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High-T_c superconductivity by phase cloning

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Abstract

We consider a BCS-type model in the spin formalism and argue that the structure of the interaction provides a mechanism for control over directions of the spin $\overrightarrow{S}$ other than S_z, which is being controlled via the conventional chemical potential. We also find the conditions for the appearance of a high-T_c superconducting phase.

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Introduction

Twenty years after the discovery of high-temperature superconductivity [1] there is still neither consensus nor clear understanding of the mechanism or mechanisms which are behind this exciting and with innumerable practical applications phenomenon.The initial discussions (see, e.g. [2]) have led to the formation of some main conceptual stream, as presented in [3], however other view points are continuously being argued, just to mention a recent one [4].

In [5] we proposed a combination of a BCS and a mean-field Hamiltonian where the transition temperature could become arbitrarily high. This happened without increasing the interaction indefinitely but by a small denominator. In this note we investigate this effect more closely and find that another important ingredient is a chemical potential which breaks the electron conservation. We give a model for this phenomenon by the interaction with a reservoir of quasi-particles which do not have a definite electron number. Since such objects play an important role in the theory of the Josephson currents [6], we think that this possibility is not purely academic.

To avoid a terminological misunderstanding, we recall that the (quantum-mechanical) mean-field theory and the BCS-theory of superconductivity correspond to essentially different physical situations. A mean-field theory means that the particle density $\rho(x) = \psi^*(x)\psi(x)$ (in second quantization) tends to a c-number in a suitable scaling limit. With an appropriate smearing, from the operator-valued distribution $\rho(x)$ an unbounded operator is being produced, so that the best to be strived for remains the strong resolvent convergence in a representation where the macroscopic density is built in. In the BCS-theory pairs of creation operators with opposite momentum $\tilde{\psi}^*(k)\,\tilde{\psi}^*(-k)$ tend to c-numbers, so the correlations required in both cases, seem to be quite different. The main result in [5] was that both types of correlations may well co-exist in certain regions of the parameter space (temperature, chemical potential, relative values of the two coupling constants) and this appears to be the case in the KMS-state of the equivalent approximating (Bogoliubov) Hamiltonian H_B, two Hamiltonians being considered as equivalent if they lead to one and the same time evolution of the local observables [7].

In what follows, we generalize the original BCS model in the most natural way, namely by augmenting it with the missing mean-field interaction components. We show that this provides a mechanism for control over directions of the spin $\overrightarrow{S}$ other than S_z, which is being controlled via the conventional chemical potential.

1 The degenerate BCS Hamiltonian

The initial quartic BCS Hamiltonian is mainly known in terms of fermionic creation and annihilation operators [8]

$$H = \sum_k (\omega_k - \mu)(a^\dagger_{\uparrow,k} a_{\uparrow,k} + a^\dagger_{\downarrow,k} a_{\downarrow,k}) + \sum_{k,k'} V_{k,k'} a^\dagger_{\downarrow,k'} a^\dagger_{\uparrow,-k'} a_{\uparrow,-k} a_{\downarrow,k}. \tag{1.1}$$

It involves however only the algebra generated by the pair-operators $a_{\uparrow,-k}a_{\downarrow,k}$ (observe $a_\uparrow^\dagger a_\uparrow + a_\downarrow^\dagger a_\downarrow = [a_\uparrow^\dagger a_\downarrow^\dagger, a_\downarrow a_\uparrow] + 1$), so we shall only be concerned with them and shall represent them by spin matrices $a_{\uparrow,j}^\dagger a_{\downarrow,-j}^\dagger \to \sigma_{j+} = (\sigma_{j,x} + i\sigma_{j,y})/2$, $j = 1, \ldots, N$. As a weak interaction can only scratch the Fermi surface, we take $\omega_k = \omega$, $\forall k$ and incorporate the latter into μ. Finally, we set $V_{k,k'} = -2\lambda_B/N \ \forall k, k'$. With the notation $\overrightarrow{S} = \sum_{i=1}^N \overrightarrow{\sigma_i}$, Hamiltonian (1.1) becomes equivalent to

$$H = -\frac{\lambda_B}{2N}(S_x^2 + S_y^2) - \mu S_z. \tag{1.2}$$

In this form the Hamiltonian can be exactly diagonalized and the following steps are mathematically rigorous in the limit.

