Testing the DGP model with ESSENCE

Sara Rydbeck[1], Malcolm Fairbairn[2,3] and Ariel Goobar[1]

[1]Department of Physics, Stockholm University,
Albanova University Center, S–106 91 Stockholm, Sweden
[2]Perimeter Institute for Theoretical Physics, 31 Caroline Street North,
Waterloo, Ontario, Canada N2L 2Y5
[3]CERN Theory Division, CH-1211, Geneva 23, Switzerland

Abstract. We use the recent supernova data set from the ESSENCE collaboration combined with data from the Supernova Legacy Survey and nearby supernovae to test the DGP brane world model and its generalisations. Combination of this data with a flatness prior and the position of the peak of the CMB disfavours the DGP model slightly, although does not rule it out significantly. Inclusion of the baryon acoustic peak from the Sloan Digital Sky Survey would rule out the DGP model at the 3σ level, although it is not clear how self consistent this procedure would be without a re-analysis of the survey data in the framework of the DGP cosmology. Generalisations of the DGP model are tested and constraints on relevant parameters obtained.

E-mail: `sararyd@physto.se`, `malc@cern.ch`, `ariel@physto.se`

1. Introduction

Most of the energy in the universe seems to be dark, and many different observations suggest that only a minority of this energy can be dark matter, the rest apparently being some kind of energy component with negative pressure.

The simplest explanation for this dark energy component is a cosmological constant and people have attempted to explain the smallness of this energy density and hence its relatively recent dominance using anthropic arguments [1] which may concur with modern predictions from string theory [2]. Other explanations exist however, including a class of theories known as quintessence where the observed acceleration of the universe results from the stress energy of some rolling scalar field which only comes to dominance recently [3, 4].

A different approach is to ask whether General Relativity breaks down at large distances, in other words to modify the left-hand/curvature side of the Einstein equations rather than the right-hand/stress-energy side. One of the most studied examples of such a scenario is the DGP brane world model [5, 6, 7]. This paper aims to compare the predictions of this theory and its generalisations with the latest cosmological data.

arXiv:astro-ph/0701495v1   17 Jan 2007

In the DGP model, gravity is trapped on a four dimensional brane world at short distances, but is able to propagate into a higher dimensional space at large distances. The Lagrangian for the model is ($\hbar = c = 1$, $M_{Pl}^2 = (8\pi G)^{-1}$)

$$S = -\frac{M^3}{2} \int d^5x \sqrt{-g^{(5)}} R^{(5)} - \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g^{(4)}} R^{(4)} + \int d^4x \sqrt{-g^{(4)}} \mathcal{L}_M \tag{1}$$

Due to the different mass scales M and M_{Pl}, gravity propagates differently on the brane and in the bulk. The effect of gravitational leakage into the bulk will only appear at large distances ($r > L = M_{Pl}^2/2M^3$).

The tt Friedman equation in this theory takes the form

$$H^2 - \epsilon \frac{H}{L} = \frac{\rho_M}{3M_{Pl}^2} \tag{2}$$

where $\epsilon = \pm 1$. It is $\epsilon = +1$ that gives the late-time accelerating solutions that could explain dark energy. The theory also predicts very small but potentially detectable modifications to the earth-moon distance [8]. There are also changes in structure formation which could give constraints upon the theory [9].

Recently the self-accelerating branch of this theory has come under theoretical attack because it seems to possess ghost-like instabilities [10] while one of the original authors has argued that the calculational regime within which the instabilities are found is not valid [11]. In this work, we will put this issue to one side and proceed to compare the model's predictions with the latest data. Even if it turned out that the DGP model was theoretically suspect, it is interesting to see how different expansion histories may or may not be ruled out by the developing suite of data.

To test theories, we would like to make detailed comparisons with astronomical observations. There are basically two ways of doing this. As already mentioned, one is to look at the growth of perturbations in these models and see how they compare with perturbations in ΛCDM and in the observed galaxy correlation function [9]. Another way is to simply look at the space-time geometry of the universe on the largest length and time scales available in order to reconstruct the expansion history of the universe and compare it to the solutions of the modified gravity models. This latter approach is the subject of this paper.

