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Rough Neutrosophic Multi-Attribute Decision-Making Based on Grey Relational Analysis

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Abstract. This paper presents rough netrosophic multi-attribute decision making based on grey relational analysis. While the concept of neutrosophic sets is a powerful logic to deal with indeterminate and inconsistent data, the theory of rough neutrosophic sets is also a powerful mathematical tool to deal with incompleteness. The rating of all alternatives is expressed with the upper and lower approximation operator and the pair of neutrosophic sets which are characterized by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. Weight of each attribute is partially known to decision maker. We extend the neutrosophic grey relational analysis method to rough neutrosophic grey relational analysis method and apply it to multi-attribute decision making problem. Information entropy method is used to obtain the partially known attribute weights. Accumulated geometric operator is defined to transform rough neutrosophic number (neutrosophic pair) to single valued neutrosophic number. Neutrosophic grey relational coefficient is determined by using Hamming distance between each alternative to ideal rough neutrosophic estimates reliability solution and the ideal rough neutrosophic estimates un-reliability solution. Then rough neutrosophic relational degree is defined to determine the ranking order of all alternatives. Finally, a numerical example is provided to illustrate the applicability and efficiency of the proposed approach.

Keywords: Neutrosophic set, Rough Neutrosophic set, Single-valued neutrosophic set, Grey relational analysis, Information Entropy, Multi-attribute decision making.

Introduction

The notion of rough set theory was originally proposed by Pawlak [1, 2]. The concept of rough set theory [1, 2, 3, 4] is an extension of the crisp set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. It is a useful tool for dealing with uncertainty or imprecision information. It has been successfully applied in the different fields such as artificial intelligence [5], pattern recognition [6, 7], medical diagnosis [8, 9, 10, 11], data mining [12, 13, 14], image processing [15], conflict analysis [16], decision support systems [17,18], intelligent control [19], etc. In recent years, the rough set theory has caught a great deal of attention and interest among the researchers. Various notions that combine the concept of rough sets [1], fuzzy sets [20], vague set [21], grey set [22, 23] intuitionistic fuzzy sets [24], neutrosophic sets [25] are developed such as rough fuzzy sets [26], fuzzy rough sets [27, 28, 29], generalized fuzzy rough sets [30, 31], vague rough set [32], rough grey set [33, 34, 35, 36] rough intuitionistic fuzzy sets [37], intuitionistic fuzzy rough sets [38], rough neutrosophic sets [39, 40]. However neutrosophic set [41, 42] is the generalization of fuzzy set, intuitionistic fuzzy set, grey set, and vague set. Among the hybrid concepts, the concept of rough neutrosophic sets [39, 40] is recently proposed and very interesting. Literature review reveals that only two studies on rough neutrosophic sets [39, 40] are done.

Neutrosophic sets and rough sets are two different concepts. Literature review reflects that both are capable of handing uncertainty and incomplete information. New hybrid intelligent structure called “rough neutrosophic sets” seems to be very interesting and applicable in realistic problems. It seems that the computational techniques based on any one of these structures alone will not always provide the best results but a fusion of two or more of them can often offer better results [40].

Decision making process evolves through crisp environment to the fuzzy and uncertain and hybrid environment. Its dynamics, adaptability, and flexibility continue to exist and reflect a high degree of survival value. Approximate reasoning, fuzziness, greyness, neutrosophics and dynamic readjustment characterize this process. The decision making paradigm evolved in modern society must be strategic, powerful and pragmatic rather than retarded. Realistic model cannot be constructed without genuine understanding of the most advanced decision making model evolved so far i.e. the human decision making process. In order to perform this, very new hybrid concept such as rough neutrosophic set must be introduced in decision making model.

Decision making that includes more than one measure of performance in the evaluation process is termed as multi-attribute decision making (MADM). Different methods of MADM are available in the literature. Several methods of MADM have been studied for crisp, fuzzy, intuitionistic fuzzy, grey and neutrosophic environment. Among these, the most popular MADM methods are Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) proposed by Hwang & Yoon [43], Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) proposed by Brans et al. [44], VIšekriterijumsko KOmpromisno Rangiranje (VIKOR) developed by Opricovic & Tzeng [45], ELimination Et Choix Traduisant la REalité (ELECTRE) studied by Roy [46], ELECTRE II proposed by Roy and Bertier [47], ELECTREE III proposed by(Roy [48], ELECTRE IV proposed by Roy and Hugonnard [49), Analytical Hierarchy Process(AHP) developed by Satty [50], fuzzy AHP developed by Buckley [51], Analytic Network Process (ANP) studied by Mikhailov [52], Fuzzy TOPSIS proposed by Chen [53], single valued neutrosophic multi criteria decision making studied by Ye [54, 55, 56], neutrosophic MADM studied by Biswas et al. [57], Entropy based grey relational analysis method for MADM studied by Biswas et al. [58]. A small number of applications of neutrosophic MADM are available in the literature. Mondal and Pramanik [59] used neutrosophic multicriteria decision making for teacher selection in higher education. Mondal and Pramanik [60] also developed model of school choice using neutrosophic MADM based on grey relational analysis. However, MADM in rough neutrosophic environment is yet to appear in the literature. In this paper, an attempt has been made to develop rough neutrosophic MADM based on grey relational analysis.

Rest of the paper is organized in the following way. Section 2 presents preliminaries of neutrosophic sets and rough neutrosophic sets. Section 3 is devoted to present rough neutrosophic multi-attribute decision-making based on grey relational analysis. Section 4 presents a numerical example of the proposed method. Finally, section 5 presents concluding remarks and direction of future research.