We assume that the thermal state $\langle A \rangle = \mathrm{Tr}\, \mathrm{A}\mathrm{e}^{-\beta \mathrm{H}} / \mathrm{Tr}\, \mathrm{e}^{-\beta \mathrm{H}}$ is such that the length of $\overrightarrow{S}$ is much bigger than the fluctuations around it, $\langle (\overrightarrow{S} - \langle \overrightarrow{S} \rangle)^2 \rangle$. This means that in the identity

$$S^2 = (S - \langle S \rangle)^2 + 2\langle S \rangle S - \langle S \rangle^2$$

the first term is small compared to the second one. Since the last term is a c-number, Hamiltonian (1.2) becomes equivalent to the following one, linear in $\overrightarrow{S}$

$$H_B = -\lambda_B \left(\frac{\langle S_x \rangle}{N} S_x + \frac{\langle S_y \rangle}{N} S_y \right) - \mu S_z. \tag{1.3}$$

Of course, the original Hamiltonian is invariant under rotations around the z-axis, but the spin-vector $\overrightarrow{S}$ will point into some direction (with $-\mu S_z$ contribution taken into account) and we shall call this resulting spin direction S_B, that is Eq. (1.3) can be rewritten as

$$H_B = -\lambda_B \frac{\langle S_x \rangle}{N} S_x - \mu S_z =: -W_0 S_B, \tag{1.4}$$

where

$$W_0 = \sqrt{(\lambda_B \langle S_x \rangle / N)^2 + \mu^2} > |\mu| \tag{1.5}$$

$$S_B = b S_x + \sqrt{1 - b^2} S_z, \tag{1.6}$$

and the mixing parameter of the Bogoliubov transformation [9] is defined through

$$b = \lambda_B \langle S_x \rangle / N W_0. \tag{1.7}$$

This rotation can be inverted and if $S_\perp$ is in the $x - z$ plane orthogonal to S_B, we have (Figure 1)

$$S_x = b S_B - \sqrt{1 - b^2} S_\perp.$$

Since $H = -W_0 S_B$ is the sum of N spins in the B-direction, Eq.(1.6), the thermal expectation values are the usual ones

$$\frac{\langle S_B \rangle}{N} = \tanh \frac{W_0}{2T}, \qquad \langle S_\perp \rangle = 0. \tag{1.8}$$

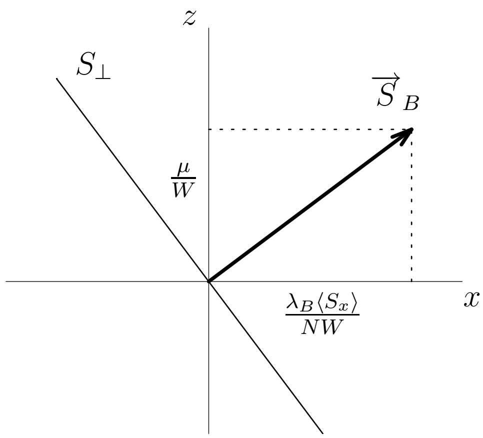

Figure 1: The total-spin decomposition.

The self-consistency of the system is expressed by the so-called "gap-equation"

$$\frac{\langle S_x \rangle}{N} = b \tanh \frac{W_0}{2T} = \frac{\lambda_B}{W} \frac{\langle S_x \rangle}{N} \tanh \frac{W_0}{2T}. \tag{1.9}$$

This gap-equation has two solutions

(A) *a normal state*

$$\langle S_x \rangle = 0 \quad \forall T \tag{1.10}$$

(B) *a superconducting state*

$$\langle S_x \rangle \neq 0, \quad \frac{W_0}{\lambda_B} = \tanh \frac{W_0}{2T}, \tag{1.11}$$

$$\text{for } T = \frac{\lambda_B}{2} F\left(\frac{W_0}{2T}\right) < T_c, \tag{1.12}$$

where the characteristic function $0 < F(\alpha) \leq 1$ is given by

$$F(\alpha) = \frac{\tanh \alpha}{\alpha} \begin{array}{l} \nearrow \quad 1 - \alpha^2/3, \quad \alpha \to 0 \\ \searrow \quad 1/\alpha, \qquad\quad \alpha \to \infty \end{array}$$