Studies regarding the comparison between data and the DGP model based upon expansion history are to date contradictory. The first paper on the subject was presented by two of the current authors [12] using the 2005 data release from the SuperNova Legacy Survey (SNLS) [13]. In that work it was argued that the SNLS data combined with the position of the baryon acoustic peak in the Sloan Digital Sky Survey were less consistent with a flat universe in the DGP model than in the ΛCDM model. These conclusions were backed up when Maartens and Majoretto performed the same test using the SNLS data, the baryon acoustic peak and the CMB shift parameter [14].

Later, a new "gold" data set of supernovae was released [15]. Analysis of this data and the CMB shift parameter suggested that a flat universe was completely consistent with the DGP model [16], a conclusion the present authors can confirm, also upon

addition of the data from the baryon acoustic peak. It is interesting to note that it has been suggested that there may be an inconsistency between the different parts of this gold data set [17].

The two sets of data come from different instruments - the SNLS supernovae are detected using a combination of imaging at the Canada-France-Hawaii Telescope and spectroscopic studies on large ground based instruments, in particular Gemini, VLT and Keck. The Riess et al. 07 gold sample contains supernovae from the Supernova Cosmology Project, SNLS, the High-Z Team, the GOODS transient survey, and includes 21 new supernovae at extremely high redshift obtained with the Hubble Space Telescope. In all cases, low redshift supernovae from independent surveys have been included in the data sets.

Furthermore the supernova magnitudes are obtained from the two different data sets using two different methods, the SALT algorithm which is used by the SNLS group fits supernovae based upon their light curve and their colour [18] as does the MLCS method favoured by Riess et al. , the most recent version being called MLCS2k2 [19]. The parameters in these two algorithms are obtained by training on low redshift supernovae before they are used to obtain the magnitudes of high redshift supernovae. SALT and MLCS2k2 differ both in the details of the brightness-shape relation correction and in how extinction/colour corrections are applied.

Recently the ESSENCE group has released a set of 60 supernovae at intermediate to high redshifts [20]. This data has been combined with the SNLS data and data at low redshift to form a new data set spanning a large redshift range in detail. The supernovae in this data set have been analysed using both methods - SALT and MLCS. In this paper, we combine this new data with the CMB and baryon oscillation data in order to see if the DGP model is favoured or disfavoured.

While the model as it stands has one extra space dimension, it is possible to imagine generalisations of the model with higher dimensional bulks, although because of the curvature singularity an infinitely thin 4D brane would create in more than 5 space-time dimensions, the theory would need to be regularised at the location of the brane [21]. We will present a parametrisation of these higher dimensional generalisations as has been done in previous work [22, 23, 12] and we will introduce an extra degree of freedom which may go some way to modeling the regularisations.

2. Observational status of Expansion History.

Observations of high-redshift Type Ia supernovae have been used for over a decade to map the expansion history of the universe [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 13, 39]. These surveys provided the first direct evidence that the universe expands at an accelerated rate. Cross-cutting techniques involving the anisotropies in the cosmic microwave radiation (CMB) [40, 41, 42, 43, 44], mass density estimates from X-ray observations of clusters [45] and more recently weak lensing [46], baryon acoustic oscillations (BAO) [47, 48] and the integrated Sachs-Wolfe effect [49, 50] are

apparently continuing to confirm our current understanding of the universe, namely it being spatially flat, as predicted by inflation, and the current expansion being dominated by dark energy.

The ESSENCE group have analysed supernovae at intermediate redshifts ($0.1 < z < 0.8$) using both the MLCS2k2 and the SALT methods to obtain the supernova magnitudes from the light curves and spectra. When using the MLCS2k2, the group adopt what they refer to as the 'glosz' prior to model A_V - the extinction in the V band of the supernova in the host galaxy [20]. They have also fitted higher redshift supernovae ($0.11 < z < 1.1$) from the SNLS survey [13] and a set of nearby supernovae ($z < 0.11$) again using both the SALT and MLCS2k2 method to obtain magnitudes of supernovae over a large redshift range.

Measurements of the brightness of Type Ia supernovae as a function of redshift are sensitive to the cosmological model via the integration over expansion history in the expression for the luminosity distance ($c = 1$)

$$d_L = \frac{1+z}{H_0\sqrt{|\Omega_k|}}\mathcal{S}\left(\sqrt{|\Omega_k|}\int_0^z \frac{d\bar{z}}{E(\bar{z})}\right) \qquad (3)$$

where the function $\mathcal{S}(x)$ is defined as $\sin(x)$ for $\Omega_k < 0$ (*closed Universe*), $\sinh(x)$ for $\Omega_k > 0$ (*open Universe*) and $\mathcal{S}(x) = x$, and the factor $\sqrt{|\Omega_k|}$ is removed for the *flat Universe*. The parameter $E(z) = H(z)/H_0$.

