\,,\\
(h_2+1)\;\; Z_I\equiv\tilde{\zeta}_I &\;\;\longrightarrow\;\; \text{fixed by (56)}\,,\\
2\,(h_1-h_2)\;\; \tilde{\zeta}_\lambda,\,\zeta^\lambda &\;\;\longrightarrow\;\; \text{flat directions for }\mathscr{V}\,.
\end{aligned}\tag{57}$$

Upon implementation of conditions (56), the first term in the scalar potential (54) vanishes, and the resulting effective potential $\mathscr{V}_{eff}$, as a function of the remaining scalar fields, acquires the following mirror symmetric expression

$$\mathscr{V}_{eff}(\phi,w,\bar{w},z,\bar{z}) \;=\; \mathscr{V}_{|\frac{\partial\mathscr{V}}{\partial Z^A}=0}=-\frac{2}{\phi}\,\overline{V}_1{}^T\tilde{Q}^T\mathscr{M}(\mathscr{N}_{SK})\,\tilde{Q}V_1-\frac{2}{\phi}\,\overline{V}_2{}^T\,Q\,\mathscr{M}(\mathscr{N}_{KS})\,Q^T\,V_2-$$
$$-\frac{8}{\phi}\,\overline{V_1}{}^T\,\mathbb{C}^T\,Q^T\,(V_2\,\overline{V}_2{}^T+\overline{V}_2V_2^T)\,Q\,\mathbb{C}\,V_1\,. \tag{58}$$

The above formula for $\mathscr{V}$ is manifestly invariant if we exchange $\mathcal{M}_{SK}$ with $\mathcal{M}_{KS}$ and Q with $\tilde{Q}^T$.

4. Formulation in terms of an $N=1$ superpotential

In this section we show that the expression for $\mathscr{V}$ in (54) can be described in terms of the $N=1$ superpotential proposed in [17]

$$W \;=\; e^{-\frac{K_{SK}+K_{KS}}{2}}\,V_2^T\,Q\,\mathbb{C}\,V_1\,, \tag{59}$$

where $K_{SK}(w,\bar{w})$ and $K_{KS}(z,\bar{z})$ are the Kähler potentials on $\mathcal{M}_{SK}$ and $\mathcal{M}_{KS}$ defined in (12) . The scalars of the $N=1$ theory are $S, \bar{S}$, $w^i, \bar{w}^{\bar{\imath}}$, z^a, $\bar{z}^{\bar{a}}$ and span a Kähler manifold with Kähler potential given in (12). The $N=1$ scalar potential reads

$$\mathscr{V}_{N=1} \quad = \quad e^{K_{tot}} \left(g^{a\bar{b}}\, D_a W\, D_{\bar{b}} \overline{W} + g^{i\bar{\jmath}}\, D_i W\, D_{\bar{\jmath}} \overline{W} + g^{S\bar{S}}\, D_S W\, D_{\bar{S}} \overline{W} - 3\, |W|^2 \right) , \tag{60}$$

where the covariant derivatives are defined as $D_x W = \partial_x W + \partial_x K_{tot}\, W$, where $x = i, a, S$. Note that W is S-independent and therefore

$$g^{S\bar{S}}\, D_S W\, D_{\bar{S}} \overline{W} \quad = \quad g^{S\bar{S}}\, D_S K_S\, D_{\bar{S}} K_S\, |W|^2 = |W|^2 \,. \tag{61}$$