2 Mathematical Preliminaries

2.1 Definitions on neutrosophic Set

The concept of neutrosophy set is originated from the new branch of philosophy, namely, neutrosophy. Neutrosophy [25] gains very popularity because of its capability to deal with the origin, nature, and scope of neutralities, as well as their interactions with different conceptional spectra.

Definition2.1.1: Let E be a space of points (objects) with generic element in E denoted by y. Then a neutrosophic set $N1$ in E is characterized by a truth membership function T_{N1} , an indeterminacy membership function I_{N1} and a falsity membership function F_{N1}. The functions T_{N1} and F_{N1} are real standard or non-standard subsets of $]^-0,1^+[$ that is $T_{N1}: E \to]^-0,1^+[$; $I_{N1}: E \to]^-0,1^+[$; $F_{N1}: E \to]^-0,1^+[$.

It should be noted that there is no restriction on the sum of $T_{N1}(y)$, $I_{N1}(y)$, $F_{N1}(y)$ i.e.

$$^-0 \le T_{N1}(y) + I_{N1}(y) + F_{N1}(y) \le 3^+$$

Definition2.1.2: (complement) The complement of a neutrosophic set A is denoted by $N1^c$ and is defined by

$$T_{N1^c}(y) = \{1^+\} - T_{N1}(y);\ I_{N1^c}(y) = \{1^+\} - I_{N1}(y)$$

$$F_{N1^c}(y) = \{1^+\} - F_{N1}(y)$$

Definition2.1.3: (Containment) A neutrosophic set $N1$ is contained in the other neutrosophic set $N2$, $N1 \subseteq N2$ if and only if the following result holds.

$$\inf T_{N1}(y) \le \inf T_{N2}(y),\ \sup T_{N1}(y) \le \sup T_{N2}(y)$$

$$\inf I_{N1}(y) \ge \inf I_{N2}(y),\ \sup I_{N1}(y) \ge \sup I_{N2}(y)$$

$$\inf F_{N1}(y) \ge \inf F_{N2}(y),\ \sup F_{N1}(y) \ge \sup F_{N2}(y)$$

for all y in E.

Definition2.1.4: (Single-valued neutrosophic set). Let E be a universal space of points (objects) with a generic element of E denoted by y.

A single valued neutrosophic set [61] S is characterized by a truth membership function $T_N(y)$, a falsity membership function $F_N(y)$ and indeterminacy membership function $I_N(y)$ with $T_N(y)$, $F_N(y)$, $I_N(y) \in [0,1]$ for all y in E.

When E is continuous, a SNVS S can be written as follows:

$$S = \int_y \langle T_S(y), F_S(y), I_S(y) \rangle / y, \forall y \in E$$

and when E is discrete, a SVNS S can be written as follows:

$$S = \sum \langle T_S(y), F_S(y), I_S(y) \rangle / y, \forall y \in E$$

It should be observed that for a SVNS S,

$$0 \le \sup T_S(y) + \sup F_S(y) + \sup I_S(y) \le 3,\ \forall y \in E$$

Definition2.1.5: The complement of a single valued neutrosophic set S is denoted by S^c and is defined by

$$T_S^c(y) = F_S(y);\ I_S^c(y) = 1 - I_S(y);\ F_S^c(y) = T_S(y)$$

Definition2.1.6: A SVNS S_{N1} is contained in the other SVNS S_{N2} , denoted as $S_{N1} \subseteq S_{N2}$ iff, $T_{S_{N1}}(y) \le T_{S_{N2}}(y)$; $I_{S_{N1}}(y) \ge I_{S_{N2}}(y)$; $F_{S_{N1}}(y) \ge F_{S_{N2}}(y)$, $\forall y \in E$.

Definition2.1.7: Two single valued neutrosophic sets S_{N1} and S_{N2} are equal, i.e. $S_{N1}= S_{N2}$, iff, $S_{N1} \subseteq S_{N2}$ and $S_{N1} \supseteq S_{N2}$

Definition2.1.8: (Union) The union of two SVNSs S_{N1} and S_{N2} is a SVNS S_{N3} , written as $S_{N3} = S_{N1} \cup S_{N2}$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to S_{N1} and S_{N2} as follows:

$T_{S_{N3}}(y) = max(T_{S_{N1}}(y), T_{S_{N2}}(y))$;

$I_{S_{N3}}(y) = max(I_{S_{N1}}(y), I_{S_{N2}}(y))$;

$F_{S_{N3}}(y) = min(F_{S_{N1}}(y), F_{S_{N2}}(y))$ for all y in E.

Definition2.1.9: (Intersection) The intersection of two SVNSs $N1$ and $N2$ is a SVNS $N3$, written as $N3 = N1 \cap N2$. Its truth membership, indeterminacy membership and falsity membership functions are related to $N1$ an $N2$ as follows:

$T_{S_{N3}}(y) = \min(T_{S_{N1}}(y), T_{S_{N2}}(y))$;

$I_{S_{N3}}(y) = \max(I_{S_{N1}}(y), I_{S_{N2}}(y))$;

$F_{S_{N3}}(y) = \max(F_{S_{N1}}(y), F_{S_{N2}}(y))$, $\forall y \in E$.

Distance between two neutrosophic sets.