In our analysis we fit the cosmological model to the supernova data using the luminosity distance above but we also fit to the baryon acoustic peak detected in the SDSS Luminous Red Galaxy survey (LRG) of Eisenstein el al (2005) [47, 44], which constrains the following combination of parameters

$$\frac{\sqrt{\Omega_M}}{E(z_1)^{\frac{1}{3}}}\left[\frac{1}{z_1\sqrt{|\Omega_k|}}\mathcal{S}\left(\sqrt{|\Omega_k|}\int_0^{z_1}\frac{dz}{E(z)}\right)\right]^{\frac{2}{3}} = 0.472 \pm 0.017, \qquad (4)$$

where $z_1 = 0.35$ and $\mathcal{S}$ and Ω_k are defined as in Eq.(3). The quoted uncertainty corresponds to one standard deviation, where a Gaussian probability distribution has been assumed.

There is some debate [16, 52] as to whether or not one should use the baryon acoustic peak to constrain models of dark energy which behave differently to a cosmological constant, for two reasons. The first is that the reconstruction from redshift space to co-moving space required to accurately identify the position of the acoustic peak has been done assuming a constant equation of state [47]. While one would expect the change in the position of the acoustic peak in an alternative dark energy model where the equation of state is a function of redshift to be small, the correction is difficult to quantify without detailed study for each model in question. Secondly, in modified models of gravity, one would expect structure formation to proceed differently [9]. Although at the first approximation this would not change the physical co-moving size of the acoustic peak feature in the correlation function, such an effect might create systematic distortions in its reconstruction from redshift space.

For these reasons we will constrain the models with and without the SDSS baryon acoustic peak, we leave it to the reader to decide if they want to pay attention to the BAO constraints on parameter space or not.

Finally, a cleaner measure of geometry is the CMB shift parameter [51, 14] - the expansion history of the universe has to be such that the observed position of the CMB peak corresponds to the physical horizon size at last scattering. The photons used to measure this angular size have passed through the integrated geometry created by the particular dark energy model in question. The angular size of the first peak of the CMB as measured by the WMAP 3 year data constrains the ratio [53, 54]

$$\sqrt{\Omega_M} \int_0^z \frac{d\bar{z}}{E(\bar{z})} = 1.70 \pm 0.03 \qquad (5)$$

which can then be applied as another cut to parameter space for each model.

Having described the observations that we will compare with the theoretical models, we can move on the expansion predictions for brane world gravity.

3. Comparison of the DGP model with the data.

In this section we will compare the predictions of the DGP model with the latest cosmological expansion history data set described in the previous section. As we have discussed in the introduction, the DGP model in its simplest form with one extra flat dimension (1) gives rise to a modified Friedman equation as written in equation (2).

We have made a distinction between best model fit and parameter determination - when we talk about 'confidence levels' we are referring to the regions within which the χ^2 values change from the minimum χ^2 value by the critical amounts that are usually used ($\chi^2_{min} + 2.3$ for 68% etc.). If we use the term '1 σ', '2 σ' or '3 σ' we are referring to respectively the 68%, 95% and 99% significances obtained by comparing the χ^2 to the number of degrees of freedom. If the errors were Gaussian *and* the model being fitted was a good one then those two measures of statistical significance should be equivalent. However, since we do not know if both those criteria are exactly fulfilled, it is important that the reader understands the distinction between the different quantities.

3.1. Mathematical preliminaries

It will be interesting to also consider generalisations of the DGP model which might result from having a higher dimensional bulk. Although such models have not been derived explicitly, it is possible to guess at their possible form and the way that the Friedman equation would be modified as a function of the number of extra dimensions [22]. Actual realisations of higher dimensional models of this kind have problems which arise when quantities reach infinity on the brane [21]. This is solved assuming a non-zero thickness of the brane, followed by some regularisation procedure, of which one kind leads to an generalised equation of the form [55]

$$H^2 - \frac{1}{L^2(\beta + (LH)^{n-2})} = \frac{\rho_M}{3M_{Pl}^2} \qquad (6)$$

for zero spatial curvature on the brane. Here L corresponds to the crossover length scale and n is the number of extra dimensions. The extra term β parameterises the regularisation in cases where $n \neq 1$.