(see Figure 2) and the critical temperature (at which the gap opens) is

$$T_c = \frac{\lambda_B}{2} F\left(\frac{|\mu|}{2T_c}\right) \leq \frac{|\lambda_B|}{2}. \tag{1.13}$$

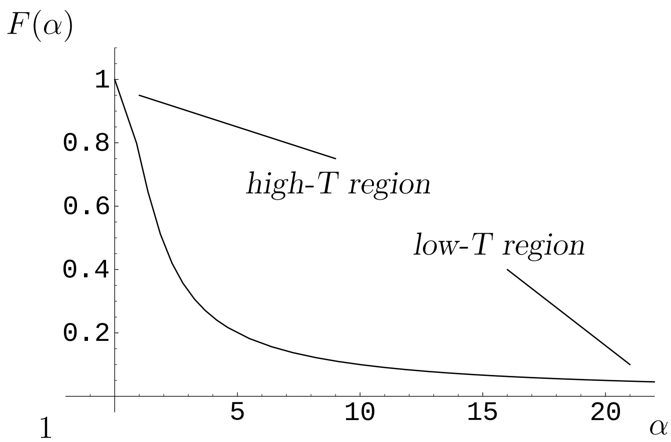

Figure 2: The characteristic function $F(\alpha)$: high-T region corresponds to small α.

On Figure 3, the pure BSC-situation is shown: the plot of both sides of Eq.(1.11), for $T = \lambda_B/4, \lambda_B/2, 3\lambda_B/4$. The limit (1.13) becomes obvious.

Relations (1.5), (1.12) in fact suggest some possibilities for high-T_c generation. A realistic mechanism should result in a deviation from the proportionality relation (1.12) as far as the characteristic-function part is considered (as $F(\alpha)$ is always less than 1). Also, it should aim a modification of the quasiparticle dispersion relation (1.5), as is e.g. the case of the gossamer superconductor [10]. We rather target the appearance of an effective chemical potential, whose variation would provide a means of influence on the transition temperature.

2 On the role of the chemical potential

The chemical potential is a control parameter which adjusts the number of Cooper pairs, in our formalism S_z. We shall now argue that the BCS-interaction gives us a handle to control also other directions of $\overrightarrow{S}$ and in this way to clone the Josephson phase of R.

Suppose our system interacts strongly with a superconducting reservoir R such that Eq.(1.3) holds for the ensemble:

$$H = -\frac{\lambda_B}{2}\left((\overrightarrow{S} + \overrightarrow{S}^{(R)})\frac{\langle \overrightarrow{S} + \overrightarrow{S}^{(R)}\rangle}{N + N_R} - (S_z + {S_z}^{(R)})\frac{\langle S_z + {S_z}^{(R)}\rangle}{N + N_R}\right). \tag{2.1}$$

The cross-term

$$\frac{\overrightarrow{S}\langle \overrightarrow{S}^{(R)}\rangle}{N + N_R} \tag{2.2}$$

induces control parameters which just copy on the system the situation in R, if R is dominant [7, 11, 12]. By the coupling with the reservoir, the $\overrightarrow{S}$-direction is cloned. We

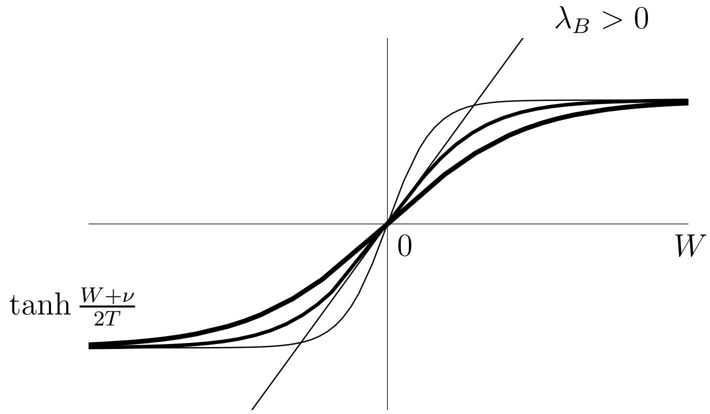

Figure 3: The pure BCS situation: plot of both sides of Eq.(1.11), for $T = \lambda_B/4, \lambda_B/2, 3\lambda_B/4$ (the line thickness increases with λ_B).