The general SVNS can be presented in the follow form $S = \{(y/(T_S(y), I_S(y), F_S(y))) : y \in E\}$

Finite SVNSs can be represented as follows:

$$S = \begin{Bmatrix} (y_1/(T_S(y_1), I_S(y_1), F_S(y_1))), \cdots, \\ (y_m/(T_S(y_m), I_S(y_m), F_S(y_m))) \end{Bmatrix}, \forall y \in E \quad (1)$$

Definition 2.1.10: Let

$$S_{N1} = \begin{Bmatrix} (y_1/(T_{S_{N1}}(y_1), I_{S_{N1}}(y_1), F_{S_{N1}}(y_1))), \cdots, \\ (y_n/(T_{S_{N1}}(y_n), I_{S_{N1}}(y_n), F_{S_{N1}}(y_n))) \end{Bmatrix} \quad (2)$$

$$S_{N2} = \begin{Bmatrix} (x_1/(T_{S_{N2}}(y_1), I_{S_{N2}}(y_1), F_{S_{N2}}(y_1))), \cdots, \\ (x_n/(T_{S_{N2}}(y_n), I_{S_{N2}}(y_n), F_{S_{N2}}(y_n))) \end{Bmatrix} \quad (3)$$

be two single-valued neutrosophic sets, then the Hamming distance [57] between two SNVS $N1$and $N2$ is defined as follows:

$$d_S(S_{N1}, S_{N2}) = \sum_{i=1}^{n} \left\langle \begin{matrix} |T_{S_{N1}}(y) - T_{S_{N2}}(y)| + \\ |I_{S_{N1}}(y) - I_{S_{N2}}(y)| + \\ |F_{S_{N1}}(y) - F_{S_{N2}}(y)| \end{matrix} \right\rangle \quad (4)$$

and normalized Hamming distance [58] between two SNVSs S_{N1} and S_{N2} is defined as follows:

$${}^N d_S(S_{N1}, S_{N2}) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \begin{matrix} |T_{S_{N1}}(y) - T_{S_{N2}}(y)| + \\ |I_{S_{N1}}(y) - I_{S_{N2}}(y)| + \\ |F_{S_{N1}}(y) - F_{S_{N2}}(y)| \end{matrix} \right\rangle \quad (5)$$

with the following properties

1. $0 \le d_S(S_{N1}, S_{N2}) \le 3n$ (6)
2. $0 \le {}^N d_S(S_{N1}, S_{N2}) \le 1$ (7)

2.2 Definitions on rough neutrosophic set

There exist two basic components in rough set theory, namely, crisp set and equivalence relation, which are the mathematical basis of RSs. The basic idea of rough set is based on the approximation of sets by a couple of sets known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. Rough neutrosophic sets [39, 40] is the generalization of rough fuzzy set [26] and rough intuitionistic fuzzy set [37].

Definition2.2.1: Let Z be a non-null set and R be an equivalence relation on Z. Let P be neutrosophic set in Z with the membership function T_P, indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of P in the approximation (Z, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$\underline{N}(P) = \left\langle \begin{matrix} < x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > / \\ z \in [x]_R, x \in Z \end{matrix} \right\rangle, \quad (8)$$

$$\overline{N}(P) = \left\langle \begin{matrix} < x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) > / \\ z \in [x]_R, x \in Z \end{matrix} \right\rangle \quad (9)$$

Here, $T_{\underline{N}(P)}(x) = \wedge_z \in [x]_R \, T_P(z)$,

$I_{\underline{N}(P)}(x) = \wedge_z \in [x]_R \, I_P(z)$, $F_{\underline{N}(P)}(x) = \wedge_z \in [x]_R \, F_P(z)$,

$T_{\overline{N}(P)}(x) = \vee_z \in [x]_R \, T_P(z)$, $I_{\overline{N}(P)}(x) = \vee_z \in [x]_R \, T_P(z)$,

$F_{\overline{N}(P)}(x) = \vee_z \in [x]_R \, I_P(z)$

So, $0 \le T_{\underline{N}(P)}(x) + I_{\underline{N}(P)}(x) + F_{\underline{N}(P)}(x) \le 3$

$0 \le T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \le 3$

Here $\vee$ and $\wedge$ present the "max" and the "min'' operators respectively. $T_P(z)$, $I_P(z)$ and $F_P(z)$ present the membership, indeterminacy and non-membership of z with respect to P. It is very easy to observe that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Z. Therefore, the NS mapping $\underline{N}$, $\overline{N}$: $N(Z) \rightarrow N(Z)$ presents the lower and upper rough NS approximation operators. The pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set [40] in (Z, R).

Based on the above definition, it is observed that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence clases of R if $\underline{N}(P) = \overline{N}(P)$

i.e $T_{\underline{N}(P)}(x) = T_{\overline{N}(P)}(x)$, $I_{\underline{N}(P)}(x) = I_{\overline{N}(P)}(x)$,

$F_{\underline{N}(P)}(x) = F_{\overline{N}(P)}(x)$.

P is said to be a definable neutrosophic set in the approximation (*Z, R*). It can be easily proved that zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets [40].