Let us now use the following properties of special geometry

$$\begin{aligned} g^{a\bar{b}}\, D_a V_1 D_{\bar{b}} \overline{V_1} \quad &= \quad -\frac{1}{2}\, \mathbb{C}^T \mathscr{M}(\mathscr{N}_{KS}) \mathbb{C} - \overline{V_1}\, V_1^T \,, \\ g^{i\bar{\jmath}}\, D_i V_2 D_{\bar{\jmath}} \overline{V}_2 \quad &= \quad -\frac{1}{2}\, \mathbb{C}^T \mathscr{M}(\mathscr{N}_{SK}) \mathbb{C} - \overline{V}_2\, V_2^T \,, \end{aligned} \tag{62}$$

and write the relevant terms in $V_{N=1}$

$$\begin{aligned} g^{a\bar{b}}\, D_a W\, D_{\bar{b}} \overline{W} \quad &= \quad e^{-\frac{K_{SK}+K_{KS}}{2}} \left(-\frac{1}{2}\, V_2^T\, Q\, \mathscr{M}(\mathscr{N}_{KS})\, Q^T\, \overline{V}_2 - V_1^T \mathbb{C}^T Q^T \overline{V}_2 V_2^T Q \mathbb{C} \overline{V}_1 \right) , \\ g^{i\bar{\jmath}}\, D_i W\, D_{\bar{\jmath}} \overline{W} \quad &= \quad e^{-\frac{K_{SK}+K_{KS}}{2}} \left(-\frac{1}{2}\, V_1^T \tilde{Q}^T\, \mathscr{M}(\mathscr{N}_{SK})\, \tilde{Q}\, \overline{V}_1 - V_1^T \mathbb{C}^T Q^T \overline{V}_2 V_2^T Q \mathbb{C} \overline{V}_1 \right) , \\ -2\, |W|^2 \quad &= \quad -2\, e^{-\frac{K_{SK}+K_{KS}}{2}}\, V_1^T \mathbb{C}^T Q^T V_2 \overline{V}_2^T Q \mathbb{C} \overline{V}_1 \,. \end{aligned} \tag{63}$$

The scalar potential therefore can be recast in the following form

$$\begin{aligned} \mathscr{V}_{N=1} \quad = \quad & e^{K_S} \left(-\frac{1}{2}\, \overline{V}_1^{\,T} \tilde{Q}^T \mathscr{M}(\mathscr{N}_{SK})\, \tilde{Q} V_1 - \frac{1}{2}\, \overline{V}_2^{\,T}\, Q\, \mathscr{M}(\mathscr{N}_{KS})\, Q^T\, V_2 - \right. \\ & \left. -2\, \overline{V_1}^{\,T}\, \mathbb{C}^T\, Q^T\, (V_2\, \overline{V}_2^{\,T} + \overline{V}_2 V_2^T)\, Q\, \mathbb{C}\, V_1 \right) , \end{aligned} \tag{64}$$

which coincides with the expression in (54) provided $\mathrm{Im}S = -\exp(-K_S)/2 = -\phi/8$.

5. Conclusions

We have derived the scalar potential for an $N=2$ supergravity theory with general electric and magnetic gauging of an abelian subalgebra of the Heisenberg isometry algebra of a special quaternionic Kähler manifold. Although we have only discussed the bosonic action, by applying the results of [13], the full Lagrangian, including fermionic terms and the transformation laws are known.This Lagrangian is supposed to describe the effective theory for a compactification of Type II superstring on a generalized Calabi-Yau manifold, which, in this context, is viewed as a deformation of a Calabi-Yau manifold when general fluxes are turned on. One limitation of this description is that classical c-map has been used to obtain a manifest $\mathrm{Sp}(2\,h_2+2)\times \mathrm{Sp}(2\,h_1+2)$-symmetric description. It would be interesting to describe a situation in which a quantum c-map [19], encompassing both perturbative and non-perturbative effects for the quaternionic geometry, is used in this context of generalized geometries.

6. Acknowledgements

Work supported in part by the European Community's Human Potential Program under contract MRTN-CT-2004-005104 'Constituents, fundamental forces and symmetries of the universe', in which R. D'A. and M.T. are associated to Torino University. The work of S.F. has been supported in part by European Community's Human Potential Program under contract MRTN-CT-2004-005104 'Constituents, fundamental forces and symmetries of the universe' and the contract MRTN- CT-2004-503369 'The quest for unification: Theory Confronts Experiments', in association with INFN Frascati National Laboratories and by D.O.E. grant DE-FG03-91ER40662, Task C.

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