shall call this spin-coaxial exchange. Thus, if the reservoir is in the normal state, $\overrightarrow{S}^{(R)}$ is in the z-direction and we get the usual chemical potential. If R is superconducting, we get a chemical-potential coupled $S_B^{(R)}$ (a $\nu S_B^{(R)}$-term in the Hamiltonian), which represents the quasi-particles — the elementary excitations of the superconductor. If the reservoir dictates a Josephson phase, e.g. in the x-direction, this means that in the cross-term (2.2) $\langle S_x^{(R)}\rangle \sim \nu$, correspondingly the Hamiltonian becomes

$$H - \nu S_x = -W S_B, \quad \nu > 0 \tag{2.3}$$

with dispersion relation

$$W = \sqrt{\left(\nu + \lambda_B \frac{\langle S_x\rangle}{N}\right)^2 + \mu^2}$$

and mixing parameter

$$c = \frac{\nu + \lambda_B \langle S_x\rangle/N}{W}.$$

The gap-equation reads:

$$\frac{\langle S_x\rangle}{N} = \frac{\nu + \lambda_B \langle S_x\rangle/N}{W} \tanh \frac{W}{2T} \tag{2.4}$$

and the temperature at which the gap opens becomes

$$T = \frac{1}{2}\left(\frac{\nu}{\langle S_x\rangle/N} + \lambda_B\right) F(\frac{W}{2T}). \tag{2.5}$$

So with this choice

(i) expectedly, there is no normal phase, i.e. solution with $\langle S_x\rangle = 0$;

(ii) $\forall T$ there exists a solution with $\langle S_x \rangle > 0$.

Thus there is no phase transition $\forall\, \nu > 0$. It is quenched, since the symmetry is broken externally and not spontaneously.

With all this taken into account, for the description of a system which exhibits a phase transition towards a high-temperature superconducting phase, we should aim constructing a Hamiltonian of the form

$$H = aS_x + dS_z + kaS_x + kdS_z = -W(1+k)(bS_x + \sqrt{1-b^2}S_z), \tag{2.6}$$

where

$$W^2 = a^2 + d^2, \qquad b = -a/W,$$

a and d are parameters to be determined later on and k characterizes the strength of the interaction between the two subsystems. Among others, this particular form of the Hamiltonian means that the spins of the system and the reservoir are parallel, both pointing into the B-direction, as defined in (1.6). Correspondingly, for $S_\perp$ we get

$$S_\perp = -\sqrt{1-b^2}\, S_x + bS_z \tag{2.7}$$

and the gap-equations read:

$$\begin{aligned} \frac{\langle S_x \rangle}{N} &= -\frac{a}{W} \tanh \frac{W(1+k)}{2T} \\ \frac{\langle S_z \rangle}{N} &= \frac{\sqrt{W^2 - a^2}}{W} \tanh \frac{W(1+k)}{2T}. \end{aligned} \tag{2.8}$$

The parameters a and d have to comply with some conditions, in order to achieve the desired behaviour. Thus a, being related to $\langle S_x \rangle$, has to be monomial in it to allow for a phase transition and has to depict the BCS-coupling strength, so to be proportional to λ_B. In turn, d, being related to $\langle S_z \rangle$, should account for two different features: on one hand, for the fact that in our formalism S_z represents the Cooper pairs and their number is controlled by the chemical potential μ, and on the other hand, for the role of the mean field, whose contribution is quadratic in the electron density, so represented by a S_z^2-term in the Hamiltonian. These requirements allow to fix the parameters a and b up to an overall constant:

$$\begin{aligned} a &= -\frac{\lambda_B \langle S_x \rangle}{N} \\ d &= \mu - \frac{\lambda_M \langle S_z \rangle}{N} \end{aligned} \tag{2.9}$$