Definition2.2.2 If $N(P) = (\underline{N}(P), \overline{N}(P))$ is a rough neutrosophic set in (*E, R*) , the rough complement of $N(P)$ [40] is the rough neutrosophic set denoted by $\sim N(P) = (\underline{N}(P)^c, \overline{N}(P)^c)$,where $\underline{N}(P)^c, \overline{N}(P)^c$ represent the complements of neutrosophic sets of $\underline{N}(P), \overline{N}(P)$ respectively.

$$\underline{N}(P)^c = \left\langle {< x, T_{\underline{N}(P)}(x), 1 - I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) >/ \atop , x \in Z} \right\rangle, \text{ ,and}$$

$$\overline{N}(P)^c = \left\langle {< x, T_{\underline{N}(P)}(x), 1 - I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) >/ \atop , x \in Z} \right\rangle \qquad (10)$$

Definition2.2.3. If $N(P_1)$ *and* $N(P_2)$ are two rough neutrosophic sets of the neutrosophic sets respectively in *Z*, then Bromi et al. [40] defined the following definitions.

$$N(P_1) = N(P_2) \Leftrightarrow \underline{N}(P_1) = \underline{N}(P_2) \wedge \overline{N}(P_1) = \overline{N}(P_2)$$

$$N(P_1) \subseteq N(P_2) \Leftrightarrow \underline{N}(P_1) \subseteq \underline{N}(P_2) \wedge \overline{N}(P_1) \subseteq \overline{N}(P_2)$$

$$N(P_1) \cup N(P_2) = < \underline{N}(P_1) \cup \underline{N}(P_2), \overline{N}(P_1) \cup \overline{N}(P_2) >$$

$$N(P_1) \cap N(P_2) = < \underline{N}(P_1) \cap \underline{N}(P_2), \overline{N}(P_1) \cap \overline{N}(P_2) >$$

$$N(P_1) + N(P_2) = < \underline{N}(P_1) + \underline{N}(P_2), \overline{N}(P_1) + \overline{N}(P_2) >$$

$$N(P_1) \,.\, N(P_2) = < \underline{N}(P_1) \,.\, \underline{N}(P_2), \overline{N}(P_1) \,.\, \overline{N}(P_2) >$$

If *N, M, L* are rough neutrosophic sets in (*Z, R*), then the following proposition [40] are stated from definitions

Proposition i:

1. $\sim N(\sim N) = N$
2. $N \cup M = M \cup N, M \cup N = N \cup M$
3. $(L \cup M) \cup N = L \cup (M \cup N)$,
   $(L \cap M) \cap N = L \cap (M \cap N)$
4. $(L \cup M) \cap N = (L \cup M) \cap (L \cup N)$,
   $(L \cap M) \cup N = (L \cap M) \cup (L \cap N)$

Proposition ii:

De Morgan's Laws are satisfied for neutrosophic sets

1. $\sim (N(P_1) \cup N(P_2)) = (\sim N(P_1)) \cap (\sim N(P_2))$
2. $\sim (N(P_1) \cap N(P_2)) = (\sim N(P_1)) \cup (\sim N(P_2))$

Proposition iii:

If P_1 and P_2 are two neutrosophic sets in *U* such that $P_1 \subseteq P_2$ *, then* $N(P_1) \subseteq N(P_2)$

1. $N(P_1 \cap P_2) \subseteq N(P_2) \cap N(P_2)$
2. $N(P_1 \cup P_2) \supseteq N(P_2) \cup N(P_2)$

Proposition iv:

1. $\underline{N}(P) = \sim \overline{N}(\sim P)$
2. $\overline{N}(P) = \sim \underline{N}(\sim P)$
3. $\underline{N}(P) \subseteq \overline{N}(P)$

rough neutrosophic multi-attribute decision-making based on grey relational analysis

3. Rough neutrosophic multi-attribute decision-making based on grey relational analysis

We consider a multi-attribute decision making problem with m alternatives and n attributes. Let $A_1, A_2, ..., A_m$ and $C_1, C_2, ..., C_n$ represent the alternatives and attributes respectively.

The rating reflects the performance of the alternative A_i against the attribute C_j. For MADM weight vector $W = \{w_1, w_2,...,w_n\}$ is assigned to the attributes. The weight w_j (j = 1, 2, ..., n) reflects the relative importance of the attribute C_j (j = 1, 2, ..., m) to the decision making process. The weights of the attributes are usually determined on subjective basis. They represent the opinion of a single decision maker or accumulate the opinions of a group of experts using group decision technique. The values associated with the alternatives for MADM problems are presented in the Table 1.

Table1: Rough neutrosophic decision matrix

$$D = \langle \underline{d}_{ij}, \overline{d}_{ij} \rangle_{m \times n} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \langle \underline{d}_{11}, \overline{d}_{11} \rangle & \langle \underline{d}_{12}, \overline{d}_{12} \rangle & \ldots & \langle \underline{d}_{1n}, \overline{d}_{1n} \rangle \\ A_2 & \langle \underline{d}_{21}, \overline{d}_{21} \rangle & \langle \underline{d}_{22}, \overline{d}_{22} \rangle & \ldots & \langle \underline{d}_{2n}, \overline{d}_{2n} \rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ A_m & \langle \underline{d}_{m1}, \overline{d}_{m1} \rangle & \langle \underline{d}_{m2}, \overline{d}_{m2} \rangle & \ldots & \langle \underline{d}_{mn}, \overline{d}_{mn} \rangle \end{array} \qquad (11)$$

Where $\langle \underline{d}_{ij}, \overline{d}_{ij} \rangle$ is rough neutrosophic number according to the *i*-th alternative and the *j*-th attribute.

Grey relational analysis [GRA] [62] is a method of measuring degree of approximation among sequences according to the grey relational grade. Grey system theory deals with primarily on multi-input, incomplete, or uncertain information. GRA is suitable for solving problems with complicated relationships between multiple factors and variables. The theories of grey relationalanalysis have already caught much attention and interest among the researchers [63, 64]. In educational field, Pramanik and

Mukhopadhyaya [65] studied grey relational analysis based intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. Rough neutrosophic multi-attribute decision-making based on grey relational analysis is presented by the following steps.

Step1: Determination the most important criteria.