Dividing through by H_0^2, we have

$$E^2(z) - \frac{\Omega_L}{\beta + \sqrt{\Omega_L}^{(2-n)} E(z)^{(n-2)}} = \Omega_M(1+z)^3 \tag{7}$$

where $\Omega_L = (H_0 L)^{-2}$ and $E(z) = H(z)/H_0$. It is easy to see that when β dominates the denominator‡, the Ω_L-term will behave very much like a cosmological constant term. As usual, we can constrain one of the parameters in terms of the others by setting $z = 0$ in (7), giving

$$\beta = \frac{\Omega_L}{1 - \Omega_M} - \sqrt{\Omega_L}^{(2-n)} \tag{8}$$

which is the equivalent of the equation $\Omega_\Lambda + \Omega_M = 1$ in flat ΛCDM.

3.2. Models with $\beta = 0$, including original DGP model

The first models that we constrain are those with the regularisation parameter β set to zero. If there are too many parameters in a dark energy model it is usually rather easy to fit the data for some combination of those parameters, so one can assume priors like imposing a flat universe in order to obtain interesting constraints. Here, because we have the same number of free parameters§ in these $\beta = 0$ models as in the case of ΛCDM, we are able to include curvature and still get interesting constraints. We do this simply by replacing the Hubble Constant squared H^2 with $H^2 + ka^{-2}$ [12]. This gives us

$$H^2 + \frac{k}{a^2} - L^{-2} \left[\beta + \left(L\sqrt{H^2 + \frac{k}{a^2}} \right)^{n-2} \right]^{-1} = \frac{\rho_M}{3M_{Pl}^2} \tag{9}$$

which for the case of the original DGP model leads to

$$E^2(z) = \Omega_K(1+z)^2 + \left(\sqrt{\Omega_M(1+z)^3 + \Omega_L/4} + \sqrt{\Omega_L/4} \right)^2 \tag{10}$$

where the normal definition $\Omega_K = -k/(aH_0)^2$ has been used.

The different models were compared with the two sets of data, one of which contained supernovae that had been analysed using the SALT method of determining magnitudes while the other using the MLCS2k2 method. The data-set analysed by the ESSENCE collaboration with SALT gave rise to anomalously large values of χ^2 seemingly due to a few outlying data points. Since the data analysed using MLCS gave χ^2 per degree of freedom rather close to one we have more confidence in this data set and have restricted ourselves to using it.