Finally, if we take k in the form ν/W, which is no restriction whatsoever, the model Hamiltonian obtains the form

$$H = (W + \nu) \left[\frac{\lambda_B \langle S_x \rangle}{NW} S_x + \frac{1}{W} \left(\mu - \frac{\lambda_M \langle S_z \rangle}{N} \right) S_z \right], \tag{2.10}$$

thus giving rise to the following system of coupled gap-equations:

$$\frac{\langle S_x\rangle}{N} = \frac{\lambda_B\langle S_x\rangle}{NW}\tanh\frac{W+\nu}{2T} \tag{2.11}$$

$$\frac{\langle S_z\rangle}{N} = \frac{\mu-\lambda_M\langle S_z\rangle/N}{W}\tanh\frac{W+\nu}{2T} \tag{2.12}$$

with

$$W = \sqrt{\frac{\lambda_B^2\langle S_x\rangle^2}{N^2}+\left(\mu-\frac{\lambda_M\langle S_z\rangle}{N}\right)^2}$$

$$= \sqrt{\mu_{eff}^2+\frac{\lambda_B^2\langle S_x\rangle^2}{N^2}} \geq |\mu_{eff}| \tag{2.13}$$

This system has both solutions, corresponding to normal and to superconducting phases, so with $\langle S_x\rangle = 0$, resp. $\langle S_x\rangle \neq 0$. In the latter case, from Eq.(2.11) the transition temperature is found to be

$$T_c = \frac{|\lambda_B|}{2}\,\frac{|\nu+\mu_{\rm eff}|}{|\mu_{\rm eff}|}. \tag{2.14}$$

As μ_{eff} can be made arbitrarily small, this means that for given values of the parameters μ, ν and λ_B the critical temperature can become arbitrarily high, as suggested in [5].

The second (coupled) gap-equation provides a relation between the model parameters that determines the relevant parameter range:

$$\frac{\langle S_z\rangle}{N} = \frac{\mu}{\lambda_B+\lambda_M}.$$

The existence of further order parameters is of severe importance for the physical content of the models under consideration [13]. Even in the simple model above, the presence of a second order parameter leads to an enrichment of the structure and to new effects.

Let us discus the superconducting solution more in detail. In the mean-field enhanced model, for $\langle S_x\rangle \neq 0$, Eq.(2.11) reduces to

$$\frac{W}{\lambda_B} = \tanh\frac{W+\nu}{2T}. \tag{2.15}$$

We have chosen the positive eigenvalues of H, Eq.(1.5). This is not really a restriction, since the consideration of the opposite situation will give the conjugate picture. Thus, tanh(...) and λ_B must always have the same sign. Also, Eq.(2.13), the lower bound for the values of W is determined through the effective chemical potential in the z-direction, $\mu_{\rm eff}$. Depending on the coupling of the system to the reservoir (the value and the sign of ν), we are led to the following situations:

(A) Spin-coaxial exchange, $\nu > 0$ (Figure 4)

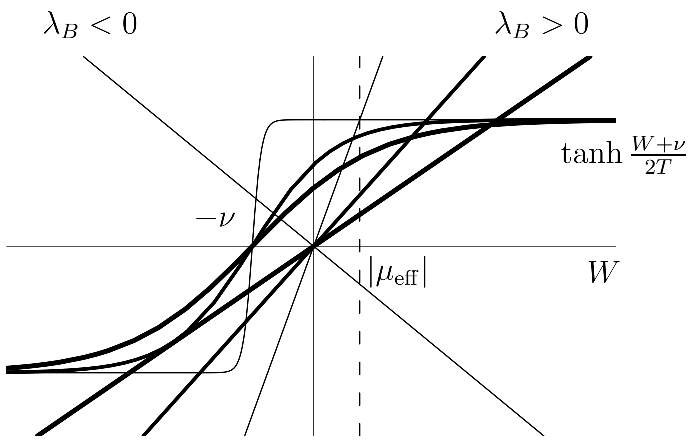

Figure 4: Mean-field enhanced BCS: spin-coaxial exchange (plot of both sides of Eq.(2.15); the line thickness increases with T and λ_B).