Many attributes may be involved in decision making problems. However, all attributes are not equally important. So it is important to select the proper criteria for decision making situation. The most important criteria may be selected based on experts' opinions.

Step2: Data pre-processing

Considering a multiple attribute decision making problem having m alternatives and n attributes, the general form of decision matrix can be presented as shown in Table-1. It may be mentioned here that the original GRA method can effectively deal mainly with quantitative attributes. There exists some complexity in the case of qualitative attributes. In the case of a qualitative attribute (i.e. quantitative value is not available), an assessment value is taken as rough neutrosophic number.

Step3: Construction of the decision matrix with rough neutrosophic form

For multi-attribute decision making problem, the rating of alternative A_i (i = 1, 2,…m) with respect to attribute C_j (j = 1, 2,…n) is assumed to be rough neutrosophic sets. It can be represented with the following form.

$$A_i=\left[\begin{array}{l} C_1 \Big/ \left\langle \underline{N}_1(\underline{T}_{i1}\,\underline{I}_{i1}\,\underline{F}_{i1}), \overline{N}_1(\overline{T}_{i1}\,\overline{I}_{i1}\,\overline{F}_{i1})\right\rangle, \\ C_2 \Big/ \left\langle \underline{N}_2(\underline{T}_{i2}\,\underline{I}_{i2}\,\underline{F}_{i2}), \overline{N}_2(\overline{T}_{i2}\,\overline{I}_{i2}\,\overline{F}_{i2})\right\rangle, \cdots, \\ C_n \Big/ \left\langle \underline{N}_n(\underline{T}_{in}\,\underline{I}_{in}\,\underline{F}_{in}), \overline{N}_n(\overline{T}_{in}\,\overline{I}_{in}\,\overline{F}_{in})\right\rangle : C_j \in C \end{array}\right]$$

$$= \left[C_j \Big/ \left\langle \underline{N}_j(\underline{T}_{ij}\,\underline{I}_{ij}\,\underline{F}_{ij}), \overline{N}_j(\overline{T}_{ij}\,\overline{I}_{ij}\,\overline{F}_{ij})\right\rangle : C_j \in C \right] \textit{ for } \quad (12)$$

$$j = 1, 2, \cdots, n$$

Here $\overline{N}$ and $\underline{N}$ are neutrosophic sets with

$\left\langle \overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij} \right\rangle$ *and* $\left\langle \underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij} \right\rangle$

are the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative A_i satisfying the attribute C_j, respectively where

$0 \le \underline{T}_{ij}, \overline{T}_{ij} \le 1$, $\quad 0 \le \underline{I}_{ij}, \overline{I}_{ij} \le 1$, $\quad 0 \le \underline{F}_{ij}, \overline{F}_{ij} \le 1$

$0 \le \underline{T}_{ij} + \underline{I}_{ij} + \underline{F}_{ij} \le 3$, $\; 0 \le \overline{T}_{ij} + \overline{I}_{ij} + \overline{F}_{ij} \le 3$

The rough neutrosophic decision matrix (see Table 2) can be presented in the following form:

Table 2. Rough neutrosophic decision matrix

$d_{\tilde{N}} = \left\langle \underline{N}_{ij}(F), \overline{N}_{ij}(F) \right\rangle_{m\times n} =$

	C_1	C_2	...	C_n
A_1	$\langle \underline{N}_{11}, \overline{N}_{11} \rangle$	$\langle \underline{N}_{12}, \overline{N}_{12} \rangle$	...	$\langle \underline{N}_{1n}, \overline{N}_{1n} \rangle$
A_2	$\langle \underline{N}_{21}, \overline{N}_{21} \rangle$	$\langle \underline{N}_{22}, \overline{N}_{22} \rangle$	...	$\langle \underline{N}_{2n}, \overline{N}_{2n} \rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle \underline{N}_{n1}, \overline{N}_{n1} \rangle$	$\langle \underline{N}_{n2}, \overline{N}_{n2} \rangle$	...	$\langle \underline{N}_{mn}, \overline{N}_{mn} \rangle$

(13)

Where $\underline{N}_{ij}$ *and* $\overline{N}_{ij}$ are lower and upper approximations of the neutrosophic set *P*.

Step4: Determination of the accumulated geometric operator.

Let us consider a rough neutrosophic set as $\left\langle \underline{N}_{ij}(\underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij}), \overline{N}_{ij}(\overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij}) \right\rangle$

We transform the rough neutrosophic number to SVNSs by the following operator. The Accumulated Geometric Operator (AGO) is defined in the following form:

$$N_{ij}\left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle =$$

$$N_{ij}\left\langle \left(\underline{T}_{ij}.\overline{T}_{ij}\right)^{0.5}, \left(\underline{I}_{ij}.\overline{I}_{ij}\right)^{0.5}, \left(\underline{F}_{ij}.\overline{F}_{ij}\right)^{0.5}, \right\rangle \quad (14)$$

The decision matrix (see Table 3) is transformed in the form of SVNSs as follows:

Table 3. Transformed decision matrix in the form SVNS

$d_S = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m\times n} =$

	C_1	C_2	...	C_n
A_1	$\langle T_{11}, I_{11}, F_{11} \rangle$	$\langle T_{12}, I_{12}, F_{12} \rangle$	...	$\langle T_{1n}, I_{1n}, F_{1n} \rangle$
A_2	$\langle T_{21}, I_{21}, F_{21} \rangle$	$\langle T_{22}, I_{22}, F_{22} \rangle$	...	$\langle T_{2n}, I_{2n}, F_{2n} \rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle T_{m1}, I_{m1}, F_{m1} \rangle$	$\langle T_{m2}, I_{m2}, F_{m2} \rangle$	...	$\langle T_{mn}, I_{mn}, F_{mn} \rangle$

(15)

Step5: Determination of the weights of criteria.