---

‡ try saying that five times quickly
§ we have theoretical motivation for n being a discrete integer and each n corresponding to a different model
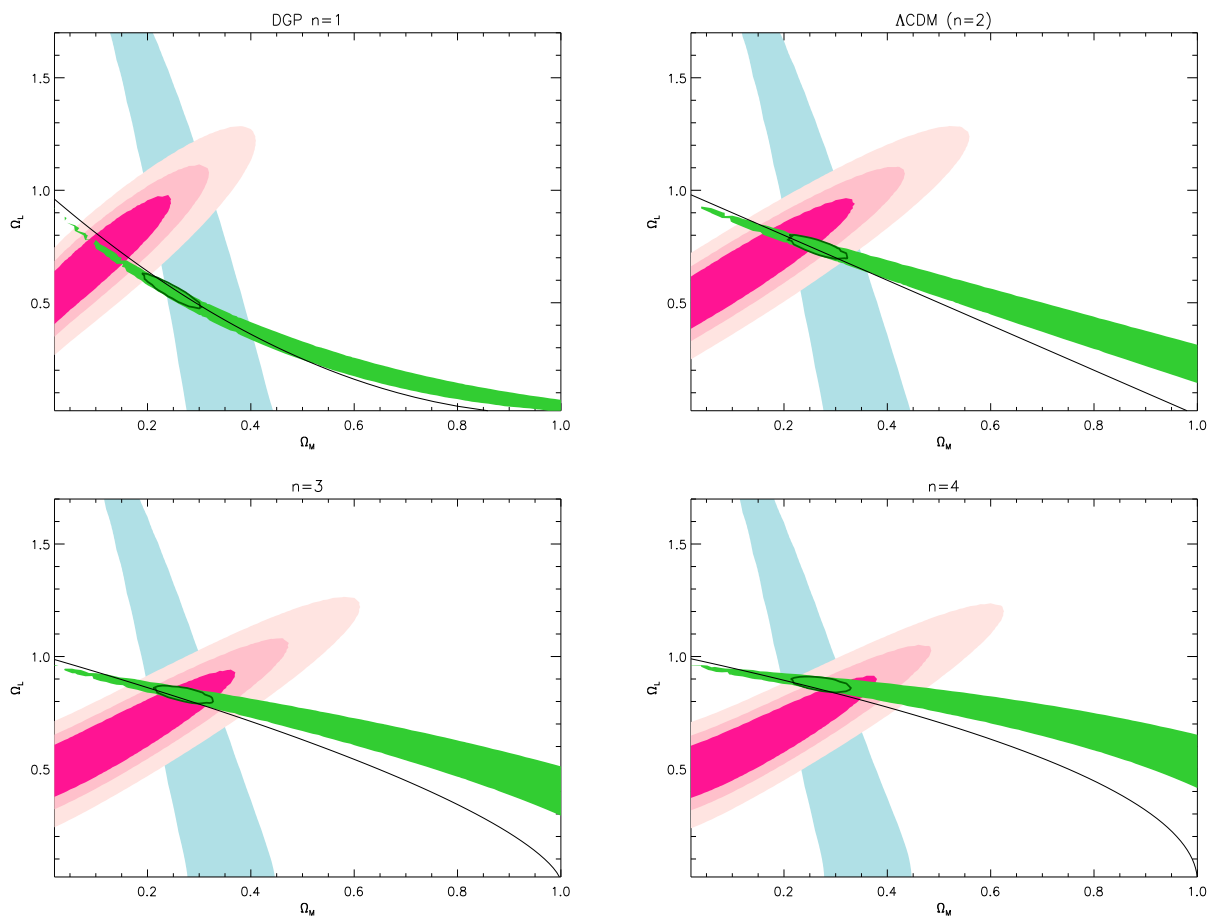
Figure 1. Supernovae from ESSENCE, SNLS and nearby sample analysed with the MLCS2k2 method compared with models described by equation 9 with $\beta = 0$ which include the basic DGP model ($n = 1$). Because ΛCDM is at this level identical to the $n = 2$ case we plot the different models in order of increasing n. The pink concentric ellipses correspond to the 68%, 95% and 99% confidence regions from the latest supernova data (3), the blue bands border the 99% confidence region for the baryon acoustic peak data (4) while the green region borders the 99% confidence region for the CMB shift parameter (5). The black line corresponds to spatially flat universes.

Figure 1 shows the comparison of the data (supernovae, galaxy survey baryon oscillation and CMB) with the DGP model and its variants where the supernova data has been analysed using the MLCS2k2 method. Note that at this level of expansion history the $n = 2$ DGP variant is identical to ΛCDM although presumably perturbations would grow very differently in the two models. The actual χ^2 values corresponding to the two different models DGP (n=1) and ΛCDM (n=2), as well as the higher dimensional generalisations for n=3 and n=4 are listed in the table 1.

All models can fit the supernova data alone equally well, and more or less equally can fit the data assuming flatness. The requirement of having to fit the CMB data singles out ΛCDM as being slightly favoured over the other models, although the χ^2 values are such that the DGP model is roughly $2\,\sigma$ disfavoured, which is not statistically

$\beta = 0$	MLCS (162 data points)			
	n=1	ΛCDM (n=2)	n=3	n=4
χ^2_{min}(SNe)	188	188	188	188
χ^2_{min}(SNe+flat)	188	188	189	189
χ^2_{min}(SNe+flat+CMB)	200	188	190	197
χ^2_{min}(SNe+flat+CMB+BAO)	215	192	191	197