- the BCS-coupling has to be attractive and stronger than the effective chemical potential, $\lambda_B > |\mu_{\text{eff}}|$;
- the solution (when existing) is uniquely determined;
- as also seen from Eq.(2.14), the higher-temperature solutions require also stronger BCS coupling (the thickness of the lines increases with T, resp. with λ_B; the admissible solutions have to be to the right of the dashed line).

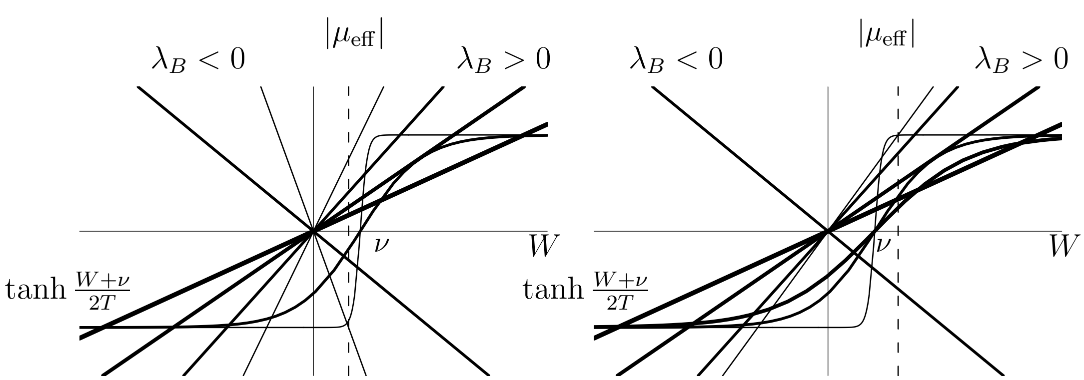

Figure 5: Mean-field enhanced BCS: spin-anticoaxial exchange. (a) $|\mu_{\text{eff}}| < |\nu|$; (b) $|\mu_{\text{eff}}| > |\nu|$ (also here, the line thickness increases with T, resp. with λ_B; the admissible solutions have to be to the right of the dashed line)

(B) Spin-anticoaxial exchange, $\nu < 0$

In this case the relative values of $|\mu_{\text{eff}}|$ and $|\nu|$ become of importance.

When $|\mu_{\text{eff}}| < |\nu|$, (Figure 5a),

- solutions with repulsive BCS-coupling are possible and uniquely defined;
- $|\lambda_B|$, when $\lambda_B < 0$, has to dominate the effective chemical potential, $|\mu_{\text{eff}}|$;
- in the positive λ_B-coupling range, it can happen that the full system has none, one or two solutions.

When $|\mu_{\text{eff}}| > |\nu|$, (Figure 5b),

- only solutions with positive BCS-couplings are possible;
- depending on the relations between the parameters — μ_{eff}, λ_B and ν — encoded in the second gap-equation, the system can have none, one or two solutions.

As Eq.(2.14) requires small values of $|\mu_{\text{eff}}|$ in order to achieve high transition temperature, this would correspond rather to the situations depicted on Figures 4 and 5a.

3 Conclusions

We considered a BCS-type model with two order parameters, whose solvability is encoded in two coupled gap equations and which exhibits a high-temperature superconducting phase. High-T_c superconductivity models that are based on coupled gap equations, are known in the literature: such an approach is the one due to Eliashberg [14], see also [15] for recent analysis. There, the limitations on T_c also disappear and are thus interpreted as artifacts of the Bogoliubov method. However we could not identify the underlying mechanism with the one described above. Some argumentation for a higher, compared to BCS, or unlimited transition temperature comes also from the line of considerations towards an unification of the BCS and BEC pictures [16]. Our model might be relevant here as well as it exhibits an off-diagonal long-range order, ODLRO [17, 18]. Recall that its existence is the basis for the Bose–Einstein condensation, however we are dealing here with a fermion system, so the model provides a framework for analysis of BEC in a Fermi gas [19, 20].

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