During decision-making process, decision makers may often encounter with partially known or unknown attribute weights. So, it is crucial to determine attribute weight for proper decision making. Many methods are available in the literatre to determine the unknown attribute weight such as maximizing deviation method proposed by Wu and Chen [66], entropy method proposed by Wei and Tang [67], and Xu and Hui [68]), optimization method proposed by Wang and Zhang [69], Majumder and Samanta [70].

Biswas et al. [57] used entropy method [70] for single valed neutrosophic MADM.

In this paper we use an entropy method for determining attribute weight. According to Majumder and Samanta [70], the entropy measure of a SVNS

$S_{N1}=\left\langle T_{S_{N1}}(x_i), I_{S_{N1}}(x_i), F_{S_{N1}}(x_i)\right\rangle$

$$En_i(S_{N1}) = 1-\frac{1}{n}\sum_{i=1}^{m}(T_{S_{N1}}(x_i)+F_{S_{N1}}(x_i))\left|I_{S_{N1}}(x_i)-I^{c}{}_{S_{N1}}(x_i)\right| \quad (16)$$

which has the following properties:

1. $En_i(S_{N1})=0 \Rightarrow S_{N1}$ *is a crisp set and* $I_{S_{N1}}(x_i)=0\ \forall x\in E.$
2. $En_i(S_{N1})=1 \Rightarrow \left\langle T_{S_{N1}}(x_1), I_{S_{N1}}(x_1), F_{S_{N1}}(x_1)\right\rangle = \left\langle 0.5, 0.5, 0.5\right\rangle \forall x\in E.$
3. $En_i(S_{N1})\geq En_i(S_{N2}) \Rightarrow (T_{S_{N1}}(x_1)+F_{S_{N1}}(x_1)\leq (T_{S_{N2}}(x_1)+F_{S_{N2}}(x_1)$ *and* $\left|I_{S_{N1}}(x_1)-I^{c}{}_{S_{N1}}(x_1)\right|\leq\left|I_{S_{N2}}(x_1)-I^{c}{}_{S_{N2}}(x_1)\right|$
4. $En_i(S_{N1})=En_i(S_{N1^c})\ \forall x\in E.$

In order to obtain the entropy value En_j of the j-th attribute C_j (j = 1, 2,…, n), equation (16) can be written as follows:

$$En_j=1-\frac{1}{n}\sum_{i=1}^{m}(T_{ij}(x_i)+F_{ij}(x_i))\left|I_{ij}(x_i)-I_{ij}^{C}(x_i)\right|$$

For i = 1, 2, …, n; j = 1, 2, …, m (17)

It is observed that $E_j \in [0,1]$. Due to Hwang and Yoon [71], and Wang and Zhang [69] the entropy weight of the j-th attibute C_j is presented as:

$$W_j=\frac{1-En_j}{\sum_{j=1}^{n}\left(1-En_j\right)} \quad (18)$$

We have weight vector W= $(w_1, w_2,…,w_n)^T$ of attributes C_j (j = 1, 2, …, n) with $w_j\geq 0$ and $\sum_{i=1}^{n}w_j=1$

Step6: Determination of the ideal rough neutrosophic estimates reliability solution (IRNERS) and the ideal rough neutrosophic estimates un-reliability solution (IRNEURS) for rough neutrosophic decision matrix.

Based on the concept of the neutrosophic cube [72], maximum reliability occurs when the indeterminacy membership grade and the degree of falsity membership grade reach minimum simultaneously. Therefore, the ideal neutrosophic estimates reliability solution (INERS) $R_S^{+}=\left[r_{S_1}^{+}, r_{S_2}^{+},\cdots,r_{S_n}^{+}\right]$ is defined as the solution in which every component $r_{S_j}^{+}=\left\langle T_j^{+}, I_j^{+}, F_j^{+}\right\rangle$ is defined as follows: $T_j^{+}=\max_i\left\{T_{ij}\right\},\ I_j^{+}=\min_i\left\{I_{ij}\right\}$ and $F_j^{+}=\min_i\left\{F_{ij}\right\}$ in the neutrosophic decision matrix $D_S=\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle_{m\times n}$ (see the Table 1) for i = 1, 2, …, n and j = 1, 2, …, m.

Based on the concept of the neutrosophic cube [72], maximum un-reliability occurs when the indeterminacy membership grade and the degree of falsity membership grade reach maximum simultaneously. So, the ideal neutrosophic estimates unreliability solution (INEURS) $R_S^{-}=\left[r_{S_1}^{-}, r_{S_2}^{-},\cdots,r_{S_n}^{-}\right]$ is the solution in which every component $r_{S_j}^{-}=\left\langle T_j^{-}, I_j^{-}, F_j^{-}\right\rangle$ is defined as follows: $T_j^{-}=\max_i\left\{T_{ij}\right\},\ I_j^{-}=\min_i\left\{I_{ij}\right\}$ *and* $F_j^{-}=\min_i\left\{F_{ij}\right\}$ in the neutrosophic decision matrix $D_S=\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle_{m\times n}$ (see the Table 1)for i = 1, 2, …, n and j = 1, 2, …, m.

For the rough neutrosophic decision making matrix D = $\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle_{m\times n}$ (see Table 1), T_{ij}, I_{ij}, F_{ij} are the degrees of membership, degree of indeterminacy and degree of non membership of the alternative A_i of A satisfying the attribute C_j of C.