Table 1. Best fit χ^2 values for the fits of different data sets to the various models. SNe is the supernova data set (analysed using the MLCS2k2 procedure), next flatness is assumed then the CMB shift parameter is added to the data set. Finally we include the Baryon acoustic oscillation result (BAO) from the Sloan Digital Sky Survey.
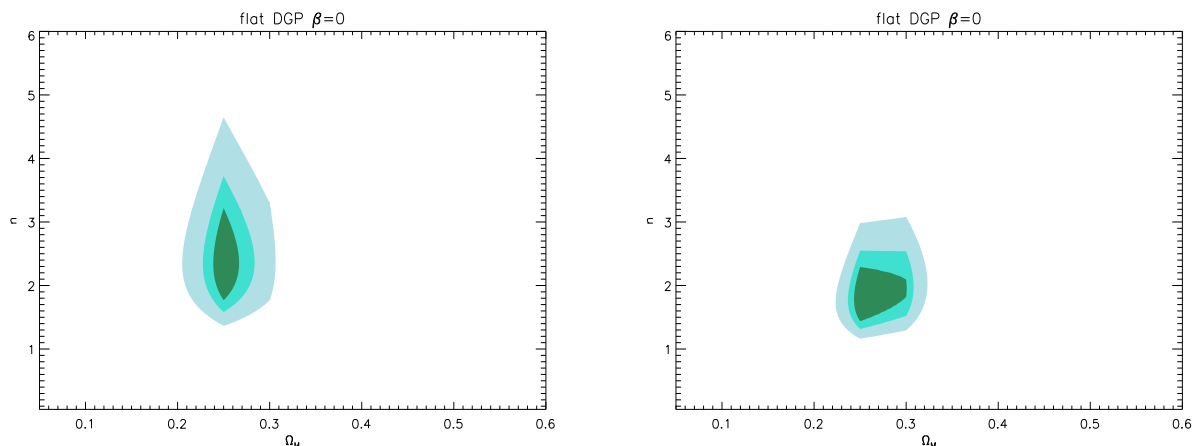

Figure 2. Best fit values of n for $\beta = 0$ and different values of the matter density Ω_M. On the left is the supernova data analysed by MLCS2k2 and on the right the data analysed by SALT.

significant enough to allow us to claim that the model is ruled out.

If we add the baryon oscillation feature to the data we find that we do seem to be able to disfavour the DGP model at the 3 σ level, but as we have already stated in section 2, the reconstruction of the peak in the correlation function from redshift space depends upon assumptions which are not valid in cosmologies where the equation of state is varying over time. At some point in the future it may be interesting to investigate such systematic effects quantitatively.

3.3. Best fit values of n.

An interesting exercise is to take the case $\beta = 0$ and allow the number of extra dimensions n to be a free, non-integer parameter. We then calculate which values of n fit the data best when we allow Ω_M to be a free parameter.

Figure 2 shows us that the best fit region when $\beta = 0$ occurs for values of n between 1.5 and 3. The case $n = 2$ corresponds exactly to a cosmological constant (whether $\beta = 0$ or not), so this tells us that without β the best fit to the data lies somewhere around
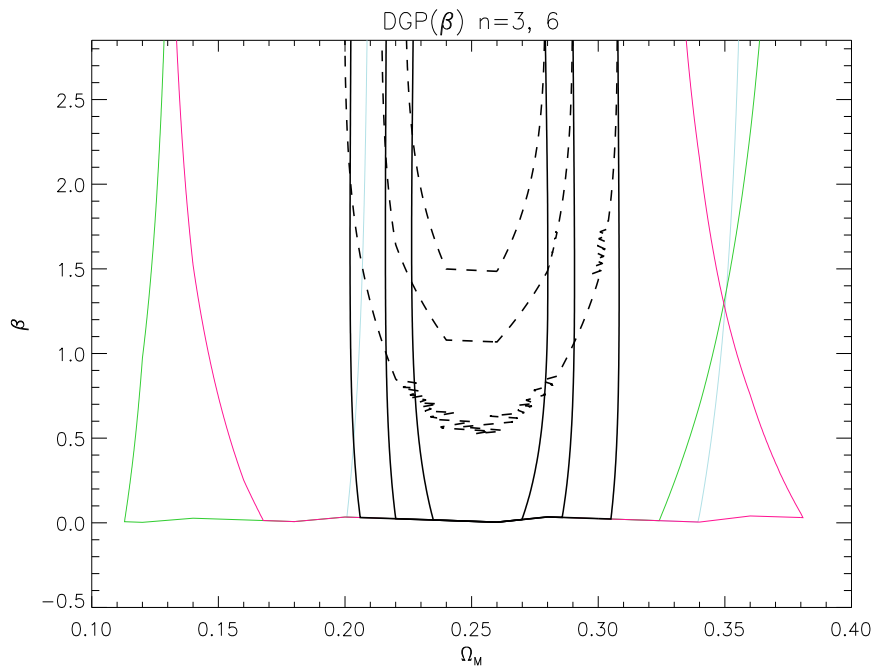
Figure 3. Constraints on the higher dimensional versions of DGP taking into account the regularisation parameter β. Plotted here is $n = 3$. The thin coloured lines are the 99% confidence levels for supernovae analysed with MLCS (red), CMB shift parameter (green) and baryon oscillation (blue). The solid black lines are the confidence levels for the combined χ^2 (see note at beginning of section). The combined χ^2 for the $n = 6$ case is also plotted for comparison using black dashed lines showing that for this case, zero β is significantly disfavoured.

the ΛCDM models, in agreement with the previous conclusions of [12] although the preferred value of n may have increased slightly.