Step7: Calculation of the rough neutrosophic grey relational coefficient of each alternative from IRNERS and IRNEURS.

Rough grey relational coefficient of each alternative from IRNERS is:

$$G_{ij}^{+}=\frac{\min_i\min_j\Delta_{ij}^{+}+\rho\max_i\max_j\Delta_{ij}^{+}}{\Delta_{ij}^{+}+\rho\max_i\max_j\Delta_{ij}^{+}}, \text{ where}$$

$\Delta_{ij}^{+}=d\left(q_{S_j}^{+}, q_{S_{ij}}\right)$, i=1, 2, …,m and j=1, 2, ….,n (19)

Rough grey relational coefficient of each alternative from IRNEURS is:

$$G_{ij}^{-}=\frac{\min_i\min_j\Delta_{ij}^{-}+\rho\max_i\max_j\Delta_{ij}^{-}}{\Delta_{ij}^{-}+\rho\max_i\max_j\Delta_{ij}^{-}}, \text{ where}$$

$\Delta_{ij}^{-}=d\left(q_{S_{ij}}, q_{S_{ij}}^{-}\right)$, i=1, 2, …,m and j=1, 2, ….,n (20)

$\rho\in[0,1]$ is the distinguishable coefficient or the identification coefficient used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. When $\rho=1$, the comparison environment is unchanged; when $\rho=0$, the comparison environment disappears. Smaller value of distinguishing coefficient reflests the large range of grey relational coefficient. Generally, $\rho=0.5$ is fixed for decision making .

Step8: Calculation of the rough neutrosophic grey relational coefficient.

Rough neutrosophic grey relational coefficient of each alternative from IRNERS and IRNEURS are defined respectively as follows:

$$G_i^+ = \sum_{j=1}^{n} w_j G_{ij}^+ \quad \text{for } i=1, 2, \ldots, m \tag{21}$$

$$G_i^- = \sum_{j=1}^{n} w_j G_{ij}^- \quad \text{for } i=1, 2, \ldots, m \tag{22}$$

Step9: Calculation of the rough neutrosophic relative relational degree.

Rough neutrosophic relative relational degree of each alternative from Indeterminacy Trthfullness Falsity Positive Ideal Soltion (ITFPIS) is defined as follows:

$$\Re_i = \frac{G_i^+}{G_i^- + G_i^+}, \text{ for } i=1, 2, \ldots, m \tag{23}$$

Step 10: Ranking the alternatives.

The ranking order of all alternatives can be determined according to the decreasing order of the rough relative relational degree. The highest value of $\Re_i$ indicates the best alternative.

4 Numerical example

In this section, rough neutrosophic MADM is considered to demonstrate the application and the effectiveness of the proposed approach. Let us consider a decision-making problem stated as follows. Suppose there is a conscious guardian, who wants to admit his/her child to a suitable school for proper education. There are three schools (possible alternatives) to admit his/her child: (1) A_1 is a Christian Missionary School; (2) A_2 is a Basic English Medium School; (3) A_3 is a Bengali Medium Kindergarten. The guardian must take a decision based on the following four criteria: (1) C_1 is the distance and transport; (2) C_2 is the cost; (3) C_3 is stuff and curriculum; and (4) C_4 is the administration and other facilites. We obtain the following rough neutrosophic decision matrix (see the Table 4) based on the experts' assessment:

Table 4. Decision matrix with rough neutrosophic number

$d_S = \langle \underline{N}_{ij}(P), \overline{N}_{ij}(P) \rangle_{3\times 4} =$

	C_1	C_2	C_3	C_4
A_1	$\langle(.6,.3,.4),(.8,.2,.2)\rangle$	$\langle(.6,.4,.3),(.8,.2,.2)\rangle$	$\langle(.6,.4,.3),(.8,.2,.1)\rangle$	$\langle(.7,.4,.4),(.9,.2,.2)\rangle$
A_2	$\langle(.7,.4,.3),(.8,.1,.2)\rangle$	$\langle(.6,.3,.3),(.8,.1,.1)\rangle$	$\langle(.7,.2,.3),(.8,.2,.1)\rangle$	$\langle(.7,.4,.4),(.8,.2,.2)\rangle$
A_3	$\langle(.7,.2,.3),(.8,.2,.1)\rangle$	$\langle(.7,.4,.3),(.8,.2,.2)\rangle$	$\langle(.7,.2,.2),(.9,.2,.1)\rangle$	$\langle(.8,.3,.2),(.9,.1,.1)\rangle$

(23)

Step2: Determination of the decision matrix in the form SVNS

Using accumulated geometric operator (AGO) from equation (13) we have the decision matrix in SVNS form is presented as follows:

	C_1	C_2	C_3	C_4
A_1	$\langle .6928, .2449, .2828 \rangle$	$\langle .6928, .2828, .2449 \rangle$	$\langle .6928, .2828, .1732 \rangle$	$\langle .7937, .2828, .2828 \rangle$
A_2	$\langle .7483, .2000, .2449 \rangle$	$\langle .6928, .1732, .1732 \rangle$	$\langle .7483, .2000, .1414 \rangle$	$\langle .7483, .2449, .2000 \rangle$
A_3	$\langle .7483, .2000, .1732 \rangle$	$\langle .7483, .2828, .2449 \rangle$	$\langle .7937, .2000, .1414 \rangle$	$\langle .8485, .1732, .1414 \rangle$

(24)

Step3: Determination of the weights of attribute

Entropy value En_j of the j-th (j = 1, 2, 3) attributes can be determined from the decision matrix d_S (15) and equation (17) as: En_1= 0.4512, En_2 = 0.5318, En_3 = 0.5096, En_4= 0.4672.