3.4. Models with non-zero β

As stated earlier, for the higher dimensional generalisations of DGP we should include the parameter β which, while required for theoretical consistency, is an extra free parameter which renders the theory less predictive and makes it easier to fit the data. For this reason, for the higher dimensional cases, we will consider spatially flat universes with non zero β.

In figure 3 we also show how the different cosmological constraints cut into parameter space by plotting the confidence regions for the four higher dimensional models. The original DGP model corresponds to $n = 1$ and in that case there is no need for a regularisation parameter β. In terms of large scale geometry of space time the $n = 2$ case is equivalent to ΛCDM with or without β. We therefore only consider $n = 3, 4, 5, 6$.

The confidence region in the parameter space of Ω_M and Ω_L centers around $\Omega_M = 0.26$ so we take this as our value for Ω_M. For each n, we obtain a minimum value

95%	$n = 3$	$n = 4$	$n = 5$	$n = 6$
$\Omega_L >$	(0.8)	0.9	1.1	1.3
$\beta >$	0	0.1	0.6	1.2

Table 2. Minimum values of Ω_L and β (via equation (8)) with 95 % confidence. For n=3, the minimum value of Ω_L is restricted only through the cutoff where β goes negative.

for Ω_L by looking at the 95% confidence level away from the best fit, corresponding to a minimum value of β that increases for increasing n, as shown in table 2. This shows us that for higher-dimensional models the fit gets worse and one will have to increase the value of β in order to get closer to the cosmological constant case which fits the data well.

4. Summary and Conclusions

Data from the Supernova legacy survey analysed with the SALT method has previously suggested that the DGP model is marginally disfavoured relative to ΛCDM [12]. At the same time, a larger data set including recent data from the Hubble space telescope and analysed using the MLCS approach has led to the conclusion that the DGP model is in fact perfectly safe [16].

In this work, we have looked at the SNLS data and new data from the ESSENCE collaboration analysed with the MLCS algorithm, and therefore avoiding any systematic effect which would depend upon the way that the SNLS magnitudes were obtained from the light curves. Combination of this data with the CMB constraint suggests that the DGP model is slightly disfavoured, and might be rather strongly dis-favoured if one can treat the baryon acoustic peak as a valid data point. The magnitude of the change in the position of the acoustic peak in the galaxy correlation function when one moves from a background cosmology with a constant equation of state to a DGP universe depends both upon the different geometries and the way that structure grows in those universes. Since the growth of structure in a DGP universe may be rather different from in ΛCDM we are not able at this stage to say whether or not the DGP model seems to be marginally or significantly disfavoured using this data. Either way, we do not find that the data and the DGP model sit together completely comfortably, as has been concluded by looking at the Riess et al. data set of gold standard simulations [16, 15].

The fact that the two data sets lead to different conclusions about the same model is very interesting and outlines the challenges which need to be overcome in order to move into the era of precision Dark Energy measurements.

Acknowledgments

[1] H. Martel, P. R. Shapiro and S. Weinberg, Astrophys. J. **492** (1998) 29 [arXiv:astro-ph/9701099].
[2] L. Susskind, arXiv:hep-th/0302219.
[3] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37** (1988) 3406.
[4] C. Wetterich, Nucl. Phys. B **302** (1988) 668.
[5] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000) [arXiv:hep-th/0005016].
[6] C. Deffayet, Phys. Lett. B **502**, 199 (2001) [arXiv:hep-th/0010186].
[7] C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65**, 044023 (2002) [arXiv:astro-ph/0105068].
[8] G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D **68** (2003) 024012 [arXiv:hep-ph/0212069].
[9] K. Koyama, JCAP **0603** (2006) 017 [arXiv:astro-ph/0601220].
[10] D. Gorbunov, K. Koyama and S. Sibiryakov, Phys. Rev. D **73** (2006) 044016 [arXiv:hep-th/0512097].
[11] G. Dvali, arXiv:hep-th/0610013.
[12] M. Fairbairn and A. Goobar, Phys. Lett. B **642** (2006) 432 [arXiv:astro-ph/0511029].
[13] P. Astier,*et al* , A&A, **447**, 31, (2006)
[14] R. Maartens and E. Majerotto, Phys. Rev. D **74** (2006) 023004 [arXiv:astro-ph/0603353].
[15] A. Riess *et al* , ApJ in press, [astro-ph/0611572]
[16] V. Barger, Y. Gao and D. Marfatia, arXiv:astro-ph/0611775.
[17] S. Nesseris and L. Perivolaropoulos, arXiv:astro-ph/0612653.
[18] J. Guy, P. Astier, S. Nobili, N. Regnault and R. Pain, arXiv:astro-ph/0506583.
[19] A. G. Riess, W. H. Press and R. P. Kirshner, Astrophys. J. **473** (1996) 88 [arXiv:astro-ph/9604143]. S. Jha, A. G. Riess and R. P. Kirshner, arXiv:astro-ph/0612666.
[20] G. Miknaitis *et al.*, arXiv:astro-ph/0701043. W. M. Wood-Vasey *et al.*, arXiv:astro-ph/0701041.
[21] P. Bostock, R. Gregory, I. Navarro and J. Santiago, Phys. Rev. Lett. **92** (2004) 221601 [arXiv:hep-th/0311074].
[22] G. Dvali and M. S. Turner, arXiv:astro-ph/0301510.
[23] D. J. H. Chung and K. Freese, Phys. Rev. D **61**, 023511 (2000) [arXiv:hep-ph/9906542].
[24] A. Goobar & S. Perlmutter, ApJ **450**, 14, (1995)
[25] S. Perlmutter*et al* , ApJL **440**, 41, (1995)
[26] S. Perlmutter*et al* , ApJ **483**, 565, (1997)
[27] P. Garanivich *et al* , 1998, ApJ **509**, 74, (1998)
[28] A. Riess *et al* , AJ **116**, 1009, (1998)
[29] S. Perlmutter *et al* , Nature **391**, 51, (1998)
[30] B. P. Schmidt *et al* , ApJ **507**, 46, (1998)
[31] S. Perlmutter *et al* , ApJ **517**, 565, (1999)
[32] R. A. Knop *et al* , ApJ **598**, 102, (2003)
[33] M. Sullivan *et al* ,MNRAS **340**, 1057, (2003)
[34] J. Tonry *et al* , ApJ **594**, 1, (2003)
[35] B. Barris *et al* , ApJ **602**, 571, (2004)
[36] A. Riess *et al* , ApJ **607** 665, (2004)
[37] K. Krisciunas*et al* , AJ **130**, 2453, (2005)

[38] S. Nobili *et al* , A&A,**437**, 789,(2005)

[39] A. Conley,*et al* , ApJ **644**, 1, (2006)

[40] de Bernardis *et al* , Nature **404**, 955,(2000)

[41] A. Melchiorri *et al* , ApJL **536**, 63, (2000)

[42] A. Balbi *et al* , ApJL **545**, 1, (2000)

[43] D. N. Spergel *et al* , ApJS **148**, 175,(2006)

[44] D. N. Spergel *et al* , [astro-ph/0603449]

[45] S. W. Allen *et al* , MNRAS **353**, 457, (2004)

[46] H. Hoekstra *et al* , ApJ **647**, 116, (2006)

[47] D. J. Eisenstein *et al.* [SDSS Collaboration], Astrophys. J. **633** (2005) 560 [arXiv:astro-ph/0501171].

[48] M. Tegmark*et al* , Phys Rev. D. in press, [astro-ph/0608632]

[49] Boughn & Crittenden, Phys. Rev. Lett. **88**, 21302, (2002)

[50] M. R. Nolta*et al* , ApJ **608**, 10,(2004)

[51] J. R. Bond, G. Efstathiou and M. Tegmark, Mon. Not. Roy. Astron. Soc. **291**, L33 (1997) [arXiv:astro-ph/9702100].

[52] J. Dick, L. Knox and M. Chu, JCAP **0607** (2006) 001 [arXiv:astro-ph/0603247].

[53] D. N. Spergel *et al.*, arXiv:astro-ph/0603449.

[54] Y. Wang and P. Mukherjee, Astrophys. J. **650** (2006) 1 [arXiv:astro-ph/0604051].

[55] G. Dvali, private communication

[56] B. Chaboyer, P. Demarque, P. J. Kernan and L. M. Krauss, Astrophys. J. **494** (1998) 96 [arXiv:astro-ph/9706128].