Then the corresponding entropy weights w_1, w_2, w_3, w_4 of all attributes according to equation (17) are obtained by w_1 = 0.2700, w_2 =0.2279, w_3 =0.2402, w_4 =0.2619 such that $\sum_{j=1}^{n} w_j = 1$

Step4: Determination of the ideal rough neutrosophic estimates reliability solution (IRNERS):

$$Q_S^+ = \langle q_{S1}^+, q_{S2}^+, q_{S3}^+, q_{S4}^+ \rangle = \begin{bmatrix} \langle \max_i\{T_{i1}\}, \min_i\{I_{i1}\}, \min_i\{F_{i1}\} \rangle, \\ \langle \max_i\{T_{i2}\}, \min_i\{I_{i2}\}, \min_i\{F_{i2}\} \rangle, \\ \langle \max_i\{T_{i3}\}, \min_i\{I_{i3}\}, \min_i\{F_{i3}\} \rangle, \\ \langle \max_i\{T_{i4}\}, \min_i\{I_{i4}\}, \min_i\{F_{i4}\} \rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle 0.7483, 0.2000, 0.1732 \rangle, \langle 0.7483, 0.1732, 0.1732 \rangle, \\ \langle 0.7937, 0.2000, 0.1414 \rangle, \langle 0.8485, 0.1732, 0.1414 \rangle \end{bmatrix}$$

Step5: Determination of the ideal rough neutrosophic estimates un-reliability solution (IRNEURS):

$$Q_S^- = \langle q_{S1}^-, q_{S2}^-, q_{S3}^-, q_{S4}^- \rangle = \begin{bmatrix} \langle \min_i\{T_{i1}\}, \max_i\{I_{i1}\}, \max_i\{F_{i1}\} \rangle, \\ \langle \min_i\{T_{i2}\}, \max_i\{I_{i2}\}, \max_i\{F_{i2}\} \rangle, \\ \langle \min_i\{T_{i3}\}, \max_i\{I_{i3}\}, \max_i\{F_{i3}\} \rangle, \\ \langle \min_i\{T_{i4}\}, \max_i\{I_{i4}\}, \max_i\{F_{i4}\} \rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle 0.6928, 0.2449, 0.2828 \rangle, \langle 0.6928, 0.2828, 0.2449 \rangle, \\ \langle 0.6928, 0.2828, 0.1732 \rangle, \langle 0.7483, 0.2828, 0.2828 \rangle \end{bmatrix}$$

Step 6: Calculation of the rough neutrosophic grey relational coefficient of each alternative from IRNERS and IRNEURS.

By using Equation (19) the rough neutrosophic grey relational coefficient of each alternative from IRNERS can be obtained

$$\left[G_{ij}^{+}\right]_{3\times4}=\begin{bmatrix}0.3333 & 0.6207 & 0.6341 & 0.5544\\ 0.7645 & 0.8075 & 0.8368 & 0.6305\\ 1.0000 & 0.6399 & 1.0000 & 1.0000\end{bmatrix} \quad (25)$$

Similarly, from Equation (20) the rough neutrosophic grey relational coefficient of each alternative from IRNEURS is

$$\left[G_{ij}^{-}\right]_{3\times4}=\begin{bmatrix}1.0000 & 1.0000 & 1.0000 & 0.5403\\ 0.5948 & 0.4752 & 0.5314 & 0.5266\\ 0.4755 & 0.6812 & 0.4690 & 0.3333\end{bmatrix} \quad (26)$$

Step7: Determine the degree of rough neutrosophic grey relational co-efficient of each alternative from INERS and IRNEURS. The required rough neutrosophic grey relational co-efficient corresponding to IRNERS is obtained by using equations (20) as:

$$G_1^{+}= 0.5290,\ G_2^{+}= 0.7566,\ G_3^{+}= 0.9179 \quad (27)$$

and corresponding to IRNEURS is obtained with the help of equation (21) as:

$$G_1^{-}= 0.8796,\ G_2^{-}= 0.5345,\ G_3^{-}= 0.4836 \quad (28)$$

Step8: Thus rough neutrosophic relative degree of each alternative from IRNERS can be obtained with the help of equation (22) as:

$$\Re_1 = 0.3756,\ \Re_2 =0.5860,\ \Re_3 =0.6549 \quad (29)$$

Step9: The ranking order of all alternatives can be determined according to the value of rough neutrosophic relational degree i.e. $\Re_3 > \Re_2 > \Re_1$. It is seen that the highest value of rough neutrosophic relational degree is R_3 therefore A_3 (Bengali Medium Kindergarten) is the best alternative (school) to admit the child.

Conclusion

In this paper, we introduce rough neutrosophic multi-attribute decision-making based on modified GRA. The concept of rough set, netrosophic set and grey system theory are fused to conduct the study first time. We define the Accumulated Geometric Operator (AGO) to transform rough neutrosophic matrix to SVNS. Here all the attribute weights information are partially known. Entropy based modified GRA analysis method is introduced to solve this MADM problem. Rough neutrosophic grey relation coefficient is proposed for solving multiple attribute decision-making problems. Finally, an illustrative example is provided to show the effectiveness and applicability of the proposed approach.

However, we hope that the concept presented here will open new approach of research in current rough neutrosophic decision-making field. The main thrsts of the paper will be in the field of practical decision-making, pattern recognition, medical diagnosis and clustering analysis.

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